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Def. $Q_1 + Q_2$ is with $\mathcal{D}(Q_1 + Q_2) = \mathcal{D}(Q_1) \cap \mathcal{D}(Q_2)$.

Def. Given a linear $\psi\colon V \to W$ and a PQ Q on W, the pullback is $(\psi^* Q)(v_1, v_2) = Q(\psi v_1, \psi v_2)$ with $\mathcal{D}(\psi^* Q) = \phi^{-1}(\mathcal{D}(Q))$.

Def. Given $\phi\colon V \to W$ and a PQ Q on V the pushforward $\phi_* Q$ is with $\mathcal{D}(\phi_* Q) = \phi(\operatorname{ann}_Q(\operatorname{rad} Q|_{\ker\phi}))$ and $(\phi_* Q)(w_1, w_2) = Q(v_1, v_2)$, where v_i are s.t. $\phi(v_i) = w_i$ and $Q(v_i, \operatorname{rad} Q|_{\ker\phi}) = 0$.

Thm(?). ψ^* and ϕ_* are well-defined and functorial, and if $\alpha/\!\!/\beta = \gamma/\!\!/\delta$, then $\gamma^*/\!\!/\alpha_* = \delta_*/\!\!/\beta^*$. ψ^* is additive but ϕ_* isn't.

$$\begin{tikzcd} \bullet \arrow[r, "\alpha"] \arrow[d, "\gamma"'] & \bullet \arrow[d, "\beta"] \\ \bullet \arrow[r, "\delta"'] \arrow[ur, dotted] & \bullet \end{tikzcd}$$

Thm(?). Over $\mathbb{R}$, given $\phi\colon V \to W$ and PQs Q on V and C on W,

$$\operatorname{sign}_V(Q + \phi^* C) = \operatorname{sign}_{\ker\phi}(\iota^* Q) + \operatorname{sign}_W(C + \phi_* Q).$$

221228 **Missing.** A fully defined theory of pushing forward Gaussians (better with determinants and signatures).

In AP/People/LiuJ/PQ.nb:

*space

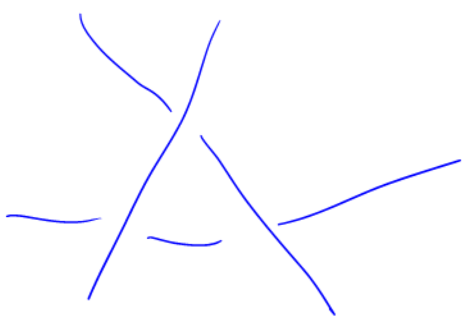
*subspace

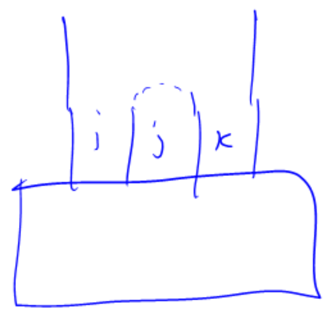
*map

* PQ

* ψ^*

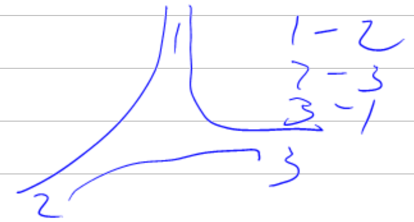
* ϕ_*

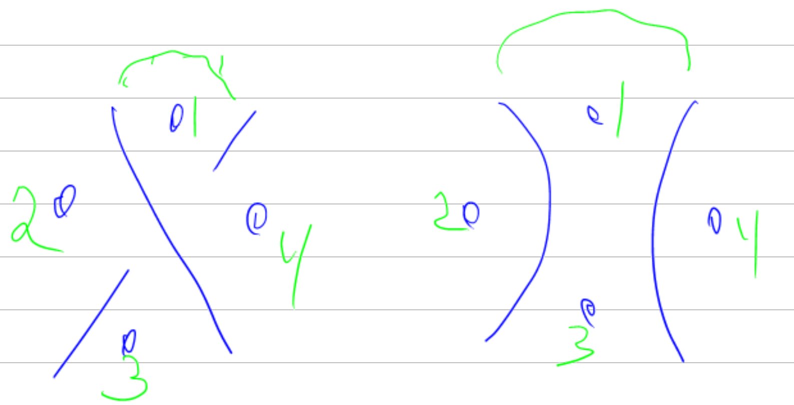




$$\begin{matrix} 1 & 0 & 0 & a_{11} & * & * & * \\ 0 & 1 & 0 & a_{12} & * & * & * \\ 0 & 0 & 1 & a_{13} & * & * & * \\ -a_{11} & -a_{12} & -a_{13} & 1 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 1 & -1 & 0 \\ 1 & 1 & \\ 0 & 0 & 1 \end{matrix}$$

$Q(-, V_i)$





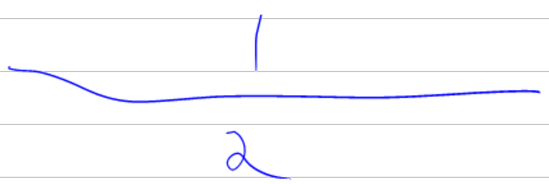
$1 = 3$

$PQ \circ$ on

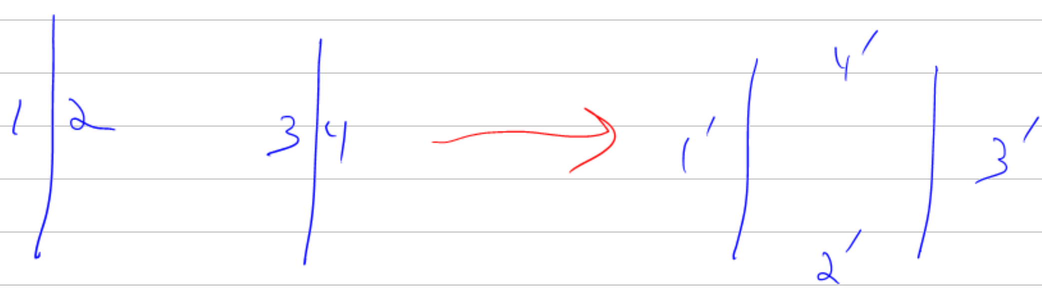
$V_1 \cdots V_4$

$\mathrm{dom} = \langle V_2, V_4, V_1 - V_3 \rangle$

push $V_1 \to 0$



$\longrightarrow$ $PQ \circ$ on 2D space



$(12)(34)$

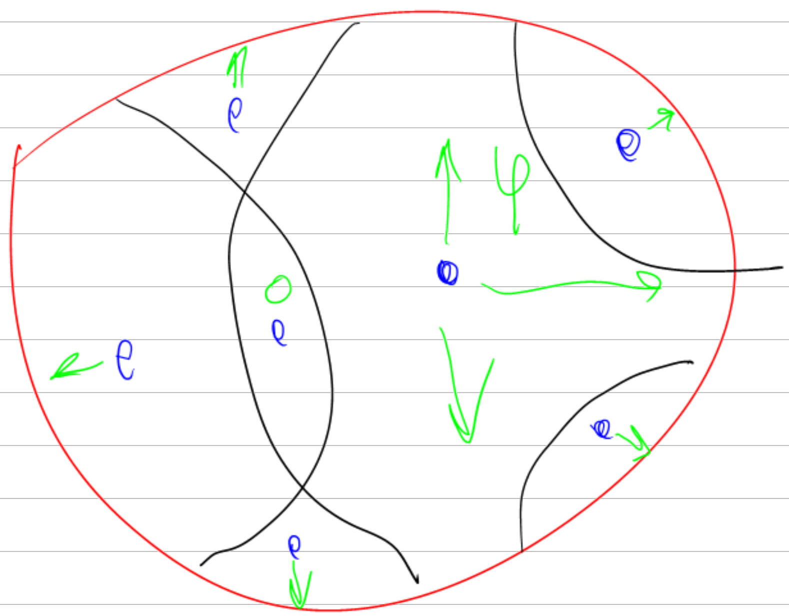
$1' \to 1$
$2' \to 2+3$
$3' \to 4$
$4' \to 2+3$

