Pensieve header: A failed attempt to replace “-i” by “OverHat[i]”: saves horizontal but wastes vertical space, and make mc more complicated.

In[•]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
 << KnotTheory`;
]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

pdf

In[•]:=
```
ω2[v_][p_] := Module[{q = Expand[p], n, c},
   If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
    c v^n + ω2[v][q - c (ω + ω^-1)^n]]];
```

pdf

In[•]:=
```
θ[x_] /; NumericQ[x] := HeavisideTheta[x]
```

pdf

In[•]:=
```
sign[ℰ_] := Module[{num, den, v, p, rs, d, k},
  {num, den} = NumeratorDenominator[ℰ]; {num, den} /= ω^(Exponent[num,ω]/2);
  p = Factor[Times @@ (ω2[v] /@ {num, den}) /. v → 4 u^2 - 2];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Return[Sign[p /. u → 0]]];
  rs = Union@(u /. rs);
  Sign[Coefficient[p, u, Exponent[p, u]]] (-1)^Exponent[p,u] + Sum[
    k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
    If[EvenQ[k], 0, 2 Sign[d]] θ[u - r],
    {r, rs}] ]
```

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In[•]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

In[•]:=
```
CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_ | η̄_, ∞]},
  Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) :→ Factor[c] Times @@ ηs^ps] ]
```

pdf

In[•]:=
```
CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs] ]
```

pdf

In[∘]:=
```
(ℰ_)* := ℰ /. {η̄ → η, η → η̄, ω → ω^-1, c_Complex :→ c*};
r_Rule⁺ := {r, r*}
```

pdf

In[∘]:=
```
RulesOf[η_i_ + rest_.] := (η_i → -rest)⁺;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

pdf

In[∘]:=
```
CF[Σ_b_[σ_, pq_]] := Σ_CF[b][σ, CF[pq]]
```

pdf

In[∘]:=
```
Format[Σ_b_B[σ_, PQ[rs_, q_]]] := Module[{ηs},
   ηs = η_# & /@ Join @@ b;
   Column[{TraditionalForm@σ,
     TableForm[Join[
       Prepend[""] /@ Table[TraditionalForm[∂_c r], {r, rs}, {c, ηs}],
       {Prepend[""][
         Join @@ (b /. {l_, m___, r_} :→ {DisplayForm@RowBox[{"(", l}],
              m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer :→ η_i ]},
       MapThread[Prepend, {Table[TraditionalForm[∂_r,c q], {r, ηs*}, {c, ηs}], ηs*}]
      ], TableAlignments → Center]
    }, Center] ];
```

pdf

In[∘]:=
```
Σ_b1_[σ1_, PQ[rs1_, q1_]] ⋃ Σ_b2_[σ2_, PQ[rs2_, q2_]] ^:=
  CF@Σ_Join[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

\par Gaps are named after the strand that follows them!

\par FM for Face Merge:

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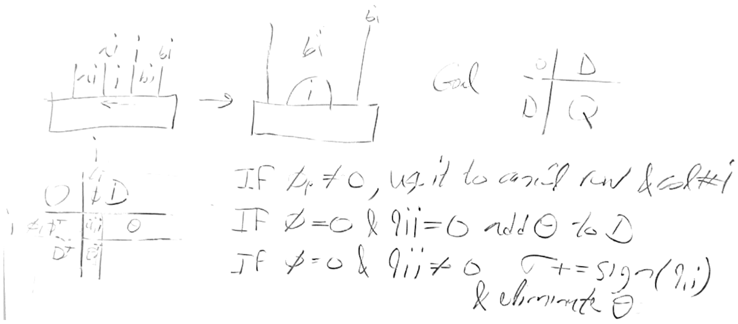
In[∘]:=
```
FM_i_,j_@Σ_B[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[rs_, q_]] :=
 CF@Σ_B[{ri,li,j,rj,lj,i},bs][σ, PQ[rs ⋃ {η_i - η_j}, q]]
```

pdf



pdf

```
In[∘]:= Cordon_i_ @ Σ_B[{li___, i_, ri___}, bs___][σ_, PQ[rs_, q_]] :=
  Module[{ϕ = ∂_{η_i} rs, nσ = σ, nrs = rs, nq = q, qii, p},
   Which[
    Or @@ ((# =!= 0) & /@ ϕ), ({p} = FirstPosition[(# === 0) & /@ ϕ, False];
     {nrs, nq} = {rs, q} /. (η_i → -rs⟦p⟧ / ϕ⟦p⟧)^+ /. (η_i → 0)^+),
    (qii = ∂_{η̄_i, η_i} q) =!= 0, (nσ += sign[qii];
     nq = q /. (η_i → -(∂_{η̄_i} q) / qii)^+ /. (η_i → 0)^+),
    qii === 0, AppendTo[nrs, ∂_{η̄_i} q]; nq = q /. (η_i → 0)^+];
   CF @ Σ_B[Most@{ri, li}, bs][nσ, PQ[nrs, nq] /. (η_Last@{ri, li} → η_First@{ri, li})^+] ]
```

tex

\par c for contract:

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```
In[∘]:= c_{i_, j_} @ t : Σ_B[{li___, i_, ri___}, {___, j_, ___}, ___][__] := t // FM_{j, First@{ri, li}} // Cordon_j
```

pdf

```
In[∘]:= c_{i_, j_} @ t : Σ_B[{___, i_, j_, ___}, ___][__] := Cordon_j @ t
c_{i_, j_} @ t : Σ_B[{j_, ___, i_}, ___][__] := Cordon_j @ t
c_{i_, j_} @ t : Σ_B[{___, j_, i_, ___}, ___][__] := Cordon_i @ t
c_{i_, j_} @ t : Σ_B[{i_, ___, j_}, ___][__] := Cordon_i @ t
```

tex

\par mc for magnetic contract:

pdf

```
In[∘]:= mc[ℰ_] := ℰ //.
   t : Σ_B[{___, i_, ___}, {___, î_, ___}, ___][__] |
     Σ_B[{___, i_, î_, ___}, ___][__] | Σ_B[{î_, ___, i_}, ___][__] ⧴ 0 ⧴ c_{i, î} @ t
```

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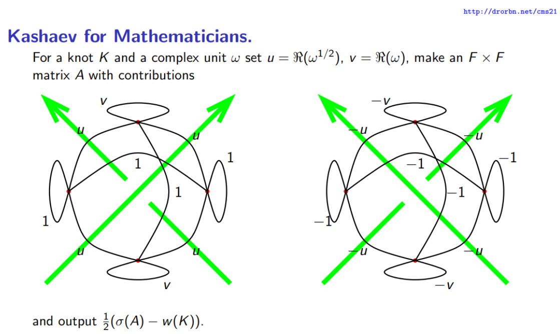
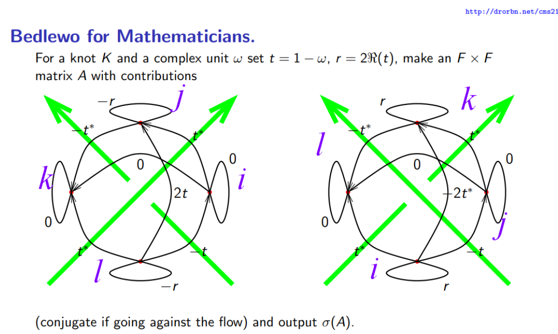
```
In[∘]:= Kas[P[i_, j_]] := CF @ Σ_B[{î, j}][0, PQ[{}, 0]];
TL[P[i_, j_]] := CF @ Σ_B[{î, j}][0, PQ[{}, 0]]
```





pdf

```
In[∘]:= Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x],
          X₊[î, j, k, l̂], X₋[ĵ, k, l, î]];
        Kas[x : (X₊ | X₋)[__]] := Module[{v = 2 u^2 - 1, p, fs, ηs, m},
          ηs = η_# & /@ (fs = List @@ x); p = Head[x] === X₊;
          m = If[p, {{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}},
                 -{{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}}];
          CF@Σ_B[fs][If[p, -1, 1], PQ[{}, ηs*.m.ηs]]]
```

pdf

