Pensieve header: An analysis of K15a55264 and K15n90489. Continues PossibleCounterexample.nb at pensieve://Projects/Signatures.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
<< Signatures`
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[•]:=
```
K = Mirror@Knot[3, 1]
Factor[Alexander[K][T]]
lhs = TLSig[K]
lhs == KasSig[K]
```

Out[•]=
Mirror[Knot[3, 1]]

Out[•]=
$$\frac{1 - T + T^2}{T}$$

Out[•]=
$$2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$$

Out[•]=
True

In[•]:=
```
K = Knot[15, Alternating, 55264]
Factor[Alexander[K][T]]
lhs = TLSig[K]
lhs == KasSig[K]
```

Out[•]=
Knot[15, Alternating, 55 264]

Out[•]=
$$\frac{\left(1 - T + T^2\right)^6}{T^6}$$

Out[•]=
$$4\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 4\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$$

Out[•]=
True

In[∘]:=
```
sign[ℰ_] := Echo@Module[{n, d, v, p, rs, e, k}, Echo@ℰ;
   {n, d} = NumeratorDenominator[ℰ];
   {n, d} /= ω^(Exponent[n,ω]/2+Exponent[n,ω,Min]/2);
   p = Factor[ω2[v]@n * ω2[v]@d /. v → 4 u^2 - 2];
   rs = Solve[p == 0, u, Reals];
   If[rs === {}, Sign[p /. u → 0],
    rs = Union@(u /. rs);
    Sign[(-1)^(e=Exponent[p,u]) Coefficient[p, u, e]] + Sum[
      k = 0; While[(d = RootReduce[∂_{u,++k} p /. u → r]) == 0];
      If[EvenQ[k], 0, 2 Sign[d]] * θ[u - r],
      {r, rs}] ]]
```

In[∘]:=
```
K = Knot[15, NonAlternating, 90489]
Factor[Alexander[K][T]]
Print["Computing KasSig..."]
rhs = KasSig[K]
Print["Computing TLSig..."]
TLSig[K] == rhs
```

Out[∘]=

Knot[15, NonAlternating, 90489]

Out[∘]=

$$\frac{\left(1-T+T^2\right)^5}{T^5}$$

Computing KasSig...

» 3

» 1

» $\frac{2}{3}\left(-3+4 u^2\right)$

» $1+2 \theta\left[-\frac{\sqrt{3}}{2}+u\right]-2 \theta\left[\frac{\sqrt{3}}{2}+u\right]$

» $\frac{1}{2}\left(3-4 u^2\right)$

» $-1-2 \theta\left[-\frac{\sqrt{3}}{2}+u\right]+2 \theta\left[\frac{\sqrt{3}}{2}+u\right]$

» $-\frac{4\left(-3+2 u^2\right)}{-3+4 u^2}$

» $-1-2 \theta\left[-\sqrt{\frac{3}{2}}+u\right]+2 \theta\left[\sqrt{\frac{3}{2}}+u\right]+2 \theta\left[-\frac{\sqrt{3}}{2}+u\right]-2 \theta\left[\frac{\sqrt{3}}{2}+u\right]$

» $-\frac{\left(-2-u+2 u^2\right)\left(-2+u+2 u^2\right)}{-3+2 u^2}$

» $-1+2\theta\left[-\sqrt{\frac{3}{2}}+u\right]-2\theta\left[\sqrt{\frac{3}{2}}+u\right]-2\theta\left[\frac{1}{4}\left(-1-\sqrt{17}\right)+u\right]-2\theta\left[\frac{1}{4}\left(1-\sqrt{17}\right)+u\right]+2\theta\left[\frac{1}{4}\left(-1+\sqrt{17}\right)+u\right]+2\theta\left[\frac{1}{4}\left(1+\sqrt{17}\right)+u\right]$

» $2\left(-1+2u^2\right)$

» $1+2\theta\left[-\frac{1}{\sqrt{2}}+u\right]-2\theta\left[\frac{1}{\sqrt{2}}+u\right]$

» $\dfrac{20-119u^2+244u^4-208u^6+64u^8}{2\left(-1+2u^2\right)\left(-2-u+2u^2\right)\left(-2+u+2u^2\right)}$

» $1-2\theta\left[-\frac{1}{\sqrt{2}}+u\right]+2\theta\left[\frac{1}{\sqrt{2}}+u\right]+2\theta\left[\frac{1}{4}\left(-1-\sqrt{17}\right)+u\right]+2\theta\left[\frac{1}{4}\left(1-\sqrt{17}\right)+u\right]-2\theta\left[\frac{1}{4}\left(-1+\sqrt{17}\right)+u\right]-2\theta\left[\frac{1}{4}\left(1+\sqrt{17}\right)+u\right]+2\theta\left[u-\boxed{-0.801\ldots}\right]-2\theta\left[u-\boxed{-0.611\ldots}\right]+2\theta\left[u-\boxed{0.611\ldots}\right]-2\theta\left[u-\boxed{0.801\ldots}\right]$

