```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
        << Signatures`
```

```
In[•]:= Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] ⋃ Kas[X[7, 8, 9, 10]] // FM_{-2,5}
```

Out[•]=

	0	0	0	0	0	0	0	−1	0	0	0	1
−1	$(\eta_{-10}$	η_{-7}	η_8	$\eta_9)$	$(\eta_{-5}$	η_3	η_6	η_5	η_2	η_{-4}	η_{-1}	$\eta_{-2})$
$\overline{\eta}_{-10}$	1	u	1	u	0	0	0	0	0	0	0	0
$\overline{\eta}_{-7}$	u	$2u^2-1$	u	1	0	0	0	0	0	0	0	0
$\overline{\eta}_8$	1	u	1	u	0	0	0	0	0	0	0	0
$\overline{\eta}_9$	u	1	u	$2u^2-1$	0	0	0	0	0	0	0	0
$\overline{\eta}_{-5}$	0	0	0	0	$1-2u^2$	$-u$	-1	$-u$	0	0	0	0
$\overline{\eta}_3$	0	0	0	0	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_6$	0	0	0	0	-1	$-u$	$1-2u^2$	$-u$	0	0	0	0
$\overline{\eta}_5$	0	0	0	0	$-u$	-1	$-u$	0	u	1	u	0
$\overline{\eta}_2$	0	0	0	0	0	0	0	u	$2u^2-1$	u	1	0
$\overline{\eta}_{-4}$	0	0	0	0	0	0	0	1	u	1	u	0
$\overline{\eta}_{-1}$	0	0	0	0	0	0	0	u	1	u	$2u^2-1$	0
$\overline{\eta}_{-2}$	0	0	0	0	0	0	0	0	0	0	0	0

```
In[•]:= Table[K → BR@K, {K, AllKnots[{3, 7}]}]
```

KnotTheory: The minimum braids representing the knots with up to 10 crossings were provided by Thomas Gittings. See arXiv:math.GT/0401051.

Out[•]=

```
{Knot[3, 1] → BR[2, {-1, -1, -1}], Knot[4, 1] → BR[3, {-1, 2, -1, 2}],
 Knot[5, 1] → BR[2, {-1, -1, -1, -1, -1}], Knot[5, 2] → BR[3, {-1, -1, -1, -2, 1, -2}],
 Knot[6, 1] → BR[4, {-1, -1, -2, 1, 3, -2, 3}], Knot[6, 2] → BR[3, {-1, -1, -1, 2, -1, 2}],
 Knot[6, 3] → BR[3, {-1, -1, 2, -1, 2, 2}], Knot[7, 1] → BR[2, {-1, -1, -1, -1, -1, -1, -1}],
 Knot[7, 2] → BR[4, {-1, -1, -1, -2, 1, -2, -3, 2, -3}],
 Knot[7, 3] → BR[3, {1, 1, 1, 1, 1, 2, -1, 2}],
 Knot[7, 4] → BR[4, {1, 1, 2, -1, 2, 2, 3, -2, 3}],
 Knot[7, 5] → BR[3, {-1, -1, -1, -1, -2, 1, -2, -2}],
 Knot[7, 6] → BR[4, {-1, -1, 2, -1, -3, 2, -3}], Knot[7, 7] → BR[4, {1, -2, 1, -2, 3, -2, 3}]}
```

In[•]:=
```
OPD[BR[n_, l_List]] := Module[{σ, f, y, r = PD[]},
  σ = Range[n]; f = Table[0, n];
  Do[y = Abs@x; ++f[[σ[[{y, y + 1}]]]]; σ[[{y, y + 1}]] = σ[[{y + 1, y}]] , {x, l}] ;
  σ = Range[n]; f = Most@FoldList[Plus, 1, f + 1];
  Do[y = Abs@x;
   AppendTo[r, If[Sign[x] > 0,
     X[f[[σ[[y + 1]]]], f[[σ[[y]]]] + 1, f[[σ[[y + 1]]]] + 1, f[[σ[[y]]]]],
     X[f[[σ[[y]]]], f[[σ[[y + 1]]]], f[[σ[[y]]]] + 1, f[[σ[[y + 1]]]] + 1]
    ]];
   ++f[[σ[[{y, y + 1}]]]]; σ[[{y, y + 1}]] = σ[[{y + 1, y}]] ,
   {x, l}];
  r]
OPD@BR[Knot[5, 2]]
```

KnotTheory: The minimum braids representing the knots with up to 10 crossings were provided by Thomas Gittings. See arXiv:math.GT/0401051.

Out[•]=
```
PD[X[1, 7, 2, 8], X[8, 2, 9, 3], X[3, 9, 4, 10],
 X[4, 13, 5, 14], X[14, 11, 15, 10], X[11, 5, 12, 6]]
```

In[•]:=
```
f = {3, 5, 7};
```

In[•]:=
```
Most@FoldList[Plus, 1, f + 1]
```

Out[•]=
```
{1, 5, 11}
```

In[•]:=
```
Kas[b_BR] := Kas@OPD@b; TL[b_BR] := TL@OPD@b;
```

In[•]:=
```
BR[Knot[7, 2]]
```

Out[•]=
```
BR[4, {-1, -1, -1, -2, 1, -2, -3, 2, -3}]
```

In[•]:=
```
TL@BR[Knot[7, 2]]
```

Out[•]=

$$\begin{pmatrix} 2\,\Theta\left(u+\frac{\sqrt{\frac{11}{3}}}{2}\right)-2\,\Theta\left(u-\frac{\sqrt{\frac{11}{3}}}{2}\right) & (\eta_{-20} & \eta_8 & \eta_{16} & \eta_{22} & \\ \overline{\eta}_{-20} & -\frac{3\,(\omega-1)^2\,(\omega^2-\omega+1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \omega-1 & -\frac{2\,(\omega-1)\,(2\,\omega^2-4\,\omega+3)}{\omega\,(3\,\omega^2-5\,\omega+3)} & -\frac{2\,(\omega-1)^2\,(2\,\omega^2-3\,\omega+3)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \frac{2}{3} \\ \overline{\eta}_{8} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & \\ \overline{\eta}_{16} & \frac{2\,(\omega-1)\,(3\,\omega^2-4\,\omega+2)}{3\,\omega^2-5\,\omega+3} & 1-\omega & -\frac{3\,(\omega-1)^2\,(\omega^2-\omega+1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \frac{(\omega-1)\,(4\,\omega-3)\,(\omega^2-\omega+1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & -\frac{2}{3} \\ \overline{\eta}_{22} & -\frac{2\,(\omega-1)^2\,(3\,\omega^2-3\,\omega+2)}{\omega\,(3\,\omega^2-5\,\omega+3)} & 0 & \frac{(\omega-1)\,(3\,\omega-4)\,(\omega^2-\omega+1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & -\frac{5\,(\omega-1)^2\,(\omega^2-\omega+1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \frac{(\omega-1)\,(4}{\omega\,(3} \\ \overline{\eta}_{19} & -\frac{2\,(\omega-1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & 0 & \frac{2\,(\omega-1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \frac{(\omega-1)\,(3\,\omega^3-5\,\omega^2+5\,\omega-4)}{\omega\,(3\,\omega^2-5\,\omega+3)} & -\frac{(\omega-1}{\omega\,(\\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 & \\ \overline{\eta}_{-9} & \frac{2\,(\omega-1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & 0 & -\frac{2\,(\omega-1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & -\frac{2\,(\omega-2)\,(\omega-1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \frac{2\,(\omega}{\omega\,(3} \\ \overline{\eta}_{-17} & \frac{(\omega-1)\,(3\,\omega-4)\,(\omega^2-\omega+1)}{\omega\,(3\,\omega^2-5\,\omega+3)} & 0 & \frac{2\,(\omega-1)\,(\omega^2-2\,\omega+2)}{\omega\,(3\,\omega^2-5\,\omega+3)} & \frac{2\,(\omega-2)\,(\omega-1)\,(\omega^2-2\,\omega+2)}{\omega\,(3\,\omega^2-5\,\omega+3)} & -\frac{2\,(\omega-1}{\omega\,(\end{pmatrix}$$

In[•]:=
```
Kas@BR[Knot[7, 2]]
```

Out[•]=

$$\begin{pmatrix} 2\,\Theta\left(u-\sqrt{-0.588\ldots}\right)-2\,\Theta\left(u-\sqrt{0.588\ldots}\right)+2\,\Theta\left(u-\sqrt{-0.951\ldots}\right)-2\,\Theta\left(u-\sqrt{0.951\ldots}\right)-2\,\Theta(u-1)\cdot \\ \overline{\eta}_{-20} \\ \overline{\eta}_{8} \\ \overline{\eta}_{16} \\ \overline{\eta}_{22} \\ \overline{\eta}_{19} \\ \overline{\eta}_{-1} \\ \overline{\eta}_{-9} \\ \overline{\eta}_{-17} \end{pmatrix}$$

In[•]:=
```
TL@BR[3, {1, 2, -2, -1}]
```

Out[•]=

$$\begin{pmatrix} & 1 & 0 & -1 & 0 & 0 & 0 \\ & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & (\eta_{-9} & \eta_{11} & \eta_8 & \eta_5 & \eta_{-1} & \eta_{-6}) \\ \overline{\eta}_{-9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{8} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-6} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[•]:= Do[Print[TL@BR[2, Table[-1, k]] /. {u → 0, ω → -1}], {k, 10, 15}];
```

$$\begin{pmatrix} 9 & (\eta_{-12} & \eta_{22} & \eta_{11} & \eta_{-1}) \\ \overline{\eta}_{-12} & \frac{2}{5} & -2 & -\frac{2}{5} & 2 \\ \overline{\eta}_{22} & -2 & 0 & 2 & 0 \\ \overline{\eta}_{11} & -\frac{2}{5} & 2 & \frac{2}{5} & -2 \\ \overline{\eta}_{-1} & 2 & 0 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 10 & (\eta_{-13} & \eta_{12} & \eta_{24} & \eta_{-1}) \\ \overline{\eta}_{-13} & \frac{4}{11} & -2 & -\frac{4}{11} & 2 \\ \overline{\eta}_{12} & -2 & 0 & 2 & 0 \\ \overline{\eta}_{24} & -\frac{4}{11} & 2 & \frac{4}{11} & -2 \\ \overline{\eta}_{-1} & 2 & 0 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 11 & (\eta_{-14} & \eta_{26} & \eta_{13} & \eta_{-1}) \\ \overline{\eta}_{-14} & \frac{1}{3} & -2 & -\frac{1}{3} & 2 \\ \overline{\eta}_{26} & -2 & 0 & 2 & 0 \\ \overline{\eta}_{13} & -\frac{1}{3} & 2 & \frac{1}{3} & -2 \\ \overline{\eta}_{-1} & 2 & 0 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 12 & (\eta_{-15} & \eta_{14} & \eta_{28} & \eta_{-1}) \\ \overline{\eta}_{-15} & \frac{4}{13} & -2 & -\frac{4}{13} & 2 \\ \overline{\eta}_{14} & -2 & 0 & 2 & 0 \\ \overline{\eta}_{28} & -\frac{4}{13} & 2 & \frac{4}{13} & -2 \\ \overline{\eta}_{-1} & 2 & 0 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 13 & (\eta_{-16} & \eta_{30} & \eta_{15} & \eta_{-1}) \\ \overline{\eta}_{-16} & \frac{2}{7} & -2 & -\frac{2}{7} & 2 \\ \overline{\eta}_{30} & -2 & 0 & 2 & 0 \\ \overline{\eta}_{15} & -\frac{2}{7} & 2 & \frac{2}{7} & -2 \\ \overline{\eta}_{-1} & 2 & 0 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 14 & (\eta_{-17} & \eta_{16} & \eta_{32} & \eta_{-1}) \\ \overline{\eta}_{-17} & \frac{4}{15} & -2 & -\frac{4}{15} & 2 \\ \overline{\eta}_{16} & -2 & 0 & 2 & 0 \\ \overline{\eta}_{32} & -\frac{4}{15} & 2 & \frac{4}{15} & -2 \\ \overline{\eta}_{-1} & 2 & 0 & -2 & 0 \end{pmatrix}$$