$2' + 4', 1', 3'$

$$y_1 \quad y_2 \qquad \eta_1 \quad \eta_2$$

$$\langle y_1 - y_2 \rangle^{\perp} = \langle \eta_1 + \eta_2 \rangle$$

$$\langle y_1 - y_2,\ y_2 - y_3,\ y_3 - y_1 \rangle^{\perp} = \langle \eta_1 + \eta_2 + \eta_3 \rangle$$



$\varphi_{\#} Q$

Jessica says:

- $\phi(\mathrm{ann}_Q(\mathrm{rad}(Q|_{\ker\phi\cap\mathcal{D}(Q)}))) = \phi(\mathrm{ann}_Q(\ker\phi\cap\mathcal{D}(Q)))$ $\neq \phi(\mathrm{rad}\, Q)$

  Proof. $\supseteq$ is clear. To prove $\subseteq$, let
  $$\ker\phi\cap\mathcal{D}(Q) \cong \mathrm{rad}(Q|_{\ker\phi\cap\mathcal{D}(Q)}) \oplus U$$
  for some complement U. Note that $u \mapsto Q(u,-)$ gives an isomorphism $U \cong U^*$. Thus for any $a \in \mathcal{D}(Q)$, there is some $u_a \in U$ such that $Q(a,U) = Q(u,U)$. If $a \in \mathrm{ann}_Q(\mathrm{rad}(Q|_{\ker\phi\cap\mathcal{D}(Q)}))$, then $a - u_a$ is in both $\mathrm{ann}_Q(\mathrm{rad}(Q|_{\ker\phi\cap\mathcal{D}(Q)}))$ and $\mathrm{ann}_Q(U)$, so $a - u_a \in \mathrm{ann}_Q(\ker\phi\cap\mathcal{D}(Q))$. Since $u_a \in \ker(\phi)$, we get $\phi(a) = \phi(u_a)$. □

Pf

$$\begin{array}{c}
\quad\ker\phi \qquad\qquad W \\
\left(\begin{array}{ccc|cc}
J & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & 0 \\
\hline
0 & I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \phi_*Q
\end{array}\right)
\end{array}$$

$J = \mathrm{diag}(\pm 1)$

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$T : V \longrightarrow W$

A COR is
Certificate of Rank

$$q = \sum a_{ij} \eta_i \eta_j \qquad \Phi = \sum \phi_{ij} \eta_i \omega_j$$

Need to compute $\Phi_* q = (D, Q)$

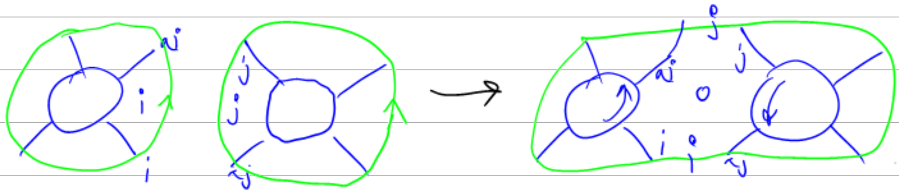
$$\begin{tikzcd}
\operatorname{Ker} A|_{k\Phi} \arrow[r] & \ker \Phi \arrow[r, "A|_{k\Phi}"'] \arrow[d, "j"] & (\ker \Phi)^* \\
 & V \arrow[r, "A"] \arrow[d, "\Phi"'] & V^* \arrow[u, "j^*"'] \\
 & W &
\end{tikzcd}
\qquad
\begin{tikzcd}
W \arrow[d] \\
V \otimes V^* \arrow[d, shift right] \\
F \arrow[u, shift right]
\end{tikzcd}$$

$$\begin{array}{c|cc}
0 & D & \\
\hline
D^T & A & B \\
 & B^T &
\end{array}$$

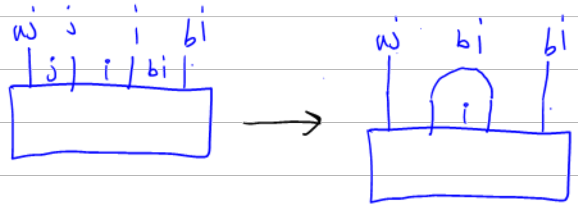
PQ: PQ[pivots, rels, q], w/ pivots: A list of the pivots of the rels. Always sorted.

