Pensieve header: Testing that two half knots match up to a full knot

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

```
MatrixSignature[A_] := Total[Sign[Select[Eigenvalues[A], Abs[#] > 10^-12 &]]];
```

```
rs_List⁺ := Union @@ (#⁺ & /@ rs)
```

```
Σ_b_[σ1_, PQ[C1_, q1_]] + Σ_b_[σ2_, PQ[C2_, q2_]] ^:= CF@Σ_b[σ1 + σ2, PQ[C1 ⋃ C2, q1 + q2]];
Flip[Σ_B[b___][σ_, PQ[C_, q_]]] := CF@Module[{b1, b2},
    b1 = Join @@ ({b} /. i_Integer ↦ γ_i);
```

```
A[Σ_b_B[σ_, PQ[C_, q_]]] := Module[{γs, r, c},
   γs = γ_# & /@ Join @@ b;
```

```
pd = PD[Knot[8, 5]]
```

PD[X[6, 2, 7, 1], X[8, 4, 9, 3], X[2, 8, 3, 7], X[14, 10, 15, 9],

```
pd1 = Take[pd, 4]
```

PD[X[6, 2, 7, 1], X[8, 4, 9, 3], X[2, 8, 3, 7], X[14, 10, 15, 9]]

```
pd2 = Complement[pd, pd1]
```

PD[X[4, 13, 5, 14], X[10, 16, 11, 15], X[12, 5, 13, 6], X[16, 12, 1, 11]]

```
TL[pd1]
```

$$2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)-2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)$$

	$(\gamma_{-14}$	γ_{10}	γ_{15}	γ_{-1}	γ_{-6}	$\gamma_4)$
$\bar\gamma_{-14}$	$\frac{2(\omega-1)^2(\omega^2+1)}{\omega(\omega^2-\omega+1)}$	$\omega-1$	$-2(\omega-1)$	0	$\frac{2(\omega-1)}{\omega(\omega^2-\omega+1)}$	$1-\omega$
$\bar\gamma_{10}$	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	0	0
$\bar\gamma_{15}$	$\frac{2(\omega-1)}{\text{[illegible]}}$	$1-\omega$	$\frac{(\omega-1)^2}{\text{[illegible]}}$	$-\frac{\omega-1}{\text{[illegible]}}$	0	0

```
Flip@TL[pd2]
```

$$-1$$

	$(\gamma_{-14}$	γ_{10}	γ_{15}	γ_{-1}	γ_{-6}	$\gamma_4)$
$\bar\gamma_{-14}$	0	$\omega-1$	0	0	0	$1-\omega$
$\bar\gamma_{10}$	$-\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2}{\omega}$	$-\frac{\omega-1}{\omega}$	0	$\frac{2(\omega-1)}{\omega}$	$\frac{2(\omega-1)^2}{\omega}$
$\bar\gamma_{15}$	0	$\omega-1$	$\frac{\omega^2+1}{\omega}$	$\frac{\omega-1}{\omega}$	-2ω	0

```
TL[pd1] + Flip@TL[pd2]
```

$$2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)-2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

	$(\gamma_{-14}$	γ_{10}	γ_{15}	γ_{-1}	γ_{-6}	$\gamma_4)$
$\bar\gamma_{-14}$	$\frac{2(\omega-1)^2(\omega^2+1)}{\omega(\omega^2-\omega+1)}$	$2(\omega-1)$	$-2(\omega-1)$	0	$\frac{2(\omega-1)}{\omega(\omega^2-\omega+1)}$	$-2(\omega-1)$
$\bar\gamma_{10}$	$-\frac{2(\omega-1)}{\omega}$	$-\frac{2(\omega-1)^2}{\omega}$	0	0	$\frac{2(\omega-1)}{\omega}$	$\frac{2(\omega-1)^2}{\omega}$
$\bar\gamma_{15}$	$\frac{2(\omega-1)}{\text{[illegible]}}$	0	$\frac{2(\omega^2-\omega+1)}{\text{[illegible]}}$	0	-2ω	0

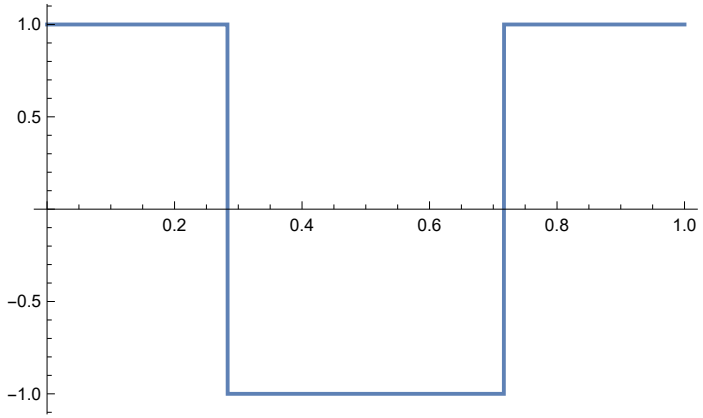
In[•]:=
```
mat = TL[pd1] + Flip@TL[pd2] // A
```
Out[•]=

$$\left\{\left\{\frac{2\,(-1+\omega)^2\,\left(1+\omega^2\right)}{\omega\,\left(1-\omega+\omega^2\right)},\ 2\,(-1+\omega),\ -2\,(-1+\omega),\ 0,\ \frac{2\,(-1+\omega)}{\omega\,\left(1-\omega+\omega^2\right)},\ -2\,(-1+\omega)\right\},\right.$$

$$\left\{-\frac{2\,(-1+\omega)}{\omega},\ -\frac{2\,(-1+\omega)^2}{\omega},\ 0,\ 0,\ \frac{2\,(-1+\omega)}{\omega},\ \frac{2\,(-1+\omega)^2}{\omega}\right\},$$

$$\left.\left\{\frac{2\,(-1+\omega)}{\omega},\ 0,\ \frac{2\,\left(1-\omega+\omega^2\right)}{\omega},\ 0,\ -2\,\omega,\ 0\right\},\ \{0,0,0,0,0,0\},\right.$$

In[•]:=
```
Plot[MatrixSignature[mat /. ω → e^(2πit)], {t, 0, 1}]
```
Out[•]=



In[•]:=
```
TLSig[pd]
```
Out[•]=

$$2\,\Theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\Theta\left[\frac{\sqrt{3}}{2}+u\right]-2\,\Theta\left[u-\sqrt{-0.630\ldots}\right]+2\,\Theta\left[u-\sqrt{0.630\ldots}\right]$$

```
Plot[MatrixSignature[mat /. ω → e^(2πit)], {t, 0, 1}]
```