```
In[•]:= Do[Print[Kas@BR[2, Table[-1, k]] /. u → 0], {k, 10, 15}];
```

$$\begin{pmatrix} 19 & (\eta_{-12} & \eta_{22} & \eta_{11} & \eta_{-1}) \\ \overline{\eta}_{-12} & \frac{1}{10} & 0 & -\frac{1}{10} & 0 \\ \overline{\eta}_{22} & 0 & -10 & 0 & -10 \\ \overline{\eta}_{11} & -\frac{1}{10} & 0 & \frac{1}{10} & 0 \\ \overline{\eta}_{-1} & 0 & -10 & 0 & -10 \end{pmatrix}$$

$$\begin{pmatrix} 21 & (\eta_{-13} & \eta_{12} & \eta_{24} & \eta_{-1}) \\ \overline{\eta}_{-13} & \frac{1}{11} & 0 & -\frac{1}{11} & 0 \\ \overline{\eta}_{12} & 0 & -11 & 0 & -11 \\ \overline{\eta}_{24} & -\frac{1}{11} & 0 & \frac{1}{11} & 0 \\ \overline{\eta}_{-1} & 0 & -11 & 0 & -11 \end{pmatrix}$$

$$\begin{pmatrix} 23 & (\eta_{-14} & \eta_{26} & \eta_{13} & \eta_{-1}) \\ \overline{\eta}_{-14} & \frac{1}{12} & 0 & -\frac{1}{12} & 0 \\ \overline{\eta}_{26} & 0 & -12 & 0 & -12 \\ \overline{\eta}_{13} & -\frac{1}{12} & 0 & \frac{1}{12} & 0 \\ \overline{\eta}_{-1} & 0 & -12 & 0 & -12 \end{pmatrix}$$

$$\begin{pmatrix} 25 & (\eta_{-15} & \eta_{14} & \eta_{28} & \eta_{-1}) \\ \overline{\eta}_{-15} & \frac{1}{13} & 0 & -\frac{1}{13} & 0 \\ \overline{\eta}_{14} & 0 & -13 & 0 & -13 \\ \overline{\eta}_{28} & -\frac{1}{13} & 0 & \frac{1}{13} & 0 \\ \overline{\eta}_{-1} & 0 & -13 & 0 & -13 \end{pmatrix}$$

$$\begin{pmatrix} 27 & (\eta_{-16} & \eta_{30} & \eta_{15} & \eta_{-1}) \\ \overline{\eta}_{-16} & \frac{1}{14} & 0 & -\frac{1}{14} & 0 \\ \overline{\eta}_{30} & 0 & -14 & 0 & -14 \\ \overline{\eta}_{15} & -\frac{1}{14} & 0 & \frac{1}{14} & 0 \\ \overline{\eta}_{-1} & 0 & -14 & 0 & -14 \end{pmatrix}$$

$$\begin{pmatrix} 29 & (\eta_{-17} & \eta_{16} & \eta_{32} & \eta_{-1}) \\ \overline{\eta}_{-17} & \frac{1}{15} & 0 & -\frac{1}{15} & 0 \\ \overline{\eta}_{16} & 0 & -15 & 0 & -15 \\ \overline{\eta}_{32} & -\frac{1}{15} & 0 & \frac{1}{15} & 0 \\ \overline{\eta}_{-1} & 0 & -15 & 0 & -15 \end{pmatrix}$$

In[•]:=
```
Echo[# → TL[#][[2, 1]] & /@ Get["../../Projects/OU/Data/B_3_3.m"];
```

» BR[3, {-2, -2, -2}] → {}

» BR[3, {-2, -2, -1}] → {}

» BR[3, {-2, -2, 1}] → {}

» BR[3, {-2, -1, -2}] → {}

» BR[3, {-2, -1, -1}] → {}

» BR[3, {-2, -1, 2}] → $\left\{\eta_{-7} - \frac{\eta_{-4}}{\omega} - \eta_6 + \frac{\eta_9}{\omega}\right\}$

» BR[3, {-2, 1, -2}] → {}

» BR[3, {-2, 1, 1}] → {}

» BR[3, {-2, 1, 2}] → $\left\{\eta_{-7} - \frac{\eta_{-4}}{\omega} - \eta_6 + \frac{\eta_9}{\omega}\right\}$

» BR[3, {-1, -2, -2}] → {}

» BR[3, {-1, -2, 1}] → $\{\eta_{-7} - \eta_{-4} - \eta_6 + \eta_9\}$

» BR[3, {-1, -1, -2}] → {}

» BR[3, {-1, -1, -1}] → {}

» BR[3, {-1, -1, 2}] → {}

» BR[3, {-1, 2, -1}] → {}

» BR[3, {-1, 2, 1}] → $\{\eta_{-7} - \eta_{-4} - \eta_6 + \eta_9\}$

» BR[3, {-1, 2, 2}] → {}

» BR[3, {1, -2, -2}] → {}

» BR[3, {1, -2, 1}] → {}

» BR[3, {1, 1, -2}] → {}

» BR[3, {1, 1, 1}] → {}

» BR[3, {1, 1, 2}] → {}

» BR[3, {1, 2, 1}] → {}

» BR[3, {1, 2, 2}] → {}

» BR[3, {2, -1, -1}] → {}

» BR[3, {2, -1, 2}] → {}

» BR[3, {2, 1, 1}] → {}

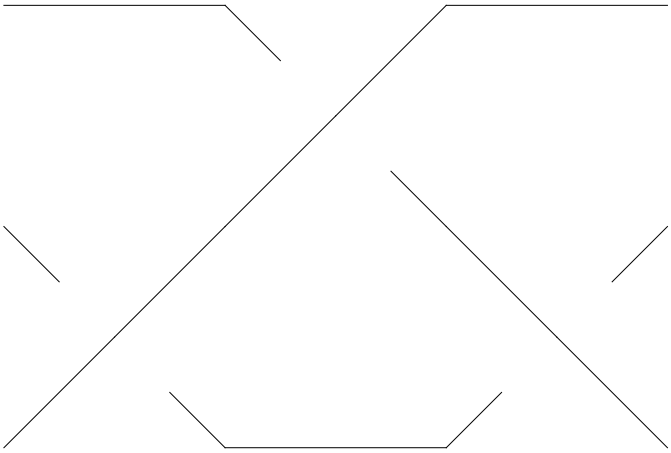
» BR[3, {2, 2, -1}] → {}

» BR[3, {2, 2, 1}] → {}

» BR[3, {2, 2, 2}] → {}

```
In[•]:= BR[3, {-2, -1, 2}] // BraidPlot
```

Out[•]=



```
In[•]:= BR[3, {-2, -1, 2}] // TL
```

Out[•]=

$$\begin{pmatrix} & 1 & 0 & -1 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} \\ 0 & (\eta_{-7} & \eta_3 & \eta_6 & \eta_9 & \eta_{-1} & \eta_{-4}) \\ \overline{\eta}_{-7} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & \frac{\omega-1}{\omega^2} & 0 & -\frac{\omega-1}{\omega^2} \\ \overline{\eta}_6 & 0 & 0 & 0 & -\frac{\omega-1}{\omega^2} & 0 & \frac{\omega-1}{\omega^2} \\ \overline{\eta}_9 & 0 & -((\omega-1)\,\omega) & (\omega-1)\,\omega & -\frac{2\,(\omega-1)^2}{\omega} & -\frac{\omega-1}{\omega} & \frac{(\omega-1)\,(2\,\omega-1)}{\omega} \\ \overline{\eta}_{-1} & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega \\ \overline{\eta}_{-4} & 0 & (\omega-1)\,\omega & -((\omega-1)\,\omega) & \frac{(\omega-2)\,(\omega-1)}{\omega} & \frac{\omega-1}{\omega} & -\frac{(\omega-1)^2}{\omega} \end{pmatrix}$$

```
In[•]:= BR[3, {1, -2, -1}] // TL
```

Out[•]=

$$\begin{pmatrix} & 1 & 0 & -1 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} \\ 0 & (\eta_{-7} & \eta_3 & \eta_6 & \eta_9 & \eta_{-1} & \eta_{-4}) \\ \overline{\eta}_{-7} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & \frac{\omega-1}{\omega^2} & 0 & -\frac{\omega-1}{\omega^2} \\ \overline{\eta}_6 & 0 & 0 & 0 & -\frac{\omega-1}{\omega^2} & 0 & \frac{\omega-1}{\omega^2} \\ \overline{\eta}_9 & 0 & -((\omega-1)\,\omega) & (\omega-1)\,\omega & -\frac{2\,(\omega-1)^2}{\omega} & -\frac{\omega-1}{\omega} & \frac{(\omega-1)\,(2\,\omega-1)}{\omega} \\ \overline{\eta}_{-1} & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega \\ \overline{\eta}_{-4} & 0 & (\omega-1)\,\omega & -((\omega-1)\,\omega) & \frac{(\omega-2)\,(\omega-1)}{\omega} & \frac{\omega-1}{\omega} & -\frac{(\omega-1)^2}{\omega} \end{pmatrix}$$

```
In[•]:= Echo[# → Kas[#][[2, 1]] & /@ Get["../../Projects/OU/Data/B_3_3.m"];
```

```
» BR[3, {-2, -2, -2}] → {}
» BR[3, {-2, -2, -1}] → {}
» BR[3, {-2, -2, 1}] → {}
» BR[3, {-2, -1, -2}] → {}
» BR[3, {-2, -1, -1}] → {}
» BR[3, {-2, -1, 2}] → {}
» BR[3, {-2, 1, -2}] → {}
» BR[3, {-2, 1, 1}] → {}
» BR[3, {-2, 1, 2}] → {}
» BR[3, {-1, -2, -2}] → {}
» BR[3, {-1, -2, 1}] → {}
» BR[3, {-1, -1, -2}] → {}
» BR[3, {-1, -1, -1}] → {}
» BR[3, {-1, -1, 2}] → {}
» BR[3, {-1, 2, -1}] → {}
» BR[3, {-1, 2, 1}] → {}
» BR[3, {-1, 2, 2}] → {}
» BR[3, {1, -2, -2}] → {}
» BR[3, {1, -2, 1}] → {}
» BR[3, {1, 1, -2}] → {}
» BR[3, {1, 1, 1}] → {}
» BR[3, {1, 1, 2}] → {}
» BR[3, {1, 2, 1}] → {}
» BR[3, {1, 2, 2}] → {}
» BR[3, {2, -1, -1}] → {}
» BR[3, {2, -1, 2}] → {}
» BR[3, {2, 1, 1}] → {}
» BR[3, {2, 2, -1}] → {}
» BR[3, {2, 2, 1}] → {}
» BR[3, {2, 2, 2}] → {}
```

In[•]:=
```
Select[Get["../../Projects/OU/Data/B_3_4.m"], Length[TL[#][[2, 1]]] > 1 &]
```

Out[•]=
```
{}
```

In[•]:=
```
Select[Get["../../Projects/OU/Data/B_3_5.m"], Length[TL[#][[2, 1]]] > 1 &]
```

Out[•]=
```
{}
```

```
In[•]:= Select[Get["../../Projects/OU/Data/B_4_4.m"], Length[TL[#][[2, 1]]] > 1 &]
```

Out[•]=

$$\{\}$$

```
In[•]:= Select[Get["../../Projects/OU/Data/B_4_5.m"], Length[TL[#][[2, 1]]] > 1 &]
```

Out[•]=

$$\{BR[4, \{-3, -2, -1, 2, 3\}], BR[4, \{-3, -2, 1, 2, 3\}],$$
$$BR[4, \{-3, 2, -1, -2, 3\}], BR[4, \{-3, 2, 1, -2, 3\}], BR[4, \{-1, -2, -3, 2, 1\}],$$
$$BR[4, \{-1, -2, 3, 2, 1\}], BR[4, \{1, -2, -3, 2, -1\}], BR[4, \{1, -2, 3, 2, -1\}]\}$$

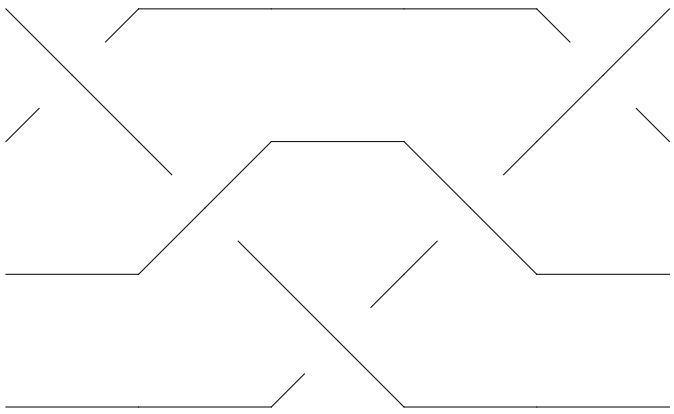
```
In[•]:= TL@BR[4, {1, -2, 3, 2, -1}]
```

Out[•]=

$$\begin{pmatrix} & 1 & 0 & -1 & 0 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 0 \\ & 0 & 0 & 0 & -1 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 1 \\ 0 & (\eta_{-11} & \eta_4 & \eta_{10} & \eta_7 & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8}) \\ \overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_4 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega^2} & 0 & -\frac{\omega-1}{\omega^2} & 0 \\ \overline{\eta}_{10} & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \overline{\eta}_7 & 0 & 0 & 0 & 0 & \frac{(\omega-1)^2}{\omega^2} & 0 & -\frac{(\omega-1)^2}{\omega^2} & 0 \\ \overline{\eta}_{14} & 0 & -((\omega-1)\,\omega) & \omega-1 & (\omega-1)^2 & 0 & -\frac{\omega-1}{\omega} & \frac{\omega-1}{\omega} & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 \\ \overline{\eta}_{-5} & 0 & (\omega-1)\,\omega & 1-\omega & -(\omega-1)^2 & 1-\omega & \frac{\omega-1}{\omega} & \frac{(\omega-1)^2}{\omega} & 0 \\ \overline{\eta}_{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[•]:= BraidPlot@BR[4, {1, -2, 3, 2, -1}]
```

Out[•]=



In[•]:=
```
Kas@BR[4, {1, -2, 3, 2, -1}]
```

Out[•]=

	1	0	−1	0	1	0	−1	0
0	$(\eta_{-11}$	η_4	η_{10}	η_7	η_{14}	η_{-1}	η_{-5}	$\eta_{-8})$
$\overline{\eta}_{-11}$	0	0	0	0	0	0	0	0
$\overline{\eta}_4$	0	0	0	-1	$-u$	0	u	1
$\overline{\eta}_{10}$	0	0	0	$-u$	$1-2u^2$	0	$2u^2-1$	u
$\overline{\eta}_7$	0	-1	$-u$	$2u^2-3$	$-u$	-1	0	1
$\overline{\eta}_{14}$	0	$-u$	$1-2u^2$	$-u$	-1	$-u$	$-2(u-1)(u+1)$	u
$\overline{\eta}_{-1}$	0	0	0	-1	$-u$	0	u	1
$\overline{\eta}_{-5}$	0	u	$2u^2-1$	0	$-2(u-1)(u+1)$	u	$4u^2-3$	0
$\overline{\eta}_{-8}$	0	1	u	1	u	1	0	$1-2u^2$