```
In[∘]:= TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x],
          X₊[î, j, k, l̂], X₋[ĵ, k, l, î]];
        TL[x : (X₊ | X₋)[__]] := Module[{t = 1 - ω, r, p, fs, ηs, m},
          r = t + t*; ηs = η_# & /@ (fs = List @@ x); p = Head[x] === X₊;
          m = If[p, {{-r, -t, 2 t, t*}, {-t*, 0, t*, 0}, {2 t*, t, -r, -t*}, {t, 0, -t, 0}},
                 {{r, -t, -2 t*, t*}, {-t*, 0, t*, 0}, {-2 t, t, r, -t*}, {t, 0, -t, 0}}];
          CF@Σ_B[fs][0, PQ[{}, ηs*.m.ηs]]]
```

pdf

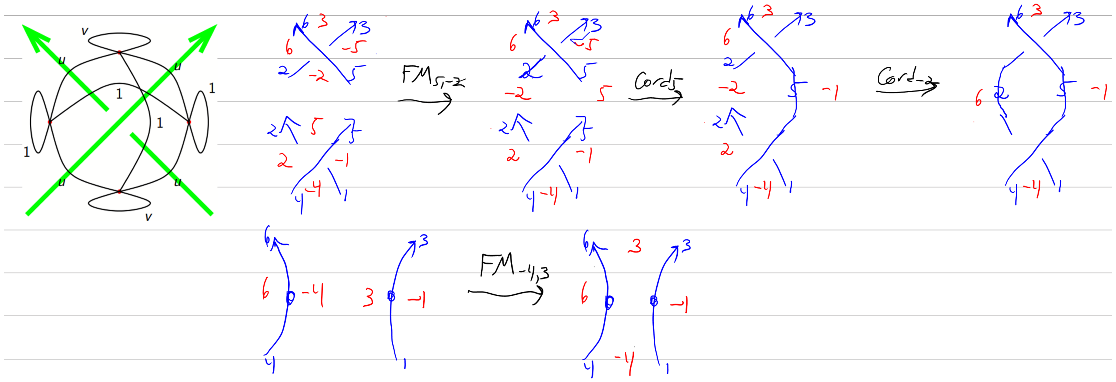
```
In[∘]:= Kas[K_] := Fold[mc[#1 ⋃ #2] &, Σ_B[][0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
        KasSig[K_] := Expand[Kas[K][[1]] / 2]
```

pdf

```
In[∘]:= TL[K_] := Fold[mc[#1 ⋃ #2] &, Σ_B[][0, PQ[{}, 0]], List @@ (TL /@ PD@K)] /.
          θ[c_ + u] /; Abs[c] ≥ 1 :> θ[c];
        TLSig[K_] := TL[K][[1]]
```

pdf

Reidemeister 2



pdf

In[∘]:=
```
{FM_{6,1̂}[TL[P[1, 3]] ⋃ TL[P[4, 6]]] == mc[TL[X[1, 5, 2, 4]] ⋃ TL[X[2, 5, 3, 6]]],
 FM_{6,1̂}[Kas[P[1, 3]] ⋃ Kas[P[4, 6]]] == mc[Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]]]}
```

Out[∘]=
pdf

```
{True, True}
```

pdf

Reidemeister 3

pdf

In[∘]:=
```
R3L = PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]];
R3R = PD[X[7, 5, 8, 4], X[8, 2, 9, 1], X[5, 3, 6, 2]];
{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R}
```

Out[∘]=
pdf

```
{True, True}
```

pdf

In[∘]:=
```
Kas@R3L
```

Out[∘]=
pdf

$$2\,\Theta\left(u-\frac{1}{2}\right)-2\,\Theta\left(u+\frac{1}{2}\right)-2$$

	$(\eta_3$	η_6	η_9	$\hat\eta_1$	$\hat\eta_4$	$\hat\eta_7)$
$\bar\eta_3$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-}{(2u-1)(2}$
$\bar\eta_6$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(}$
$\bar\eta_9$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(}$
$\bar\eta_{\hat1}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(}$
$\bar\eta_{\hat4}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-}{(2u-1)(2}$
$\bar\eta_{\hat7}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u}{(2u-1)(2}$

pdf

A Knot

pdf

In[ ]:=
```
f = TLSig[Knot[8, 5]]
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

Out[ ]=
pdf

$$2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[u-\boxed{-0.630\ldots}\right]+2\,\theta\left[u-\boxed{0.630\ldots}\right]$$

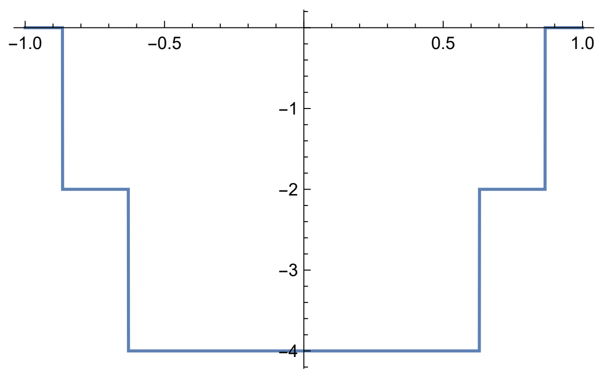
tex

{
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfOutput#1{\hfill\includegraphics[width=0.5\linewidth]{#1}}

pdf

In[ ]:=
```
Plot[f, {u, -1, 1}]
```

Out[ ]=
pdf



tex

}

tex

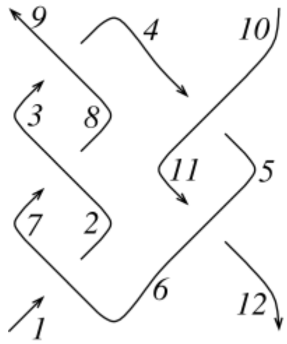
\par\parpic[r]{\includegraphics[width=2in]{CKT.pdf}}

