Expressing signatures using Heavyside theta's at their jump points; also with ω.

```
In[ ]:=  ω2[v_][p_] := Module[{q = Expand[p], n, c},
           If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
             c v^n + ω2[v][q - c (ω + ω^-1)^n]]];
```

$$In[\ ]:= \quad \omega2[v]\left[-5 + \frac{1}{\omega^3} - \frac{3}{\omega^2} + \frac{5}{\omega} + 5\,\omega - 3\,\omega^2 + \omega^3\right]$$

Out[ ]=

$$1 + 2\,v - 3\,v^2 + v^3$$

```
In[ ]:=  Θ[x_] /; NumberQ[x] := HeavisideTheta[x];
         sign[ℰ_] := Module[{num, den, v, p, rs, d, k},
           {num, den} = NumeratorDenominator[ℰ]; {num, den} /= ω^Exponent[num,ω]/2;
           p = Factor[Times @@ (ω2[v] /@ {num, den}) /. v → 2 (2 u^2 - 1)];
           rs = Solve[p == 0, u, Reals];
           If[rs === {}, Return[Sign[p /. u → 0]]];
           rs = Union@(u /. rs);
           Sign[Coefficient[p, u, Exponent[p, u]]] (-1)^Exponent[p,u] + Sum[
             k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
             If[EvenQ[k], 0, 2 Sign[d]] Θ[u - r],
             {r, rs}]
         ]
```

$$In[\ ]:= \quad \mathbf{sign}\left[1 + u^2\right]$$

Out[ ]=

1

In[ ]:= $f = \text{Expand}\left[\frac{1}{2}\left(4 + \text{sign}\left[-2\left(-1+2u^2\right)\right] + \text{sign}\left[\frac{2}{3}\left(-3+4u^2\right)\right] + \text{sign}\left[\frac{-7+8u^2}{2\left(-3+4u^2\right)}\right] + \right.\right.$

$$\text{sign}\left[-\frac{2\left(8-23u^2+16u^4\right)}{-7+8u^2}\right] + \text{sign}\left[-\frac{\left(-3+4u^2\right)^2\left(11-28u^2+16u^4\right)}{-44+155u^2-176u^4+64u^6}\right] +$$

$$\text{sign}\left[\frac{-44+155u^2-176u^4+64u^6}{8-23u^2+16u^4}\right] + \text{sign}\left[-\frac{-11+76u^2-128u^4+64u^6}{11-28u^2+16u^4}\right] +$$

$$\text{sign}\left[-\frac{\left(-29+160u^2-256u^4+128u^6\right)\left(11-170u^2+544u^4-640u^6+256u^8\right)}{4\left(-1+2u^2\right)\left(-3+4u^2\right)^2\left(-11+76u^2-128u^4+64u^6\right)}\right] +$$

$$\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)}{11-228u^2+864u^4-1152u^6+512u^8}\right] +$$

$$\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)\left(11-228u^2+864u^4-1152u^6+512u^8\right)}{2\left(-29+160u^2-256u^4+128u^6\right)\left(11-170u^2+544u^4-640u^6+256u^8\right)}\right]\right)$$

$\left.\right]$

$\text{Plot}[f, \{u, -1, 1\}, \text{Exclusions} \to \text{None}]$

Out[ ]=

$$-2\,\Theta\left[-\frac{\sqrt{3}}{2}+u\right] + 2\,\Theta\left[\frac{\sqrt{3}}{2}+u\right] + 2\,\Theta\left[u - \boxed{\curvearrowright -0.970...}\right] - 2\,\Theta\left[u - \boxed{\curvearrowright -0.910...}\right] +$$

$$2\,\Theta\left[u - \boxed{\curvearrowright -0.480...}\right] - 2\,\Theta\left[u - \boxed{\curvearrowright 0.480...}\right] + 2\,\Theta\left[u - \boxed{\curvearrowright 0.910...}\right] - 2\,\Theta\left[u - \boxed{\curvearrowright 0.970...}\right]$$