In[•]:=
```
Select[Get["../../Projects/OU/Data/B_3_3.m"], Length[Kas[#][[2, 1]]] > 0 &]
```

Out[•]=
```
{}
```

In[•]:=
```
Select[Get["../../Projects/OU/Data/B_3_4.m"], Length[Kas[#][[2, 1]]] > 0 &]
```

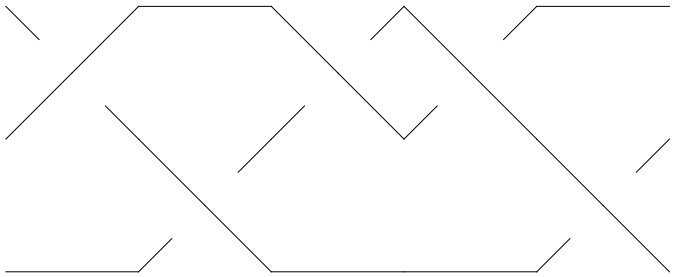
Out[•]=
```
{}
```

In[•]:=
```
Select[Get["../../Projects/OU/Data/B_3_5.m"], Length[Kas[#][[2, 1]]] > 0 &]
```

Out[•]=
```
{BR[3, {-2, -1, -1, -2, 1}], BR[3, {-2, 1, 2, 2, 1}],
 BR[3, {-1, -2, -2, -1, 2}], BR[3, {-1, 2, 1, 1, 2}]}
```

In[•]:=
```
BR[3, {-1, 2, 1, 1, 2}] // BraidPlot
```

Out[•]=



In[•]:=
```
Kas@BR[3, {-1, 2, 1, 1, 2}]
```

Out[•]=

$$\begin{pmatrix} & 1 & 0 & 1 & 2u \\ 2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)-2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1 & (\eta_{-9} & \eta_{13} & \eta_4 & \eta_8 \\ \overline{\eta}_{-9} & 0 & 0 & 0 & 0 \\ \overline{\eta}_{13} & 0 & \frac{4\,(u-1)\,(u+1)}{4u^2-3} & 0 & -\frac{2\,(u-1)\,(u+1)\,(2u-1)\,(2u+1)}{4u^2-3} \\ \overline{\eta}_4 & 0 & 0 & 4u^2-3 & u\left(4u^2-3\right) \\ \overline{\eta}_8 & 0 & -\frac{2\,(u-1)\,(u+1)\,(2u-1)\,(2u+1)}{4u^2-3} & u\left(4u^2-3\right) & \frac{\left(2u^2-1\right)\left(16u^4-16u^2+1\right)}{4u^2-3} \\ \overline{\eta}_{-1} & 0 & -\frac{8\,(u-1)\,u\,(u+1)}{4u^2-3} & 4u^2-3 & \frac{u\left(32u^4-44u^2+13\right)}{4u^2-3} \\ \overline{\eta}_{-5} & 0 & -\frac{2\,(u-1)\,(u+1)\,(2u-1)\,(2u+1)}{4u^2-3} & u\left(4u^2-3\right) & \frac{\left(2u^2-1\right)\left(16u^4-16u^2+1\right)}{4u^2-3} \end{pmatrix}$$

In[•]:=
```
TL@BR[3, {-1, 2, 1, 1, 2}]
```

Out[•]=

$$\begin{pmatrix} -1 & (\eta_{-9} & \eta_{13} & \eta_4 & \eta_8 & \eta_{-1} & \eta_{-5}) \\ \overline{\eta}_{-9} & \frac{\omega^2-\omega+1}{\omega} & \omega-1 & 1-2\,\omega & \frac{\omega-1}{\omega} & 0 & 0 \\ \overline{\eta}_{13} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 \\ \overline{\eta}_4 & \frac{\omega-2}{\omega} & 1-\omega & \frac{\omega^2-\omega+1}{\omega} & 0 & 0 & -\frac{\omega-1}{\omega} \\ \overline{\eta}_8 & 1-\omega & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & \frac{(\omega-1)\,(\omega+1)}{\omega} \\ \overline{\eta}_{-1} & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega \\ \overline{\eta}_{-5} & 0 & 0 & \omega-1 & -\frac{(\omega-1)\,(\omega+1)}{\omega} & \frac{\omega-1}{\omega} & 0 \end{pmatrix}$$

In[•]:=
```
Column@DeleteCases[_ → {0, 0}][# → {Length[TL[#]⟦2, 1⟧], Length[Kas[#]⟦2, 1⟧]} & /@
    Get["../../Projects/OU/Data/B_3_5.m"]]
```

Out[•]=

```
BR[3, {-2, -2, -1, 2, 2}] → {1, 0}
BR[3, {-2, -2, 1, 2, 2}] → {1, 0}
BR[3, {-2, -1, -1, -2, 1}] → {0, 1}
BR[3, {-2, -1, -1, -1, 2}] → {1, 0}
BR[3, {-2, -1, 2, 2, -1}] → {1, 0}
BR[3, {-2, 1, 1, 1, 2}] → {1, 0}
BR[3, {-2, 1, 2, 2, 1}] → {0, 1}
BR[3, {-1, -2, -2, -2, 1}] → {1, 0}
BR[3, {-1, -2, -2, -1, 2}] → {0, 1}
BR[3, {-1, -2, 1, 1, -2}] → {1, 0}
BR[3, {-1, -1, -2, 1, 1}] → {1, 0}
BR[3, {-1, -1, 2, 1, 1}] → {1, 0}
BR[3, {-1, 2, 1, 1, 2}] → {0, 1}
BR[3, {-1, 2, 2, 2, 1}] → {1, 0}
BR[3, {1, -2, -2, 1, 2}] → {1, 0}
BR[3, {2, -1, -1, 2, 1}] → {1, 0}
```

```
In[•]:= Column@DeleteCases[_ → {0, 0}][# → {Length[TL[#][[2, 1]]], Length[Kas[#][[2, 1]]]} & /@
          Get["../../Projects/OU/Data/B_4_3.m"]]
```

```
Out[•]=
BR[4, {-3, -2, 3}] → {1, 0}
BR[4, {-3, 2, 3}] → {1, 0}
BR[4, {-2, -3, 2}] → {1, 0}
BR[4, {-2, -1, 2}] → {1, 0}
BR[4, {-2, 1, 2}] → {1, 0}
BR[4, {-2, 3, 2}] → {1, 0}
BR[4, {-1, -2, 1}] → {1, 0}
BR[4, {-1, 2, 1}] → {1, 0}
```

```
In[•]:= Column@DeleteCases[_ → {0, 0}][# → {Length[TL[#][[2, 1]]], Length[Kas[#][[2, 1]]]} & /@
          Get["../../Projects/OU/Data/B_4_4.m"]]
```

Out[•]=

```
BR[4, {-3, -2, -2, 3}] → {1, 0}
BR[4, {-3, -2, -1, 2}] → {1, 0}
BR[4, {-3, -2, -1, 3}] → {1, 0}
BR[4, {-3, -2, 1, 2}] → {1, 0}
BR[4, {-3, -2, 1, 3}] → {1, 0}
BR[4, {-3, 2, -1, -2}] → {1, 0}
BR[4, {-3, 2, -1, 3}] → {1, 0}
BR[4, {-3, 2, 1, -2}] → {1, 0}
BR[4, {-3, 2, 1, 3}] → {1, 0}
BR[4, {-3, 2, 2, 3}] → {1, 0}
BR[4, {-2, -3, -3, 2}] → {1, 0}
BR[4, {-2, -3, 2, -1}] → {1, 0}
BR[4, {-2, -3, 2, 1}] → {1, 0}
BR[4, {-2, -1, -3, 2}] → {1, 0}
BR[4, {-2, -1, -1, 2}] → {1, 0}
BR[4, {-2, -1, 2, -3}] → {1, 0}
BR[4, {-2, -1, 2, 3}] → {1, 0}
BR[4, {-2, -1, 3, 2}] → {1, 1}
BR[4, {-2, 1, -3, 2}] → {1, 1}
BR[4, {-2, 1, 1, 2}] → {1, 0}
BR[4, {-2, 1, 2, -3}] → {1, 0}
BR[4, {-2, 1, 2, 3}] → {1, 0}
BR[4, {-2, 1, 3, 2}] → {1, 0}
BR[4, {-2, 3, 2, -1}] → {1, 0}
BR[4, {-2, 3, 2, 1}] → {1, 0}
BR[4, {-2, 3, 3, 2}] → {1, 0}
BR[4, {-1, -3, -2, 3}] → {1, 0}
BR[4, {-1, -3, 2, 3}] → {1, 0}
BR[4, {-1, -2, -3, 2}] → {1, 0}
BR[4, {-1, -2, -2, 1}] → {1, 0}
BR[4, {-1, -2, 1, -3}] → {1, 0}
BR[4, {-1, -2, 1, 3}] → {1, 0}
BR[4, {-1, -2, 3, 2}] → {1, 0}
BR[4, {-1, 2, 1, -3}] → {1, 0}
BR[4, {-1, 2, 1, 3}] → {1, 0}
BR[4, {-1, 2, 2, 1}] → {1, 0}
BR[4, {-1, 3, -2, 1}] → {1, 0}
BR[4, {-1, 3, 2, 1}] → {1, 0}
BR[4, {1, -3, -2, 3}] → {1, 0}
BR[4, {1, -3, 2, 3}] → {1, 0}
BR[4, {1, -2, -3, 2}] → {1, 0}
BR[4, {1, -2, 3, 2}] → {1, 0}
BR[4, {1, 3, -2, -1}] → {1, 0}
BR[4, {1, 3, 2, -1}] → {1, 0}
BR[4, {2, -1, -3, -2}] → {1, 0}
BR[4, {2, -1, 3, -2}] → {1, 1}
BR[4, {2, 1, -3, -2}] → {1, 1}
BR[4, {2, 1, 3, -2}] → {1, 0}
```

```
In[•]:= Column@DeleteCases[_ → {0, 0}][# → {Length[TL[#][[2, 1]]], Length[Kas[#][[2, 1]]]} & /@
          Get["../../Projects/OU/Data/B_4_5.m"]]
```

Out[•]=

```
BR[4, {-3, -3, -2, -1, 2}] → {1, 0}
BR[4, {-3, -3, -2, 1, 2}] → {1, 0}
BR[4, {-3, -3, -2, 3, 3}] → {1, 0}
BR[4, {-3, -3, 2, -1, -2}] → {1, 0}
BR[4, {-3, -3, 2, 1, -2}] → {1, 0}
BR[4, {-3, -3, 2, 3, 3}] → {1, 0}
BR[4, {-3, -2, -2, -3, 2}] → {0, 1}
BR[4, {-3, -2, -2, -2, 3}] → {1, 0}
BR[4, {-3, -2, -2, -1, 3}] → {1, 0}
BR[4, {-3, -2, -2, 1, 3}] → {1, 0}
BR[4, {-3, -2, -1, -2, 3}] → {1, 0}
BR[4, {-3, -2, -1, -1, 2}] → {1, 0}
BR[4, {-3, -2, -1, -1, 3}] → {1, 0}
BR[4, {-3, -2, -1, 2, -3}] → {1, 0}
BR[4, {-3, -2, -1, 2, 3}] → {2, 1}
BR[4, {-3, -2, -1, 3, 2}] → {0, 1}
BR[4, {-3, -2, 1, -2, 3}] → {1, 0}
BR[4, {-3, -2, 1, 1, 2}] → {1, 0}
BR[4, {-3, -2, 1, 1, 3}] → {1, 0}
BR[4, {-3, -2, 1, 2, -3}] → {1, 0}
BR[4, {-3, -2, 1, 2, 3}] → {2, 1}
BR[4, {-3, -2, 3, 3, -2}] → {1, 0}
BR[4, {-3, 2, -1, -2, -3}] → {1, 0}
BR[4, {-3, 2, -1, -2, 3}] → {2, 1}
BR[4, {-3, 2, -1, -1, -2}] → {1, 0}
BR[4, {-3, 2, -1, -1, 3}] → {1, 0}
BR[4, {-3, 2, -1, 2, 3}] → {1, 0}
BR[4, {-3, 2, 1, -3, -2}] → {0, 1}
BR[4, {-3, 2, 1, -2, -3}] → {1, 0}
BR[4, {-3, 2, 1, -2, 3}] → {2, 1}
BR[4, {-3, 2, 1, 1, -2}] → {1, 0}
BR[4, {-3, 2, 1, 1, 3}] → {1, 0}
BR[4, {-3, 2, 1, 2, 3}] → {1, 0}
BR[4, {-3, 2, 1, 3, 2}] → {0, 1}
BR[4, {-3, 2, 2, -1, 3}] → {1, 0}
BR[4, {-3, 2, 2, 1, 3}] → {1, 0}
BR[4, {-3, 2, 2, 2, 3}] → {1, 0}
BR[4, {-3, 2, 3, 3, 2}] → {0, 1}
BR[4, {-2, -3, -3, -3, 2}] → {1, 0}
BR[4, {-2, -3, -3, -2, 3}] → {0, 1}
BR[4, {-2, -3, -3, 2, -1}] → {1, 0}
BR[4, {-2, -3, -3, 2, 1}] → {1, 0}
BR[4, {-2, -3, 2, -1, -2}] → {0, 1}
BR[4, {-2, -3, 2, -1, -1}] → {1, 0}
BR[4, {-2, -3, 2, 1, 1}] → {1, 0}
BR[4, {-2, -3, 2, 2, -3}] → {1, 0}
BR[4, {-2, -2, -3, 2, 2}] → {1, 0}
```

```
BR[4, {-2, -2, -1, 2, 2}] → {1, 0}
BR[4, {-2, -2, 1, 2, 2}] → {1, 0}
BR[4, {-2, -2, 3, 2, 2}] → {1, 0}
BR[4, {-2, -1, -3, -3, 2}] → {1, 0}
BR[4, {-2, -1, -3, -2, 3}] → {0, 1}
BR[4, {-2, -1, -1, -3, 2}] → {1, 0}
BR[4, {-2, -1, -1, -2, 1}] → {0, 1}
BR[4, {-2, -1, -1, -1, 2}] → {1, 0}
BR[4, {-2, -1, -1, 2, -3}] → {1, 0}
BR[4, {-2, -1, -1, 2, 3}] → {1, 0}
BR[4, {-2, -1, -1, 3, 2}] → {1, 0}
BR[4, {-2, -1, 2, -3, -3}] → {1, 0}
BR[4, {-2, -1, 2, 2, -1}] → {1, 0}
BR[4, {-2, -1, 2, 3, 3}] → {1, 0}
BR[4, {-2, -1, 3, 2, -1}] → {0, 1}
BR[4, {-2, -1, 3, 2, 1}] → {0, 1}
BR[4, {-2, -1, 3, 3, 2}] → {1, 0}
BR[4, {-2, 1, -3, -3, 2}] → {1, 0}
BR[4, {-2, 1, -3, 2, -3}] → {0, 1}
BR[4, {-2, 1, -3, 2, 3}] → {0, 1}
BR[4, {-2, 1, 1, -3, 2}] → {1, 0}
BR[4, {-2, 1, 1, 1, 2}] → {1, 0}
BR[4, {-2, 1, 1, 2, -3}] → {1, 0}
BR[4, {-2, 1, 1, 2, 3}] → {1, 0}
BR[4, {-2, 1, 1, 3, 2}] → {1, 0}
BR[4, {-2, 1, 2, -3, -3}] → {1, 0}
BR[4, {-2, 1, 2, 2, 1}] → {0, 1}
BR[4, {-2, 1, 2, 3, 3}] → {1, 0}
BR[4, {-2, 1, 3, 3, 2}] → {1, 0}
BR[4, {-2, 3, 2, -1, -1}] → {1, 0}
BR[4, {-2, 3, 2, 1, 1}] → {1, 0}
BR[4, {-2, 3, 2, 2, 3}] → {0, 1}
BR[4, {-2, 3, 3, 2, -1}] → {1, 0}
BR[4, {-2, 3, 3, 2, 1}] → {1, 0}
BR[4, {-2, 3, 3, 3, 2}] → {1, 0}
BR[4, {-1, -3, -2, -2, 3}] → {1, 0}
BR[4, {-1, -3, 2, -1, 3}] → {1, 0}
BR[4, {-1, -3, 2, 2, 3}] → {1, 0}
BR[4, {-1, -2, -3, -3, 2}] → {1, 0}
BR[4, {-1, -2, -3, 2, -1}] → {1, 0}
BR[4, {-1, -2, -3, 2, 1}] → {2, 1}
BR[4, {-1, -2, -2, -2, 1}] → {1, 0}
BR[4, {-1, -2, -2, -1, 2}] → {0, 1}
BR[4, {-1, -2, -2, 1, -3}] → {1, 0}
BR[4, {-1, -2, -2, 1, 3}] → {1, 0}
BR[4, {-1, -2, 1, -3, -3}] → {1, 0}
BR[4, {-1, -2, 1, -3, 2}] → {0, 1}
BR[4, {-1, -2, 1, 1, -2}] → {1, 0}
BR[4, {-1, -2, 1, 3, 3}] → {1, 0}
BR[4, {-1, -2, 3, -2, 1}] → {1, 0}
```