» $\dfrac{85-475u^2+988u^4-912u^6+320u^8}{20-119u^2+244u^4-208u^6+64u^8}$

» $1-2\theta\left[u-\boxed{-0.801\ldots}\right]+2\theta\left[u-\boxed{-0.611\ldots}\right]-2\theta\left[u-\boxed{0.611\ldots}\right]+2\theta\left[u-\boxed{0.801\ldots}\right]$

» $\dfrac{-901+6028u^2-16\,176u^4+21\,824u^6-14\,848u^8+4096u^{10}}{4\left(85-475u^2+988u^4-912u^6+320u^8\right)}$

» $1-2\theta\left[u-\boxed{-0.751\ldots}\right]+2\theta\left[u-\boxed{0.751\ldots}\right]$

» $\dfrac{2\left(425-4084u^2+16\,112u^4-33\,568u^6+39\,040u^8-24\,064u^{10}+6144u^{12}\right)}{-901+6028u^2-16\,176u^4+21\,824u^6-14\,848u^8+4096u^{10}}$

» $1+2\theta\left[u-\boxed{-0.751\ldots}\right]-2\theta\left[u-\boxed{0.751\ldots}\right]-2\theta\left[u-\boxed{-0.971\ldots}\right]-2\theta\left[u-\boxed{-0.606\ldots}\right]+2\theta\left[u-\boxed{0.606\ldots}\right]+2\theta\left[u-\boxed{0.971\ldots}\right]$

» $\dfrac{\left(5-8u-4u^2+8u^3\right)\left(-5-8u+4u^2+8u^3\right)\left(-19+84u^2-128u^4+64u^6\right)}{425-4084u^2+16\,112u^4-33\,568u^6+39\,040u^8-24\,064u^{10}+6144u^{12}}$

» $1-2\theta\left[u-\boxed{-1.04\ldots}\right]+2\theta\left[u-\boxed{1.04\ldots}\right]-2\theta\left[u-\boxed{-0.969\ldots}\right]+2\theta\left[u-\boxed{0.969\ldots}\right]+2\theta\left[u-\boxed{-0.971\ldots}\right]+2\theta\left[u-\boxed{-0.606\ldots}\right]-2\theta\left[u-\boxed{0.606\ldots}\right]-2\theta\left[u-\boxed{0.971\ldots}\right]$

» $-\dfrac{2\left(-56+365u^2-964u^4+1296u^6-896u^8+256u^{10}\right)}{-19+84u^2-128u^4+64u^6}$

» $-1+2\theta\left[u-\boxed{-0.969\ldots}\right]-2\theta\left[u-\boxed{0.969\ldots}\right]-2\theta\left[u-\boxed{-0.966\ldots}\right]+2\theta\left[u-\boxed{0.966\ldots}\right]$

» $-\dfrac{4(-1+u)(1+u)\left(-1+2u^2\right)\left(7-20u^2+16u^4\right)\left(-131+852u^2-2224u^4+2912u^6-1920u^8+512u^{10}\right)}{\left(-3+4u^2\right)\left(5-8u-4u^2+8u^3\right)\left(-5-8u+4u^2+8u^3\right)\left(-56+365u^2-964u^4+1296u^6-896u^8+256u^{10}\right)}$

» $-1 + 2\,\theta[-1 + u] - 2\,\theta[1 + u] + 2\,\theta\left[-\frac{1}{\sqrt{2}} + u\right] - 2\,\theta\left[\frac{1}{\sqrt{2}} + u\right] - 2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right] + 2\,\theta[u - (-1.04\ldots)] - 2\,\theta[u - (1.04\ldots)] + 2\,\theta[u - (-0.966\ldots)] - 2\,\theta[u - (0.966\ldots)] - 2\,\theta[u - (-0.962\ldots)] + 2\,\theta[u - (0.962\ldots)]$

» $-\frac{(-3 + 4\,u^2)^5}{-131 + 852\,u^2 - 2224\,u^4 + 2912\,u^6 - 1920\,u^8 + 512\,u^{10}}$

» $-1 + 2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right] + 2\,\theta[u - (-0.962\ldots)] - 2\,\theta[u - (0.962\ldots)]$

» $-\frac{(-3 + 4\,u^2)^5}{16\,(-1 + u)\,(1 + u)\,(-1 + 2\,u^2)\,(7 - 20\,u^2 + 16\,u^4)}$

» $-1 - 2\,\theta[-1 + u] + 2\,\theta[1 + u] - 2\,\theta\left[-\frac{1}{\sqrt{2}} + u\right] + 2\,\theta\left[\frac{1}{\sqrt{2}} + u\right] + 2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$