Out[ ]=
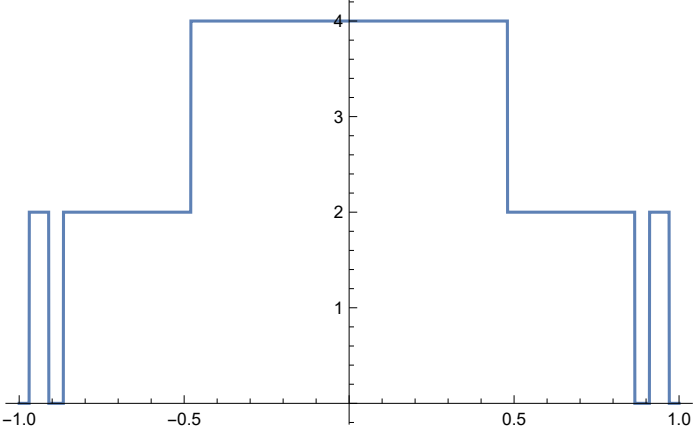
In[ ]:= $\text{sign}\left[\frac{1-3\omega+5\omega^2-5\omega^3+5\omega^4-3\omega^5+\omega^6}{\omega\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)}\right]$

Out[ ]=

$$1 - 2\,\Theta\left[u - \boxed{\curvearrowright -1.05...}\right] - 2\,\Theta\left[u - \boxed{\curvearrowright -0.630...}\right] + 2\,\Theta\left[u - \boxed{\curvearrowright 0.630...}\right] +$$

$$2\,\Theta\left[u - \boxed{\curvearrowright 1.05...}\right] + 2\,\Theta\left[u - \boxed{\curvearrowright -0.647...}\right] - 2\,\Theta\left[u - \boxed{\curvearrowright 0.647...}\right]$$

$In[\bullet]:=$ $\mathcal{E} = -\dfrac{\left(-29 + 160\,u^2 - 256\,u^4 + 128\,u^6\right)\,\left(11 - 170\,u^2 + 544\,u^4 - 640\,u^6 + 256\,u^8\right)}{4\,\left(-1 + 2\,u^2\right)\,\left(-3 + 4\,u^2\right)^2\,\left(-11 + 76\,u^2 - 128\,u^4 + 64\,u^6\right)}$

$Out[\bullet]=$

$-\dfrac{\left(-29 + 160\,u^2 - 256\,u^4 + 128\,u^6\right)\,\left(11 - 170\,u^2 + 544\,u^4 - 640\,u^6 + 256\,u^8\right)}{4\,\left(-1 + 2\,u^2\right)\,\left(-3 + 4\,u^2\right)^2\,\left(-11 + 76\,u^2 - 128\,u^4 + 64\,u^6\right)}$

$In[\bullet]:=$ **p = Expand[Numerator[$\mathcal{E}$] Denominator[$\mathcal{E}$]]**

$Out[\bullet]=$

$126\,324 - 4\,111\,536\,u^2 + 55\,241\,840\,u^4 - 418\,854\,464\,u^6 + 2\,037\,750\,016\,u^8 -$
$6\,802\,006\,016\,u^{10} + 16\,169\,902\,080\,u^{12} - 27\,900\,198\,912\,u^{14} + 35\,116\,810\,240\,u^{16} -$
$31\,970\,033\,664\,u^{18} + 20\,526\,923\,776\,u^{20} - 8\,824\,815\,616\,u^{22} + 2\,281\,701\,376\,u^{24} - 268\,435\,456\,u^{26}$

$In[\bullet]:=$ **roots = Union@(u /. Solve[p == 0, u, Reals])**

$Out[\bullet]=$

$\left\{-\dfrac{1}{\sqrt{2}},\ \dfrac{1}{\sqrt{2}},\ -\dfrac{\sqrt{3}}{2},\ \dfrac{\sqrt{3}}{2},\ \boxed{\circlearrowright\ -0.462\ldots},\ \boxed{\circlearrowright\ 0.462\ldots},\ \boxed{\circlearrowright\ -0.561\ldots},\right.$