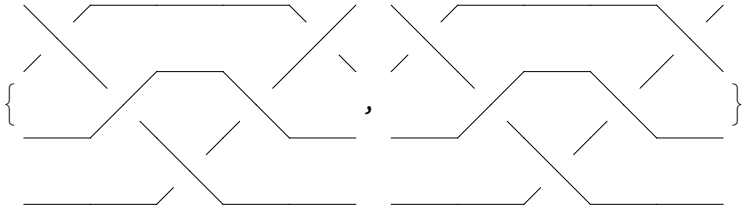
```
BR[4, {-1, -2, 3, 2, -1}] → {1, 0}
BR[4, {-1, -2, 3, 2, 1}] → {2, 1}
BR[4, {-1, -2, 3, 3, 2}] → {1, 0}
BR[4, {-1, -1, -3, -2, 3}] → {1, 0}
BR[4, {-1, -1, -3, 2, 3}] → {1, 0}
BR[4, {-1, -1, -2, -3, 2}] → {1, 0}
BR[4, {-1, -1, -2, 1, 1}] → {1, 0}
BR[4, {-1, -1, -2, 3, 2}] → {1, 0}
BR[4, {-1, -1, 2, 1, 1}] → {1, 0}
BR[4, {-1, 2, -3, 2, 1}] → {1, 0}
BR[4, {-1, 2, -1, 3, -2}] → {0, 1}
BR[4, {-1, 2, 1, -3, -3}] → {1, 0}
BR[4, {-1, 2, 1, 1, 2}] → {0, 1}
BR[4, {-1, 2, 1, 3, 2}] → {0, 1}
BR[4, {-1, 2, 1, 3, 3}] → {1, 0}
BR[4, {-1, 2, 2, 1, -3}] → {1, 0}
BR[4, {-1, 2, 2, 1, 3}] → {1, 0}
BR[4, {-1, 2, 2, 2, 1}] → {1, 0}
BR[4, {-1, 2, 3, 2, 1}] → {1, 0}
BR[4, {-1, 3, -2, -2, 1}] → {1, 0}
BR[4, {-1, 3, -2, 1, 3}] → {1, 0}
BR[4, {-1, 3, 2, 2, 1}] → {1, 0}
BR[4, {-1, 3, 3, -2, 1}] → {1, 0}
BR[4, {-1, 3, 3, 2, 1}] → {1, 0}
BR[4, {1, -3, -2, -2, 3}] → {1, 0}
BR[4, {1, -3, -2, 1, 3}] → {1, 0}
BR[4, {1, -3, 2, 2, 3}] → {1, 0}
BR[4, {1, -2, -3, -3, 2}] → {1, 0}
BR[4, {1, -2, -3, 2, -1}] → {2, 1}
BR[4, {1, -2, -3, 2, 1}] → {1, 0}
BR[4, {1, -2, -2, 1, 2}] → {1, 0}
BR[4, {1, -2, 1, -3, 2}] → {0, 1}
BR[4, {1, -2, 3, -2, -1}] → {1, 0}
BR[4, {1, -2, 3, 2, -1}] → {2, 1}
BR[4, {1, -2, 3, 2, 1}] → {1, 0}
BR[4, {1, -2, 3, 3, 2}] → {1, 0}
BR[4, {1, 1, -3, -2, 3}] → {1, 0}
BR[4, {1, 1, -3, 2, 3}] → {1, 0}
BR[4, {1, 1, -2, -3, 2}] → {1, 0}
BR[4, {1, 1, -2, 3, 2}] → {1, 0}
BR[4, {1, 2, -3, 2, -1}] → {1, 0}
BR[4, {1, 2, 3, 2, -1}] → {1, 0}
BR[4, {1, 3, -2, -2, -1}] → {1, 0}
BR[4, {1, 3, -2, -1, 3}] → {1, 0}
BR[4, {1, 3, 2, 2, -1}] → {1, 0}
BR[4, {1, 3, 3, -2, -1}] → {1, 0}
BR[4, {1, 3, 3, 2, -1}] → {1, 0}
BR[4, {2, -3, -3, 2, 3}] → {1, 0}
BR[4, {2, -1, -3, -3, -2}] → {1, 0}
BR[4, {2, -1, -1, -3, -2}] → {1, 0}
```

```
BR[4, {2, -1, -1, 2, 1}] → {1, 0}
BR[4, {2, -1, -1, 3, -2}] → {1, 0}
BR[4, {2, -1, 3, -2, 3}] → {0, 1}
BR[4, {2, -1, 3, 3, -2}] → {1, 0}
BR[4, {2, 1, -3, -3, -2}] → {1, 0}
BR[4, {2, 1, -3, -2, 1}] → {0, 1}
BR[4, {2, 1, 1, -3, -2}] → {1, 0}
BR[4, {2, 1, 1, 3, -2}] → {1, 0}
BR[4, {2, 1, 3, 3, -2}] → {1, 0}
BR[4, {3, -2, -2, 3, 2}] → {1, 0}
BR[4, {3, -2, -1, 3, 2}] → {0, 1}
BR[4, {3, -2, 1, -2, -3}] → {1, 0}
BR[4, {3, 2, -1, 2, -3}] → {1, 0}
```

In[•]:=
```
B1 = BR[4, {1, -2, 3, 2, -1}];
B2 = BR[4, {1, -2, 1, -3, 2}];
```

In[•]:=
```
BraidPlot /@ {B1, B2}
```

Out[•]=



In[•]:=
```
OPD /@ {B1, B2}
```

Out[•]=
```
{PD[X[5, 2, 6, 1], X[2, 8, 3, 9], X[11, 4, 12, 3], X[12, 10, 13, 9], X[6, 13, 7, 14]],
 PD[X[5, 2, 6, 1], X[2, 9, 3, 10], X[10, 7, 11, 6], X[3, 12, 4, 13], X[13, 8, 14, 7]]}
```

In[•]:=
```
Kas /@ OPD /@ {B1, B2}
```

Out[•]=

$$\left\{\begin{array}{c|cccccccc} & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ & (\eta_{-11} & \eta_4 & \eta_{10} & \eta_7 & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8} \\ \overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_4 & 0 & 0 & 0 & -1 & -u & 0 & u & 1 \\ \overline{\eta}_{10} & 0 & 0 & 0 & -u & 1-2u^2 & 0 & 2u^2-1 & u \\ \overline{\eta}_7 & 0 & -1 & -u & 2u^2-3 & -u & -1 & 0 & 1 \\ \overline{\eta}_{14} & 0 & -u & 1-2u^2 & -u & -1 & -u & -2(u-1)(u+1) & u \\ \overline{\eta}_{-1} & 0 & 0 & 0 & -1 & -u & 0 & u & 1 \\ \overline{\eta}_{-5} & 0 & u & 2u^2-1 & 0 & -2(u-1)(u+1) & u & 4u^2-3 & 0 \\ \overline{\eta}_{-8} & 0 & 1 & u & 1 & u & 1 & 0 & 1- \end{array}\right.$$

```
In[•]:= TL /@ OPD /@ {B1, B2}
```

Out[•]=

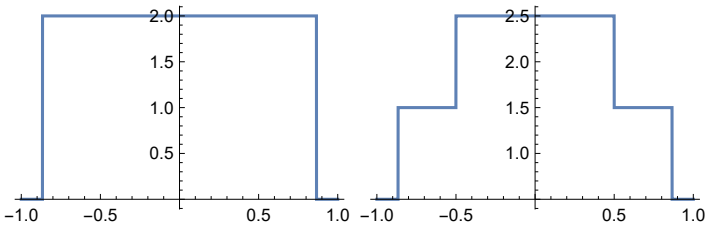
$$\left\{\begin{pmatrix} & 1 & 0 & -1 & 0 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 0 \\ & 0 & 0 & 0 & -1 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 1 \\ 0 & (\eta_{-11} & \eta_4 & \eta_{10} & \eta_7 & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8}) \\ \overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_4 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega^2} & 0 & -\frac{\omega-1}{\omega^2} & 0 \\ \overline{\eta}_{10} & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \overline{\eta}_7 & 0 & 0 & 0 & 0 & \frac{(\omega-1)^2}{\omega^2} & 0 & -\frac{(\omega-1)^2}{\omega^2} & 0 \\ \overline{\eta}_{14} & 0 & -((\omega-1)\,\omega) & \omega-1 & (\omega-1)^2 & 0 & -\frac{\omega-1}{\omega} & \frac{\omega-1}{\omega} & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 \\ \overline{\eta}_{-5} & 0 & (\omega-1)\,\omega & 1-\omega & -(\omega-1)^2 & 1-\omega & \frac{\omega-1}{\omega} & \frac{(\omega-1)^2}{\omega} & 0 \\ \overline{\eta}_{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\left\{\begin{pmatrix} & 1 & 0 & -1 & 1 & 0 & 0 & 0 & -1 \\ -1 & (\eta_{-11} & \eta_4 & \eta_{10} & \eta_7 & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8}) \\ \overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{\eta}_4 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & -\frac{\omega-1}{\omega} \\ \overline{\eta}_{10} & 0 & 0 & 0 & 1-\omega & 0 & 0 & 0 & \omega-1 \\ \overline{\eta}_7 & 0 & 1-\omega & \frac{\omega-1}{\omega} & \frac{2\,(\omega^2-\omega+1)}{\omega} & -\frac{\omega+1}{\omega} & 0 & \frac{2}{\omega} & -\frac{\omega^2-\omega+2}{\omega} \\ \overline{\eta}_{14} & 0 & 0 & 0 & -\omega-1 & \frac{\omega^2+1}{\omega} & -\frac{\omega-1}{\omega} & -\frac{2}{\omega} & 2 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 \\ \overline{\eta}_{-5} & 0 & 0 & 0 & 2\,\omega & -2\,\omega & \frac{\omega-1}{\omega} & \frac{\omega^2+1}{\omega} & -\omega-1 \\ \overline{\eta}_{-8} & 0 & \omega-1 & -\frac{\omega-1}{\omega} & -\frac{2\,\omega^2-\omega+1}{\omega} & 2 & 0 & -\frac{\omega+1}{\omega} & \frac{\omega^2+1}{\omega} \end{pmatrix}\right\}$$