Out[\#]=

$2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$

```
Computing TLSig...
```

» $\frac{2\,(-1+\omega)^2}{\omega}$

» $1+2\,\theta[-1+u]-2\,\theta[1+u]$

» 2

» 1

» $\frac{2\,(-1+\omega)^2}{\omega}$

» $1+2\,\theta[-1+u]-2\,\theta[1+u]$

» $-\frac{2\,\omega}{(-1+\omega)^2}$

» $-1-2\,\theta[-1+u]+2\,\theta[1+u]$

» $-\frac{2\,(-1+\omega)^2\,(1-3\,\omega+\omega^2)}{\omega^2}$

» $-1+2\,\theta[-1+u]-2\,\theta[1+u]-2\,\theta\left[-\frac{\sqrt{5}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{5}}{2}+u\right]$

» $-\frac{2\,(1-2\,\omega+4\,\omega^2-5\,\omega^3+4\,\omega^4-2\,\omega^5+\omega^6)}{\omega^2\,(1-3\,\omega+\omega^2)}$

» $-1-2\,\theta\left[-\frac{\sqrt{5}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{5}}{2}+u\right]-2\,\theta[u-(-0.969\ldots)]+2\,\theta[u-(0.969\ldots)]$

» $-\frac{2\,(-1+\omega)^4\,(1+\omega^2)\,(1-\omega+3\,\omega^2-\omega^3+\omega^4)}{\omega^2\,(1-2\,\omega+4\,\omega^2-5\,\omega^3+4\,\omega^4-2\,\omega^5+\omega^6)}$

» $-1+2\,\theta\left[-\frac{1}{\sqrt{2}}+u\right]-2\,\theta\left[\frac{1}{\sqrt{2}}+u\right]+2\,\theta[u-(-0.969\ldots)]-2\,\theta[u-(0.969\ldots)]$

» $-\frac{2\,(1-\omega+\omega^2)^5}{(-1+\omega)^2\,\omega\,(1+\omega^2)\,(1-\omega+3\,\omega^2-\omega^3+\omega^4)}$

» $-1-2\,\theta[-1+u]+2\,\theta[1+u]-2\,\theta\left[-\frac{1}{\sqrt{2}}+u\right]+2\,\theta\left[\frac{1}{\sqrt{2}}+u\right]+2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]$

Out[ ]=

```
True
```

In[◦]:=
```
sign[ℰ_] := Module[{n, d, v, p, rs, e, k},
  {n, d} = NumeratorDenominator[ℰ]; {n, d} /= ω^(Exponent[n,ω]/2);
  p = Factor[ω2[v]@n * ω2[v]@d /. v → 4 u^2 - 2];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Return[Sign[p /. u → 0]]];
  rs = Union@(u /. rs);
  Sign[(-1)^(e=Exponent[p,u]) Coefficient[p, u, e]] + Sum[
    k = 0; While[(d = RootReduce[∂_{u,++k} p /. u → r]) == 0];
    If[EvenQ[k], 0, 2 Sign[d]] * θ[u - r],
    {r, rs}] ]
```

In[◦]:=
```
K = Knot[16, Alternating, 144 399]
Factor[Alexander[K][T]]
lhs = TLSig[K]
lhs == KasSig[K]
```

Out[◦]=

Knot[16, Alternating, 144 399]

KnotTheory: Loading precomputed data in KnotTheory/16A.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[◦]=

$$\frac{\left(1-T+T^2\right)^2 \left(2-10 T+20 T^2-23 T^3+20 T^4-10 T^5+2 T^6\right)}{T^5}$$

Out[◦]=

$$-2\,\theta\left[u-(-0.752\ldots)\right]+2\,\theta\left[u-0.752\ldots\right]$$

Out[◦]=

True

In[◦]:=
```
K = Knot[16, NonAlternating, 225 282]
Factor[Alexander[K][T]]
lhs = TLSig[K]
lhs == KasSig[K]
```

Out[◦]=

Knot[16, NonAlternating, 225 282]

KnotTheory: Loading precomputed data in KnotTheory/16N.dts.

Out[◦]=

$$-\frac{\left(1-3 T+T^2\right) \left(1-T+T^2\right)^4}{T^5}$$

Out[◦]=

0

Out[◦]=

True

In[ ]:=
```
K = Knot[16, NonAlternating, 761158]
Factor[Alexander[K][T]]
lhs = TLSig[K]
lhs == KasSig[K]
```

Out[ ]=

Knot[16, NonAlternating, 761158]

Out[ ]=

$$\frac{\left(1 - T + T^2\right)^3 \left(1 - 4T + 7T^2 - 4T^3 + T^4\right)}{T^5}$$

Out[ ]=

$$-2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right]$$

Out[ ]=

True