rels: $\{\eta_i - \eta_j, 2\eta_7 - 3\eta_6\}$ always in RREF

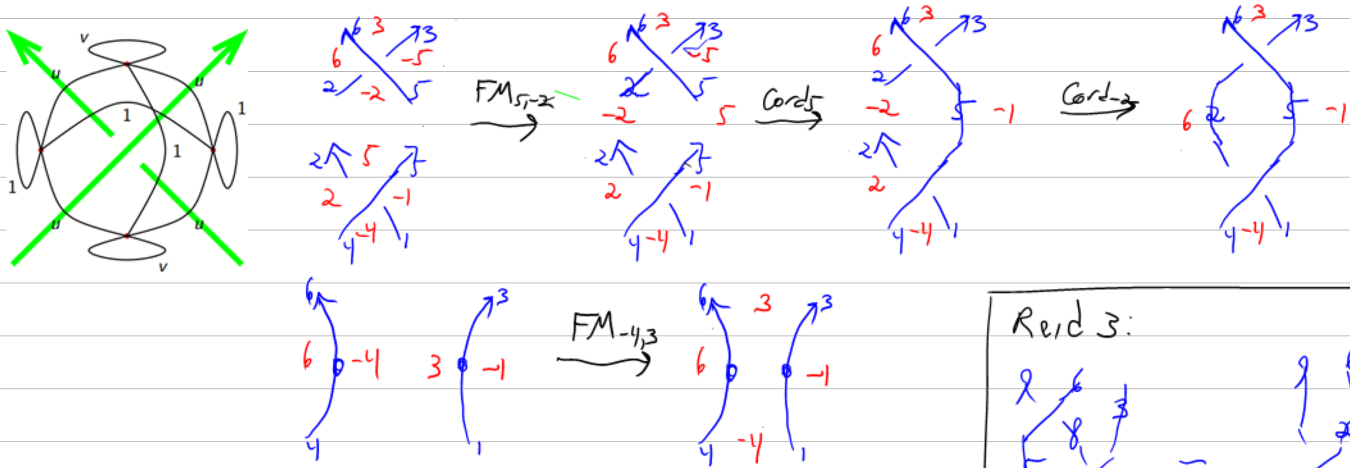
q: A quadratic containing no pivots.



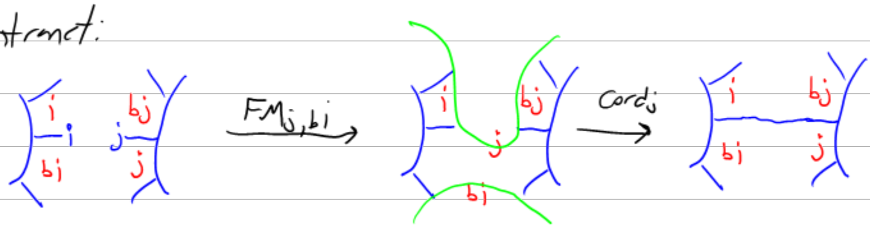
pull using $y_0 \to y_i + y_j$, then push using same. This is restriction!



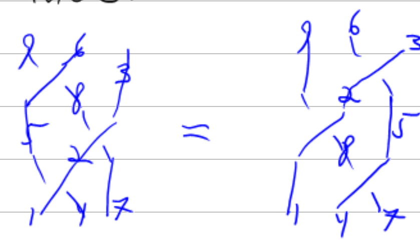
$\left.\begin{array}{l}\text{pull along } y_{bi} \mapsto y_{bi} + y_j \\ \text{push using } y_i \mapsto 0\end{array}\right\}$ commute!



Contract:



Reid 3:



Braids.

$$BR[5_2] = (3, \{-1, -1, -1, -2, 1, -2\})$$