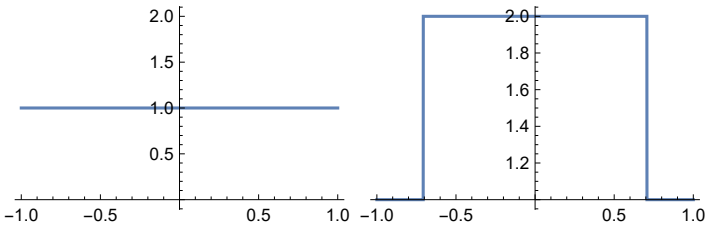
```
In[•]:= Grid@Table[
          f1 = TLSig[β]; f2 = KasSig[β];
          {β, Plot[f1, {u, -1, 1}], Plot[f2, {u, -1, 1}]},
          {β, Get["../../Projects/OU/Data/B_4_3.m"]}
         ]
```

Out[•]=

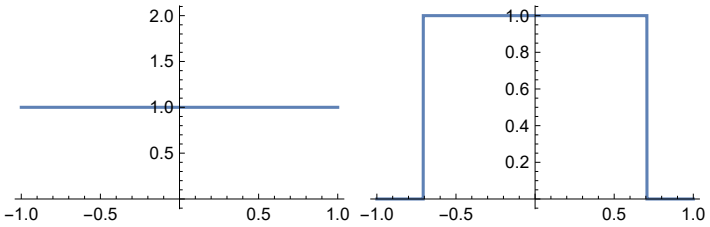
BR[4, {-3, -3, -3}]



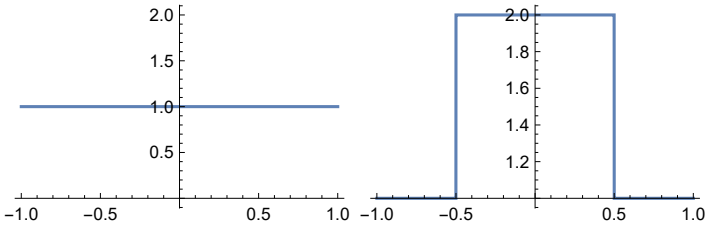
BR[4, {-3, -3, -2}]



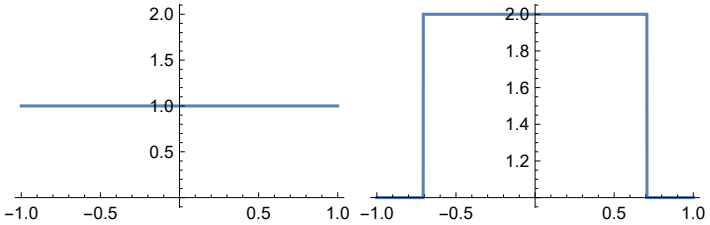
BR[4, {-3, -3, 2}]



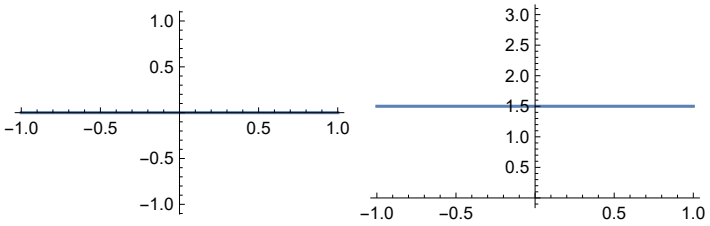
BR[4, {-3, -2, -3}]



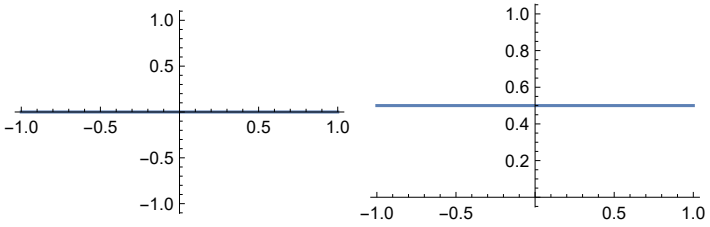
BR[4, {-3, -2, -2}]



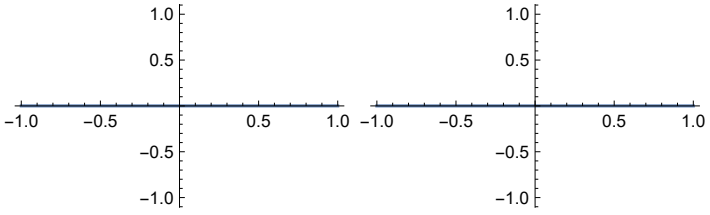
BR[4, {-3, -2, -1}]



BR[4, {-3, -2, 1}]



BR[4, {-3, -2, 3}]



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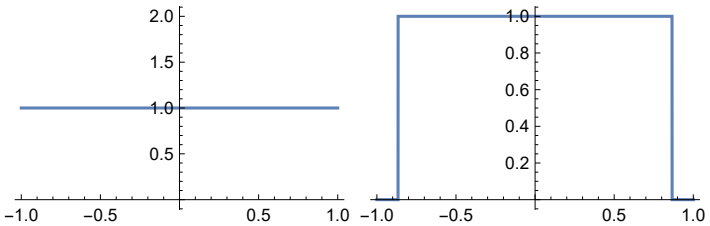
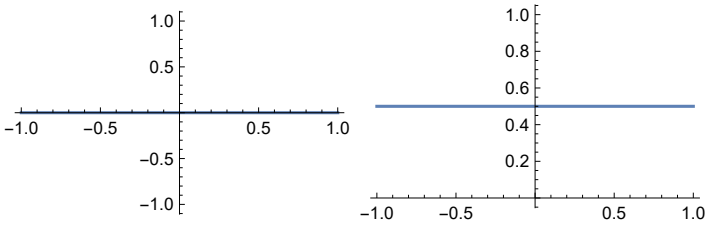


BR[4, {-3, 2, -1}]



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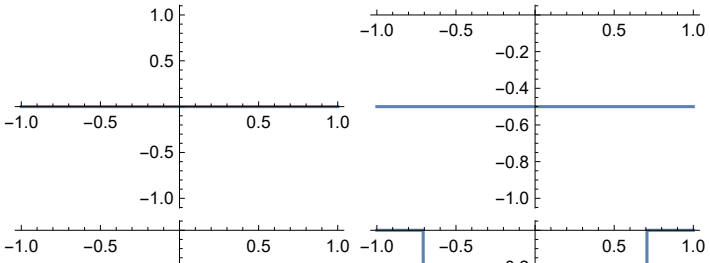
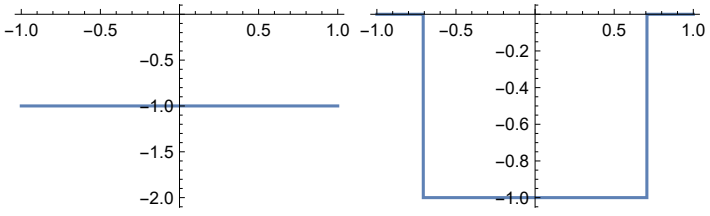


BR[4, {-3, 2, 2}]



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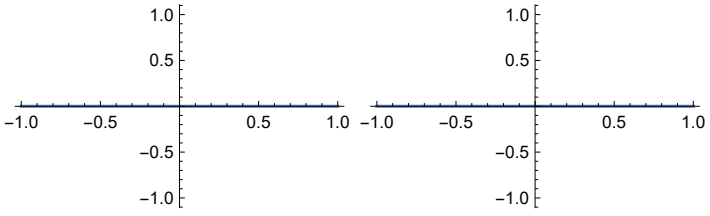
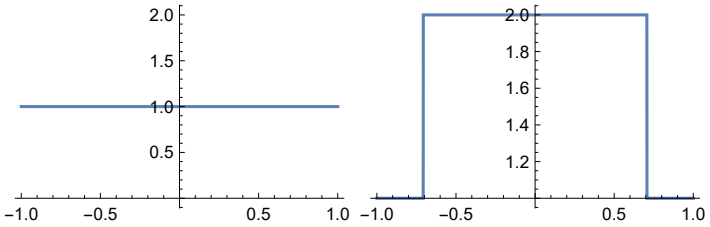


BR[4, {-2, -3, -3}]



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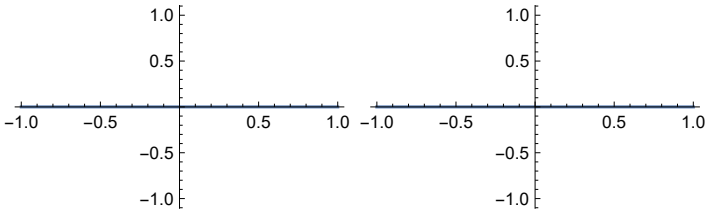

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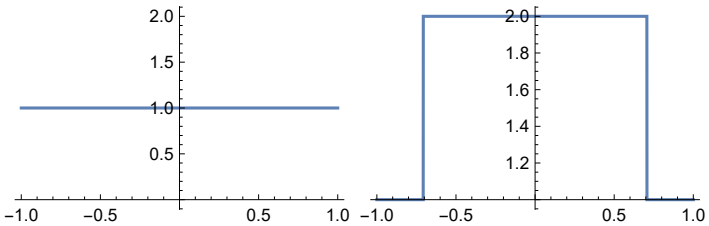
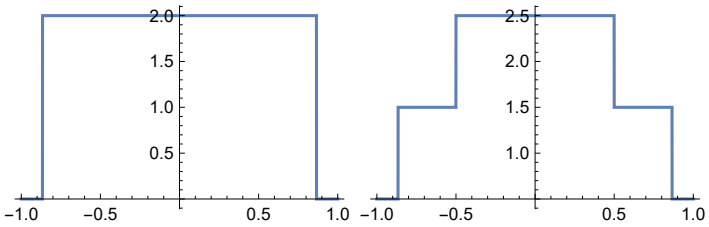


BR[4, {-2, -2, -2}]



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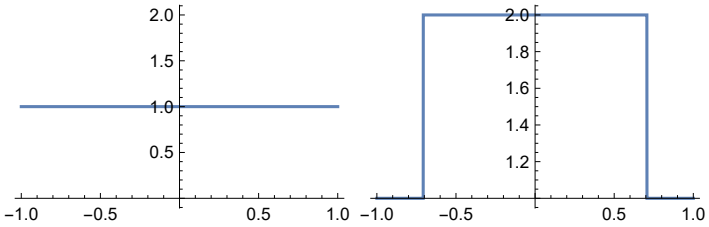

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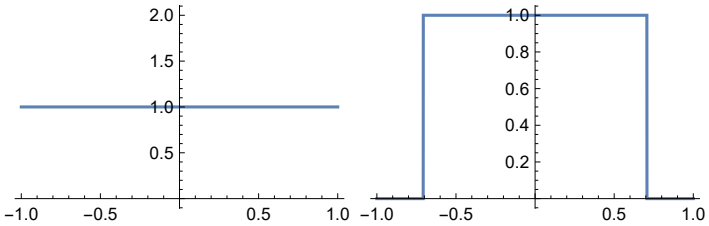

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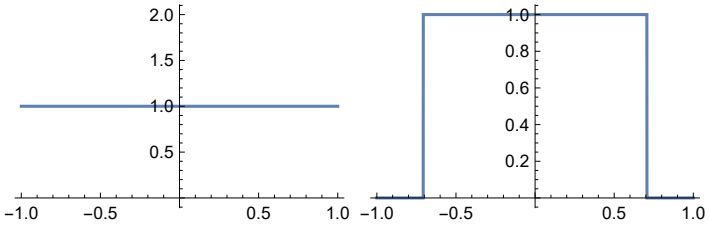

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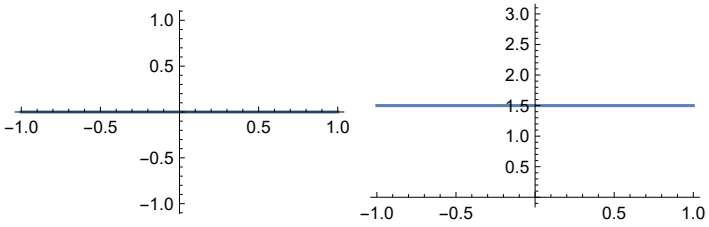

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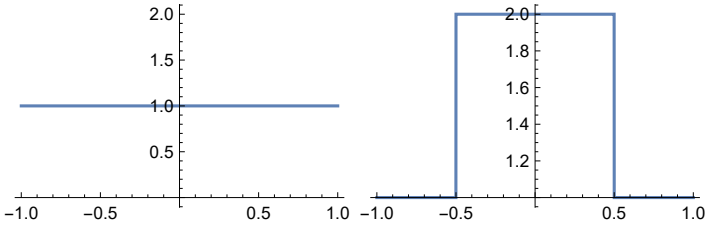

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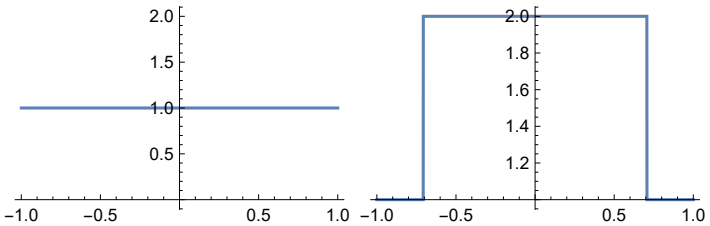
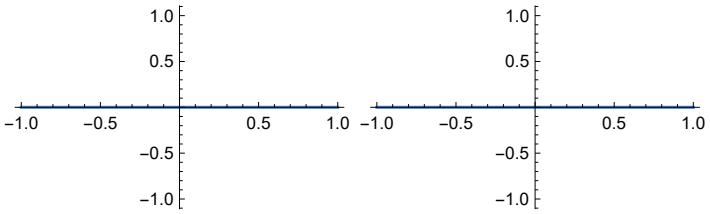