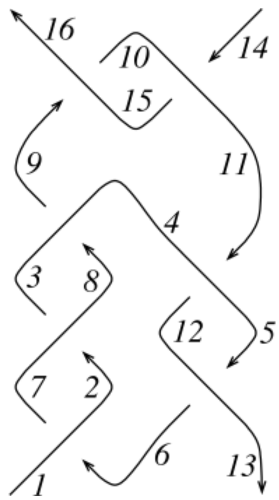
pdf

Some Tangles

tex

\par The Conway-Kinoshita-Terasaka Tangles:





pdf

In[◦]:=
```
T1 = PD[X[1, 6, 2, 7],
   X[7, 2, 8, 3],
   X[3, 8, 4, 9],
   X[11, 6, 12, 5], X[4, 11, 5, 10]];
T2 = PD[X[6, 2, 7, 1], X[2, 8, 3, 7], X[8, 4, 9, 3],
   X[5, 12, 6, 13], X[11, 4, 12, 5], X[14, 10, 15, 11], X[9, 15, 10, 16]];
```

pdf

In[◦]:= **Column@{TL[T1], Kas[T1]}**

Out[◦]=
pdf

$$2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)$$

	$(\eta_5$	$\hat{\eta}_{10}$	η_9	$\hat{\eta}_1$	η_{12}	$\hat{\eta}_5$	$\hat{\eta}_{11}$	$\eta_{11})$
$\overline{\eta}_5$	$\frac{(\omega-1)^2}{\omega}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)}{\omega}$	0	0	0	0	$1-\omega$
$\overline{\eta}_{\hat{10}}$	$\omega-1$	0	$1-\omega$	0	0	0	0	0
$\overline{\eta}_9$	$-2(\omega-1)$	$\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2}{\omega^2-\omega+1}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)\omega^2}{\omega^2-\omega+1}$	0	0	0
$\overline{\eta}_{\hat{1}}$	0	0	$\omega-1$	0	$1-\omega$	0	0	0
$\overline{\eta}_{12}$	0	0	$-\frac{2(\omega-1)}{\omega(\omega^2-\omega+1)}$	$\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2}{\omega^2-\omega+1}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)}{\omega}$	0
$\overline{\eta}_{\hat{5}}$	0	0	0	0	$\omega-1$	0	$1-\omega$	0
$\overline{\eta}_{\hat{11}}$	0	0	0	0	$-2(\omega-1)$	$\frac{\omega-1}{\omega}$	$\frac{(\omega-1)^2}{\omega}$	$\omega-1$
$\overline{\eta}_{11}$	$\frac{\omega-1}{\omega}$	0	0	0	0	0	$-\frac{\omega-1}{\omega}$	0

$$-2\,\Theta\left(u-\frac{1}{2}\right)+2\,\Theta\left(u+\frac{1}{2}\right)-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

	$(\eta_5$	$\hat{\eta}_{10}$	η_9	$\hat{\eta}_1$	η_{12}	$\hat{\eta}_5$	$\hat{\eta}_{11}$
$\overline{\eta}_5$	$2u^2-1$	u	1	0	0	0	0
$\overline{\eta}_{\hat{10}}$	u	1	u	0	0	0	0
$\overline{\eta}_9$	1	u	$\frac{2(2u^2-1)}{(2u-1)(2u+1)(4u^2-3)}$	$-\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)(4u^2-3)}$	0	0
$\overline{\eta}_{\hat{1}}$	0	0	$-\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{4u^2-3}{(2u-1)(2u+1)}$	$-\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	0	0
$\overline{\eta}_{12}$	0	0	$-\frac{1}{(2u-1)(2u+1)(4u^2-3)}$	$-\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)(4u^2-3)}$	u	1
$\overline{\eta}_{\hat{5}}$	0	0	0	0	u	1	u
$\overline{\eta}_{\hat{11}}$	0	0	0	0	1	u	$2u^2-$
$\overline{\eta}_{11}$	u	1	$\frac{2u}{(2u-1)(2u+1)}$	$-\frac{4u^2-3}{(2u-1)(2u+1)}$	$\frac{2u}{(2u-1)(2u+1)}$	1	u

pdf

In[∘]:=
```
Column@{TL[T2], Kas[T2]}
```

Out[∘]=
pdf

0

	$(\eta_{-14}$	η_{16}	η_{-1}	$\eta_{13})$
$\overline{\eta}_{-14}$	0	$1-\omega$	0	$\omega-1$
$\overline{\eta}_{16}$	$\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$
$\overline{\eta}_{-1}$	0	$\omega-1$	0	$1-\omega$
$\overline{\eta}_{13}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$	$\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$