$\left.\boxed{\circlearrowright\ 0.561\ldots},\ \boxed{\circlearrowright\ -0.787\ldots},\ \boxed{\circlearrowright\ -0.293\ldots},\ \boxed{\circlearrowright\ 0.293\ldots},\ \boxed{\circlearrowright\ 0.787\ldots}\right\}$

$In[\bullet]:=$ **Table[RootReduce[p /. u → r] > 0, {r, roots}]**

$Out[\bullet]=$

{False, False, False, False, False, False, False, False, False, False, False, False}

$In[\bullet]:=$ **RootReduce@Table[p /. u → r, {r, roots}]**

$Out[\bullet]=$

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

$In[\bullet]:=$ **Sum[**
  k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
  If[EvenQ[k], 0, Sign[d]] θ[r],
  {r, roots}]

$Out[\bullet]=$

$-\theta\left[-\dfrac{1}{\sqrt{2}}\right] + \theta\left[\dfrac{1}{\sqrt{2}}\right] - \theta\left[\boxed{\circlearrowright\ -0.462\ldots}\right] + \theta\left[\boxed{\circlearrowright\ 0.462\ldots}\right] + \theta\left[\boxed{\circlearrowright\ -0.561\ldots}\right] -$

$\theta\left[\boxed{\circlearrowright\ 0.561\ldots}\right] + \theta\left[\boxed{\circlearrowright\ -0.787\ldots}\right] + \theta\left[\boxed{\circlearrowright\ -0.293\ldots}\right] - \theta\left[\boxed{\circlearrowright\ 0.293\ldots}\right] - \theta\left[\boxed{\circlearrowright\ 0.787\ldots}\right]$

$In[\bullet]:=$ **KasSig[Knot[10, 1]]**

$Out[\bullet]=$

0

$In[\bullet]:=$ **Table[K → Expand@KasSig[K], {K, AllKnots[{3, 8}]}] // Column**

$Out[\bullet]=$

$\text{Knot}[3, 1] \to -2\,\theta\left[-\dfrac{\sqrt{3}}{2} + u\right] + 2\,\theta\left[\dfrac{\sqrt{3}}{2} + u\right]$

$\text{Knot}[4, 1] \to 0$

$\text{Knot}[5, 1] \to$
$\quad 2\,\theta\left[u - \boxed{\circlearrowright\ -0.951\ldots}\right] + 2\,\theta\left[u - \boxed{\circlearrowright\ -0.588\ldots}\right] - 2\,\theta\left[u - \boxed{\circlearrowright\ 0.588\ldots}\right] - 2\,\theta\left[u - \boxed{\circlearrowright\ 0.951\ldots}\right]$

$\text{Knot}[5, 2] \rightarrow -2\,\Theta\left[-\frac{\sqrt{\frac{7}{2}}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{\frac{7}{2}}}{2} + u\right]$

$\text{Knot}[6, 1] \rightarrow 0$

$\text{Knot}[6, 2] \rightarrow 2\,\Theta\left[u - \boxed{-0.772\ldots}\right] - 2\,\Theta\left[u - \boxed{0.772\ldots}\right]$

$\text{Knot}[6, 3] \rightarrow 0$

$\text{Knot}[7, 1] \rightarrow 2\,\Theta\left[u - \boxed{-0.975\ldots}\right] + 2\,\Theta\left[u - \boxed{-0.782\ldots}\right] +$

$\qquad 2\,\Theta\left[u - \boxed{-0.434\ldots}\right] - 2\,\Theta\left[u - \boxed{0.434\ldots}\right] - 2\,\Theta\left[u - \boxed{0.782\ldots}\right] - 2\,\Theta\left[u - \boxed{0.975\ldots}\right]$

$\text{Knot}[7, 2] \rightarrow -2\,\Theta\left[-\frac{\sqrt{\frac{11}{3}}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{\frac{11}{3}}}{2} + u\right]$

$\text{Knot}[7, 3] \rightarrow$

$\qquad -2\,\Theta\left[u - \boxed{-0.972\ldots}\right] - 2\,\Theta\left[u - \boxed{-0.656\ldots}\right] + 2\,\Theta\left[u - \boxed{0.656\ldots}\right] + 2\,\Theta\left[u - \boxed{0.972\ldots}\right]$