BR[4, {-2, -1, 2}]



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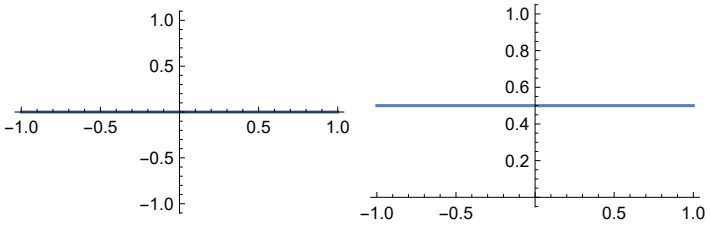

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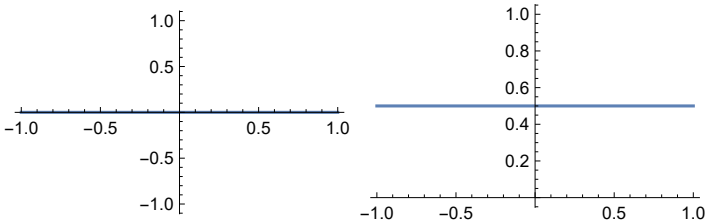

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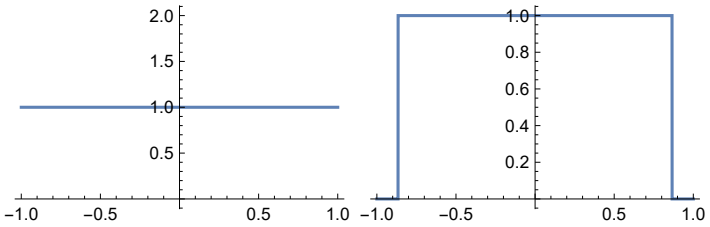
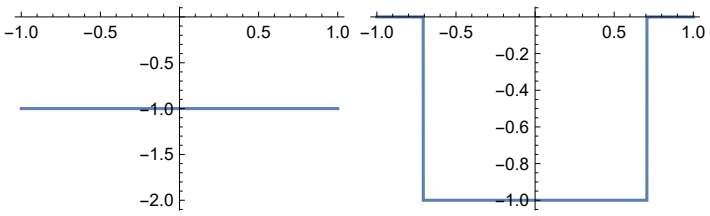
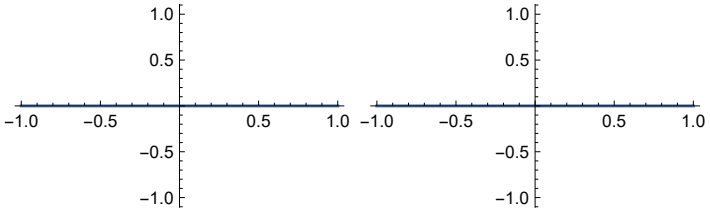


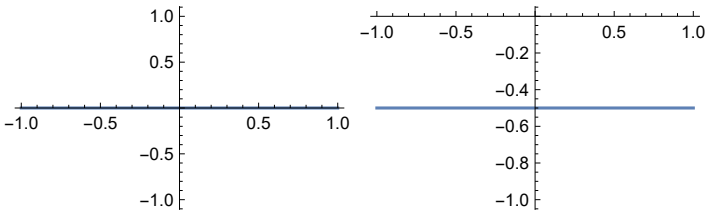
BR[4, {-2, 1, 1}]



BR[4, {-2, 1, 2}]



BR[4, {-2, 1, 3}]



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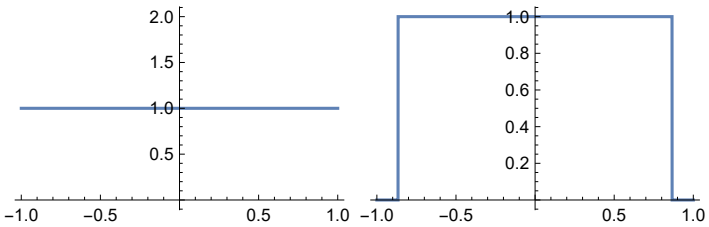

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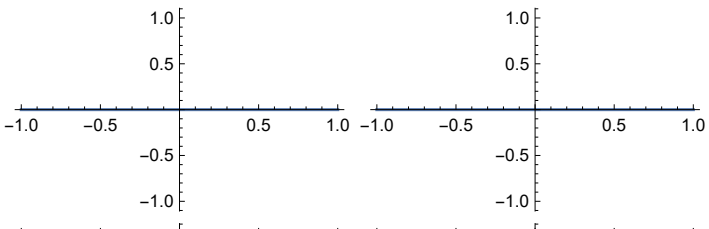

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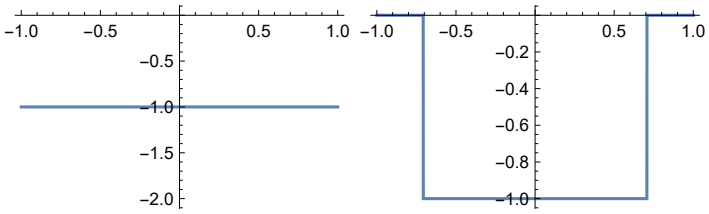
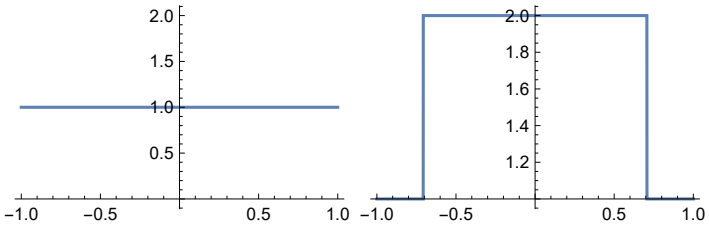


BR[4, {-1, -3, -3}]



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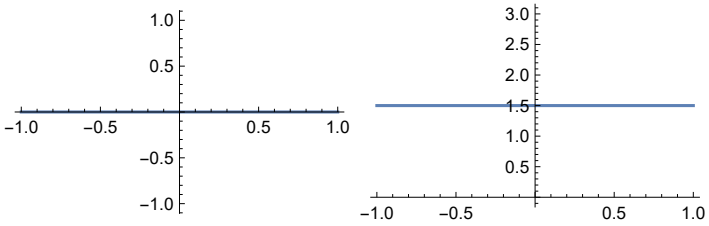

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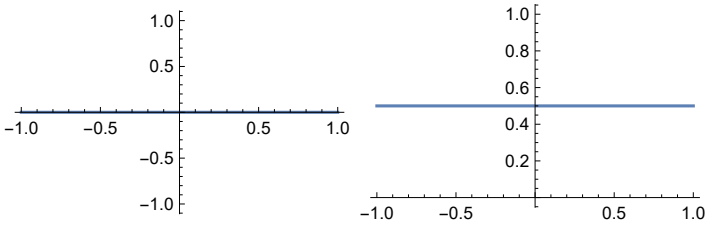

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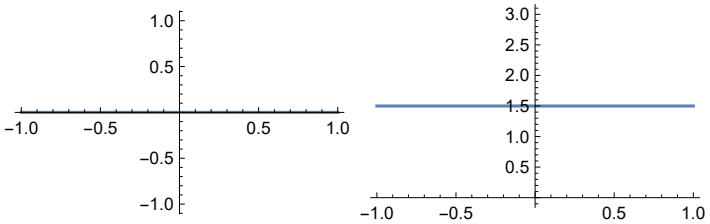

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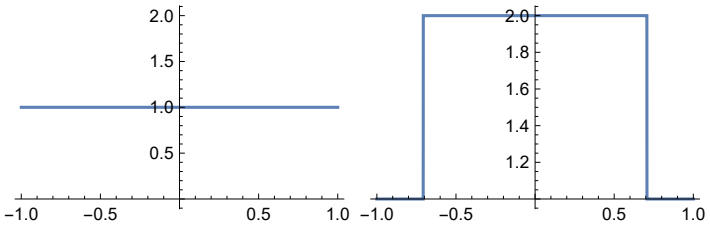

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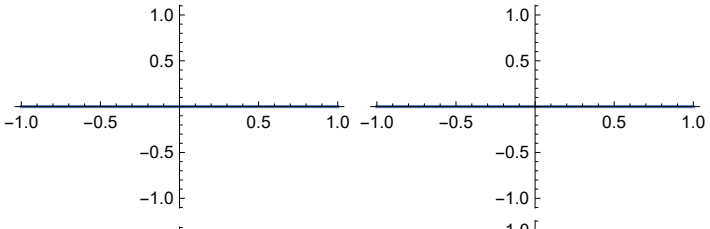
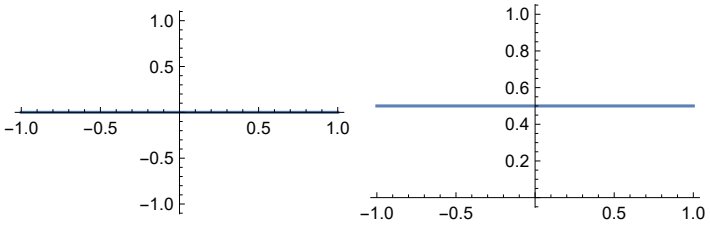
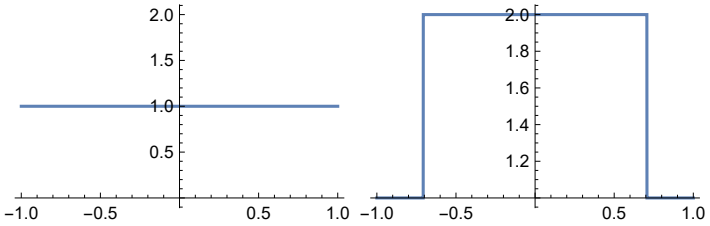


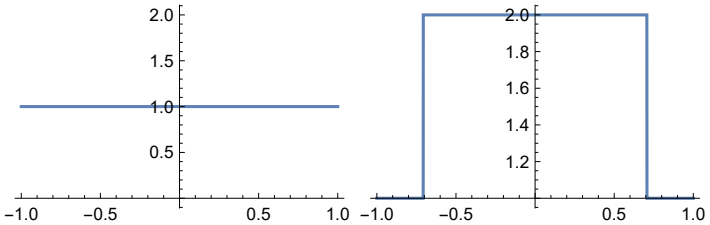
BR[4, {-1, -2, 3}]



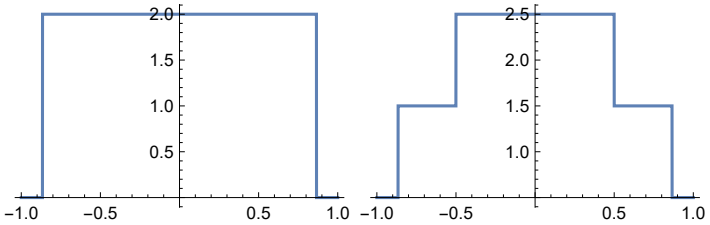
BR[4, {-1, -1, -3}]



BR[4, {-1, -1, -2}]



BR[4, {-1, -1, -1}]



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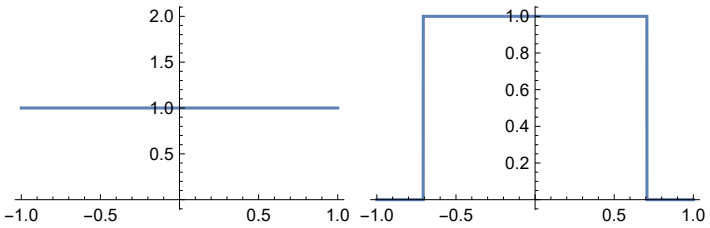

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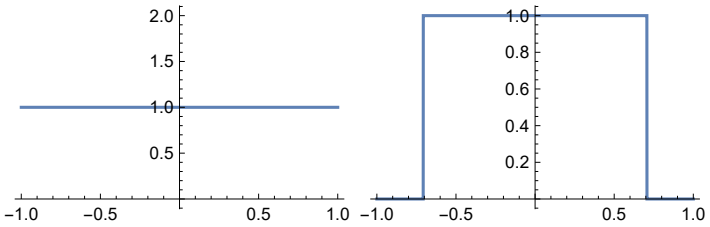

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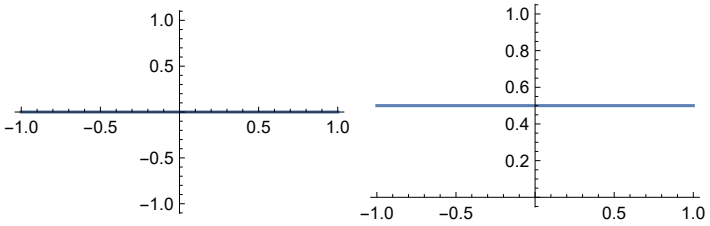

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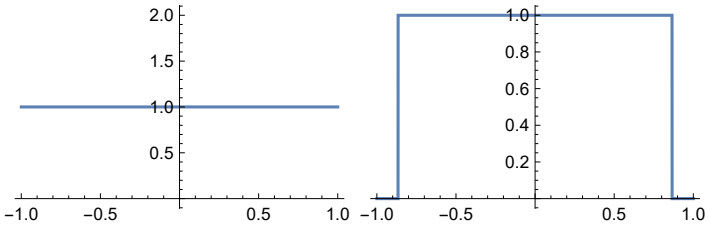

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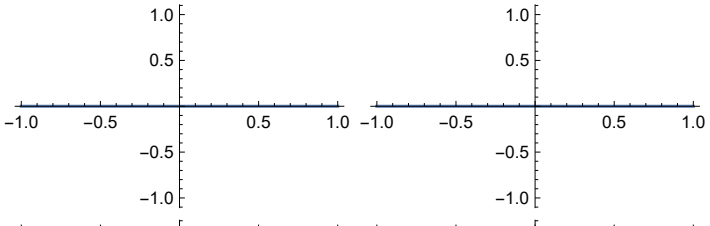

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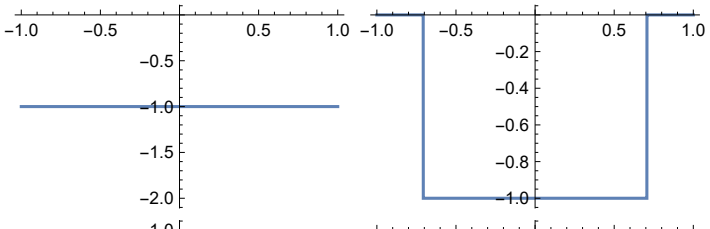

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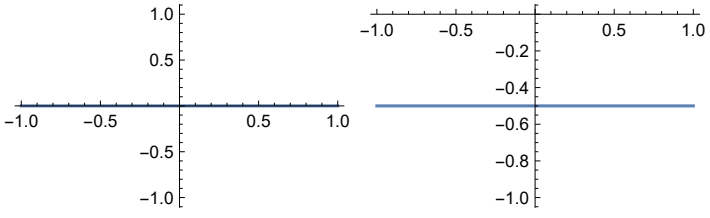

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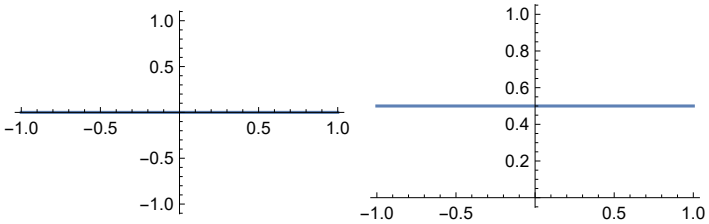

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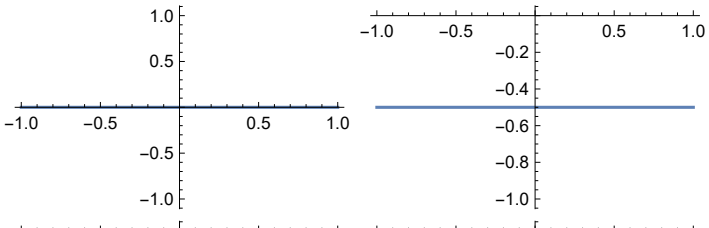

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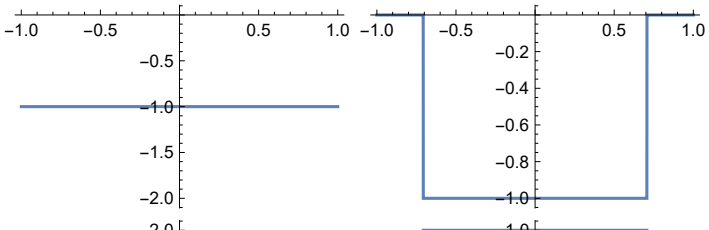

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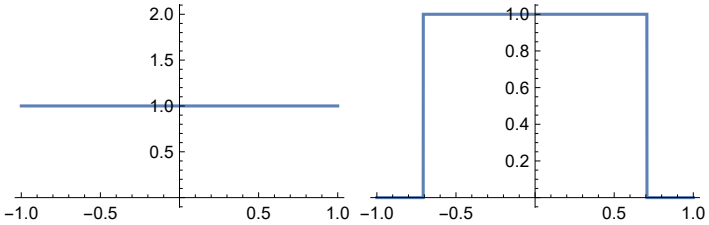

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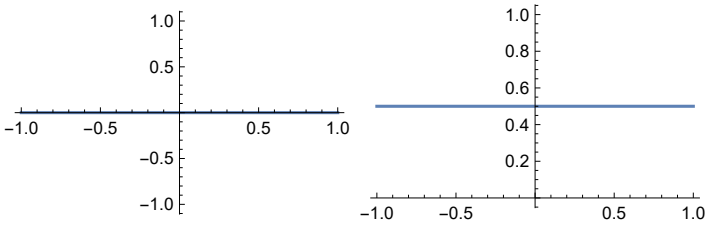

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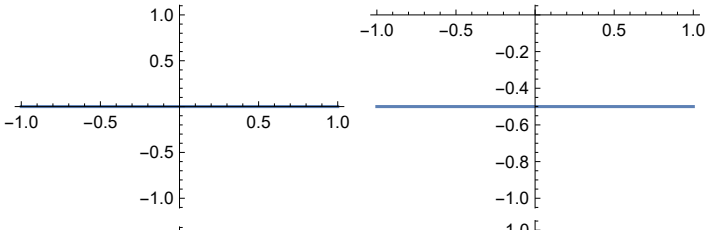

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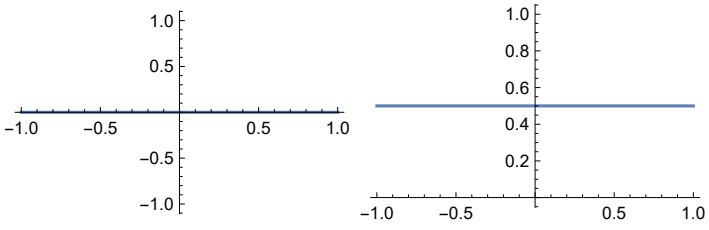

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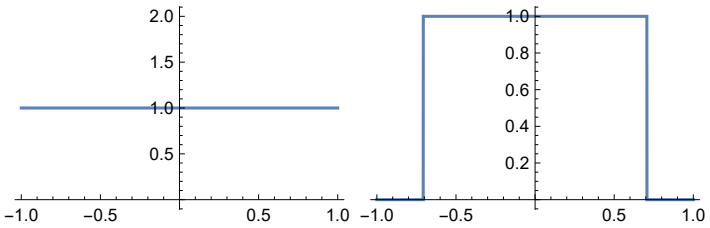

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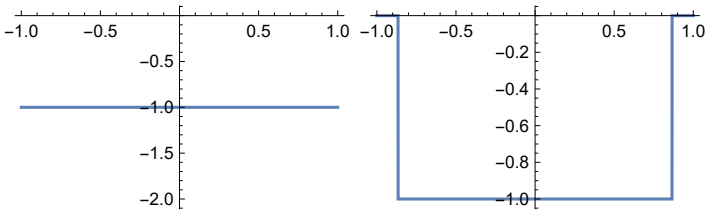

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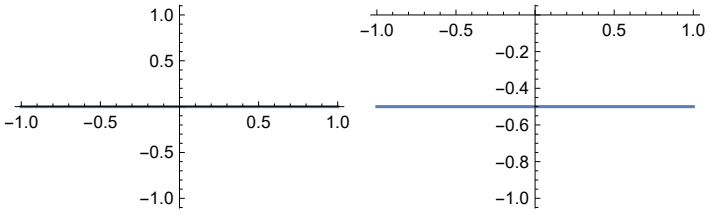

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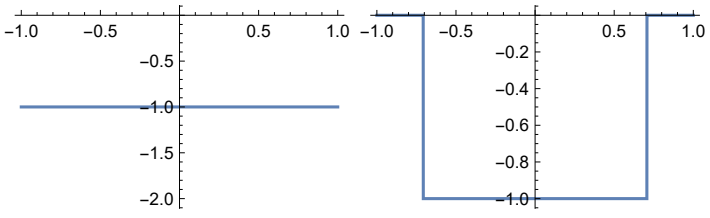

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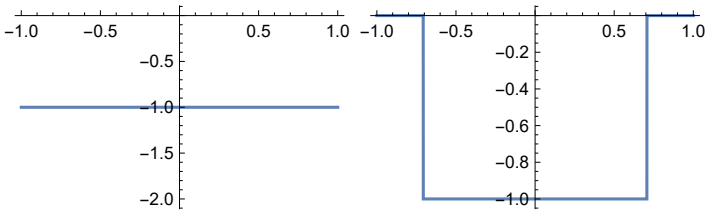

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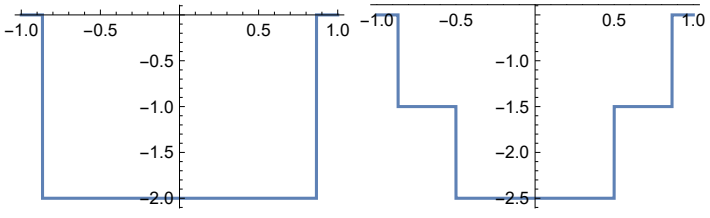

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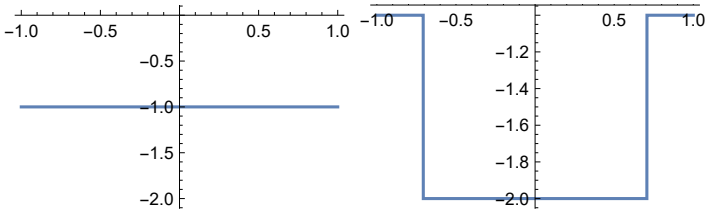

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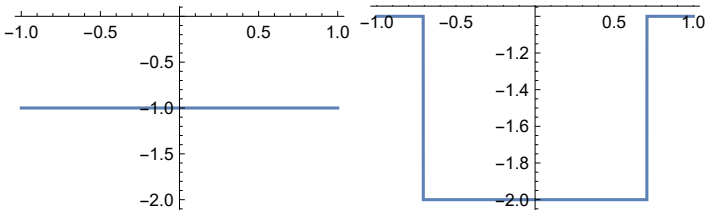

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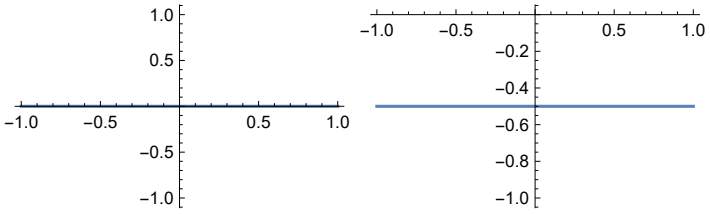

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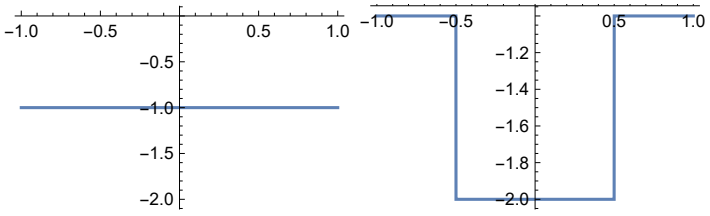

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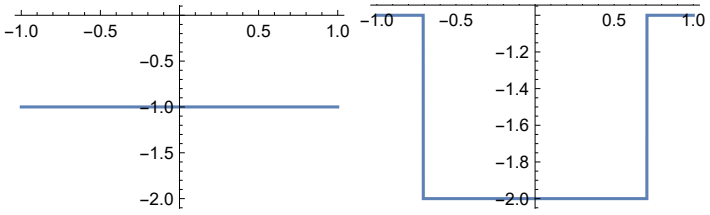

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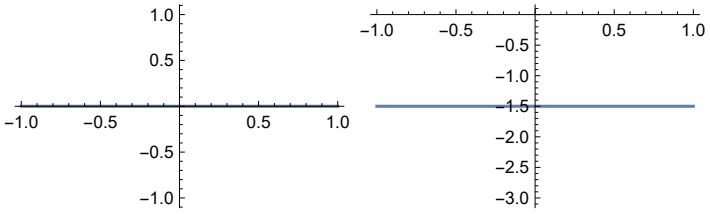

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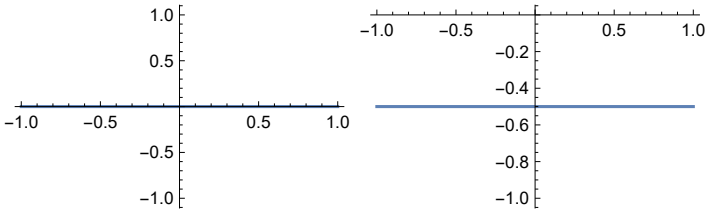

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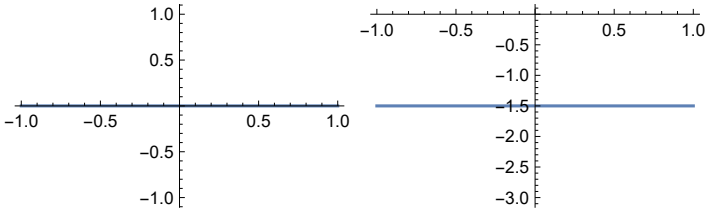

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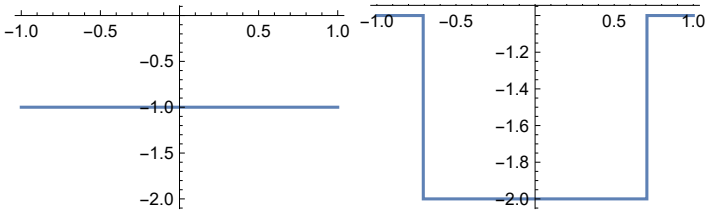

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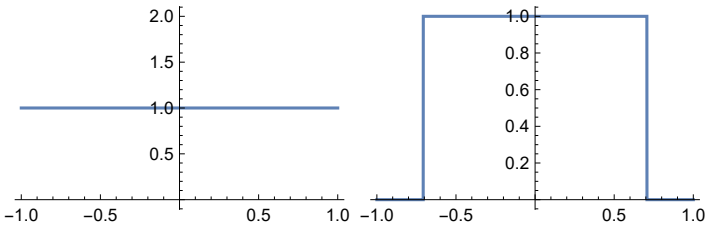