$\text{Knot}[7, 4] \rightarrow 2\,\Theta\left[-\frac{\sqrt{15}}{4} + u\right] - 2\,\Theta\left[\frac{\sqrt{15}}{4} + u\right]$

$\text{Knot}[7, 5] \rightarrow$

$\qquad 2\,\Theta\left[u - \boxed{-0.963\ldots}\right] + 2\,\Theta\left[u - \boxed{-0.757\ldots}\right] - 2\,\Theta\left[u - \boxed{0.757\ldots}\right] - 2\,\Theta\left[u - \boxed{0.963\ldots}\right]$

$\text{Knot}[7, 6] \rightarrow 2\,\Theta\left[u - \boxed{-0.920\ldots}\right] - 2\,\Theta\left[u - \boxed{0.920\ldots}\right]$

$\text{Knot}[7, 7] \rightarrow 0$

$\text{Knot}[8, 1] \rightarrow 0$

$\text{Knot}[8, 2] \rightarrow$

$\qquad 2\,\Theta\left[u - \boxed{-0.915\ldots}\right] + 2\,\Theta\left[u - \boxed{-0.529\ldots}\right] - 2\,\Theta\left[u - \boxed{0.529\ldots}\right] - 2\,\Theta\left[u - \boxed{0.915\ldots}\right]$

$\text{Knot}[8, 3] \rightarrow 0$

$\text{Knot}[8, 4] \rightarrow 2\,\Theta\left[u - \boxed{-0.745\ldots}\right] - 2\,\Theta\left[u - \boxed{0.745\ldots}\right]$

$\text{Knot}[8, 5] \rightarrow 2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\Theta\left[u - \boxed{-0.630\ldots}\right] + 2\,\Theta\left[u - \boxed{0.630\ldots}\right]$

$\text{Knot}[8, 6] \rightarrow 2\,\Theta\left[u - \boxed{-0.811\ldots}\right] - 2\,\Theta\left[u - \boxed{0.811\ldots}\right]$

$\text{Knot}[8, 7] \rightarrow -2\,\Theta\left[u - \boxed{-0.647\ldots}\right] + 2\,\Theta\left[u - \boxed{0.647\ldots}\right]$

$\text{Knot}[8, 8] \rightarrow 0$

$\text{Knot}[8, 9] \rightarrow 0$

$\text{Knot}[8, 10] \rightarrow 2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right]$

$\text{Knot}[8, 11] \rightarrow -2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right]$

$\text{Knot}[8, 12] \rightarrow 0$

$\text{Knot}[8, 13] \rightarrow 0$

$\text{Knot}[8, 14] \rightarrow 2\,\Theta\left[u - \boxed{-0.907\ldots}\right] - 2\,\Theta\left[u - \boxed{0.907\ldots}\right]$

$\text{Knot}[8, 15] \rightarrow -2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\Theta\left[-\frac{\sqrt{\frac{11}{3}}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{\frac{11}{3}}}{2} + u\right]$

$\text{Knot}[8, 16] \rightarrow 2\,\Theta\left[u - \boxed{-0.749\ldots}\right] - 2\,\Theta\left[u - \boxed{0.749\ldots}\right]$

$\text{Knot}[8, 17] \rightarrow 0$

$\text{Knot}[8, 18] \rightarrow 0$

$\text{Knot}[8, 19] \rightarrow 2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\Theta\left[u - \boxed{-0.966\ldots}\right] -$

$\qquad 2\,\Theta\left[u - \boxed{-0.259\ldots}\right] + 2\,\Theta\left[u - \boxed{0.259\ldots}\right] + 2\,\Theta\left[u - \boxed{0.966\ldots}\right]$

$$\text{Knot}[8, 20] \to 0$$

$$\text{Knot}[8, 21] \to -2\,\Theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\Theta\left[\frac{\sqrt{3}}{2} + u\right]$$

In[•]:= **Cos[5 ArcCos[x]] // TrigExpand**

Out[•]=

$$5\,x - 20\,x^3 + 16\,x^5$$