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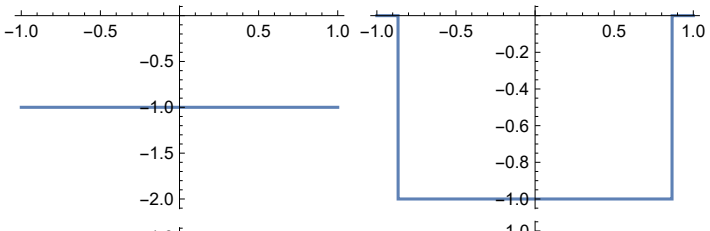

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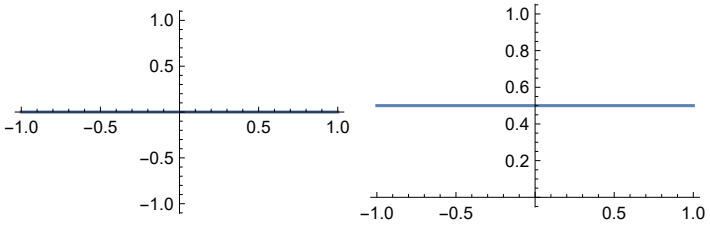

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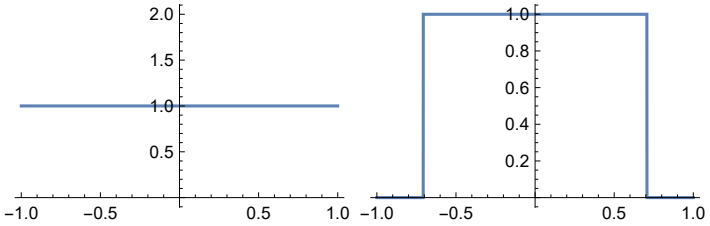

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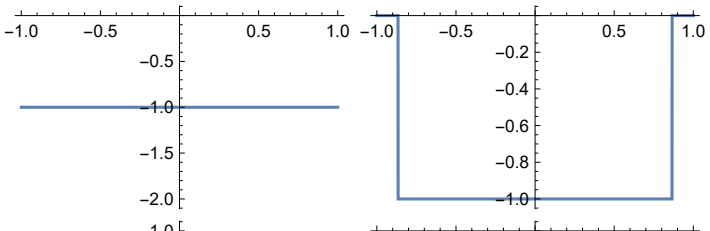

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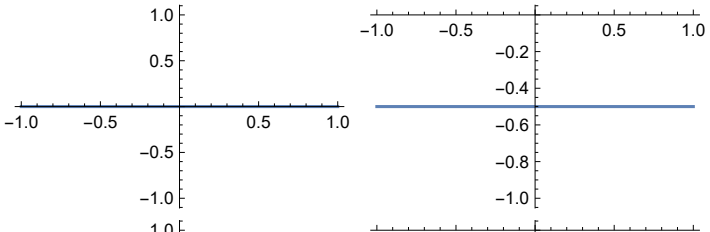

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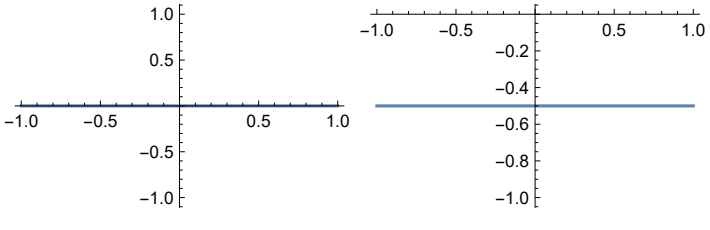

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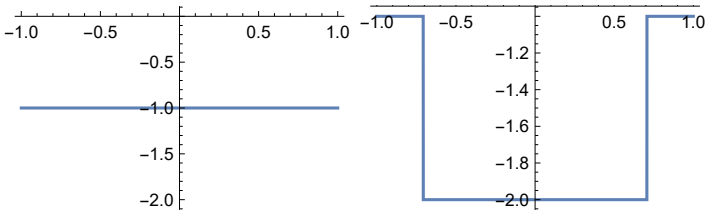

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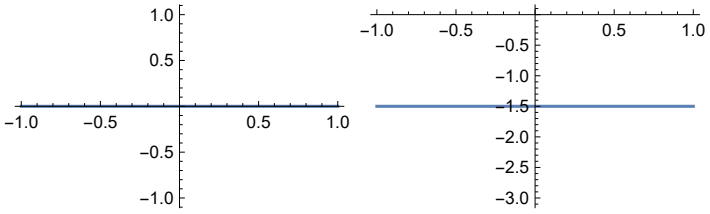

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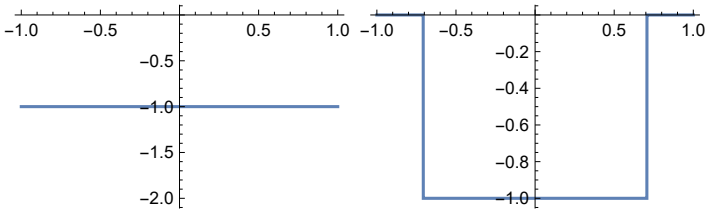

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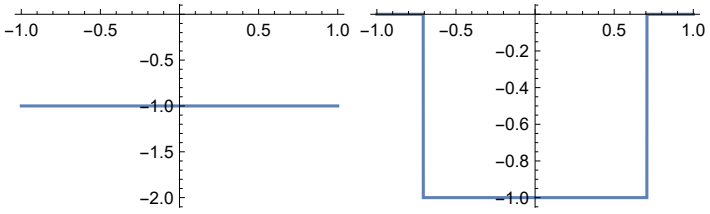

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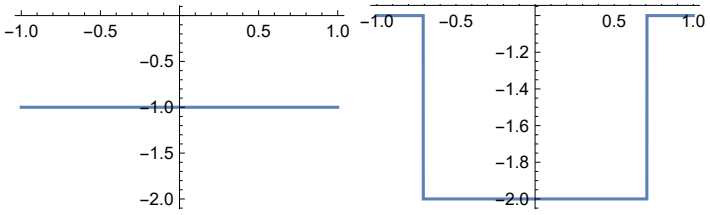

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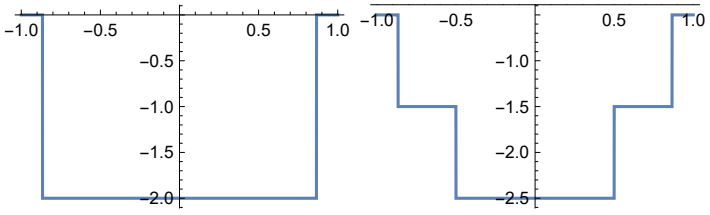

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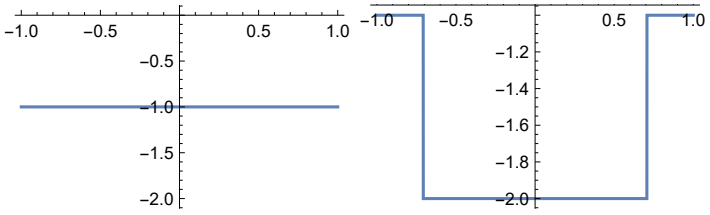

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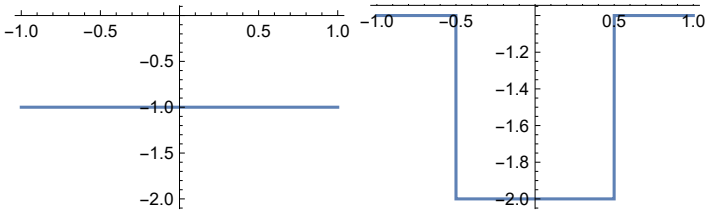

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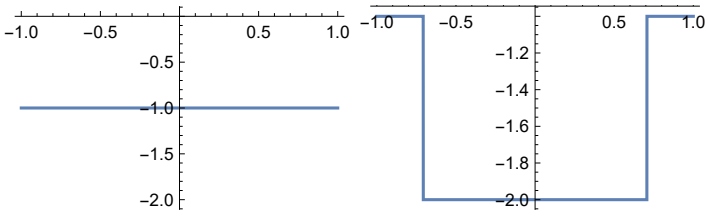

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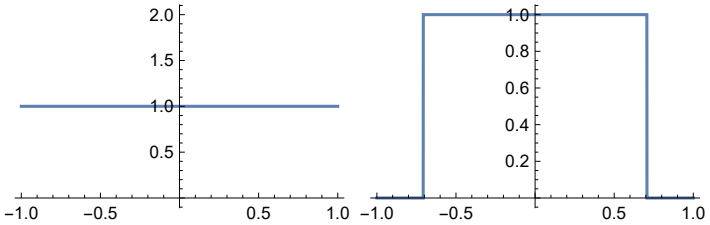

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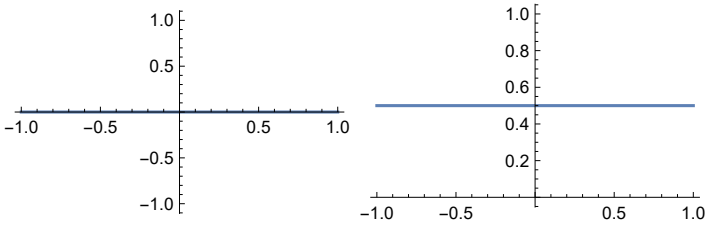

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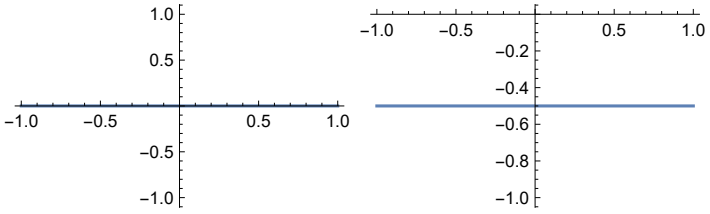

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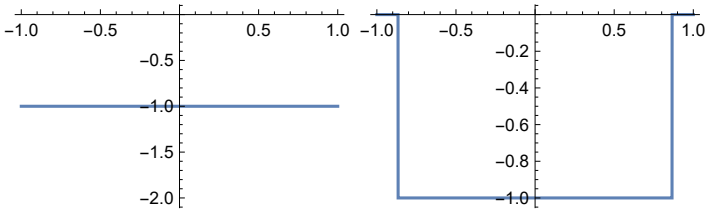

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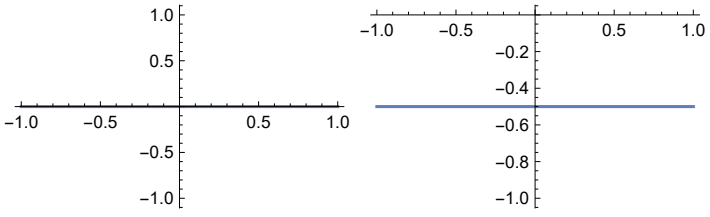

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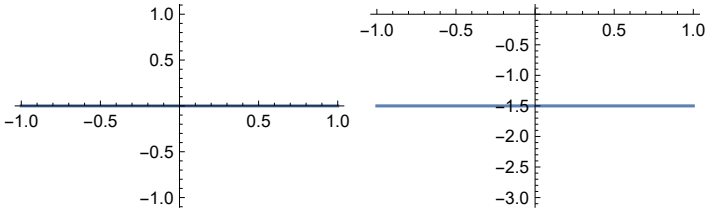

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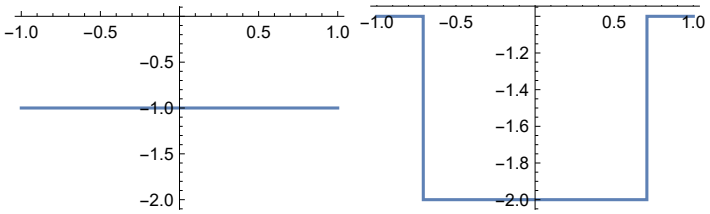

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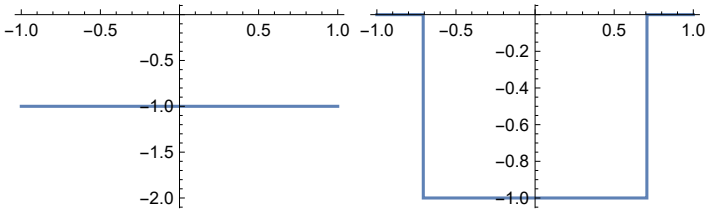

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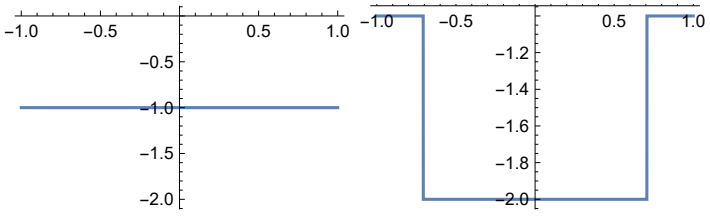
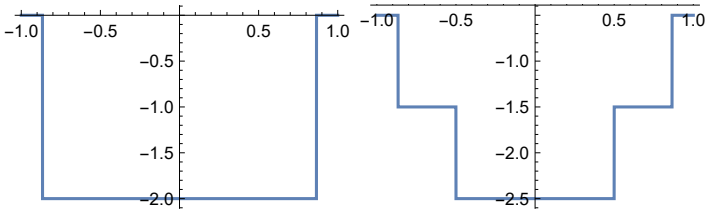
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