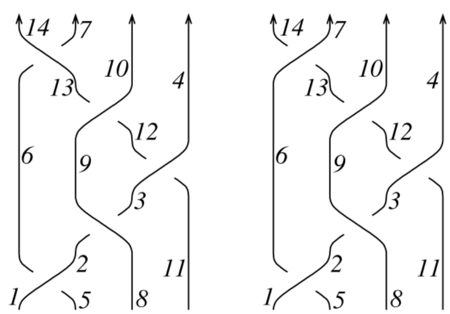
1

	$(\eta_{-14}$	η_{16}	η_{-1}	$\eta_{13})$
$\overline{\eta}_{-14}$	$\frac{1}{2}\left(-16u^4+28u^2-13\right)$	0	$\frac{1}{2}\left(16u^4-28u^2+13\right)$	0
$\overline{\eta}_{16}$	0	$-\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$	0	$\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$
$\overline{\eta}_{-1}$	$\frac{1}{2}\left(16u^4-28u^2+13\right)$	0	$\frac{1}{2}\left(-16u^4+28u^2-13\right)$	0
$\overline{\eta}_{13}$	0	$\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$	0	$-\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$

tex

\parpic[r]{\includegraphics[width=2in]{B1B2.pdf}}

Examples with non-trivial codimension:



pdf

In[∘]:=
```
B1 = PD[X[5, 2, 6, 1],
   X[2, 8, 3, 9],
   X[11, 4, 12, 3],
   X[12, 10, 13, 9],
   X[6, 13, 7, 14]];
B2 = PD[X[5, 2, 6, 1], X[2, 8, 3, 9], X[11, 4, 12, 3], X[12, 10, 13, 9], X[13, 7, 14, 6]];
```

pdf

In[∘]:= **Column@{TL[B1], Kas[B1]}**

Out[∘]=
pdf

0

	1	0	−1	0	$\frac{1}{\omega}$	0	$-\frac{1}{\omega}$	0
	0	0	0	−1	$\frac{1}{\omega}$	0	$-\frac{1}{\omega}$	1
	$(\eta_{-11}$	η_4	η_{10}	η_7	η_{14}	η_{-1}	η_{-5}	$\eta_{-8})$
$\overline{\eta}_{-11}$	0	0	0	0	0	0	0	0
$\overline{\eta}_4$	0	0	0	0	$\frac{\omega-1}{\omega^2}$	0	$-\frac{\omega-1}{\omega^2}$	0
$\overline{\eta}_{10}$	0	0	0	0	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0
$\overline{\eta}_7$	0	0	0	0	$\frac{(\omega-1)^2}{\omega^2}$	0	$-\frac{(\omega-1)^2}{\omega^2}$	0
$\overline{\eta}_{14}$	0	$-((\omega-1)\,\omega)$	$\omega-1$	$(\omega-1)^2$	0	$-\frac{\omega-1}{\omega}$	$\frac{\omega-1}{\omega}$	0
$\overline{\eta}_{-1}$	0	0	0	0	$\omega-1$	0	$1-\omega$	0
$\overline{\eta}_{-5}$	0	$(\omega-1)\,\omega$	$1-\omega$	$-(\omega-1)^2$	$1-\omega$	$\frac{\omega-1}{\omega}$	$\frac{(\omega-1)^2}{\omega}$	0
$\overline{\eta}_{-8}$	0	0	0	0	0	0	0	0

0

	1	0	−1	0	1	0	−1	0
	$(\eta_{-11}$	η_4	η_{10}	η_7	η_{14}	η_{-1}	η_{-5}	$\eta_{-8})$
$\overline{\eta}_{-11}$	0	0	0	0	0	0	0	0
$\overline{\eta}_4$	0	0	0	−1	$-u$	0	u	1
$\overline{\eta}_{10}$	0	0	0	$-u$	$1-2u^2$	0	$2u^2-1$	u
$\overline{\eta}_7$	0	−1	$-u$	$2u^2-3$	$-u$	−1	0	1
$\overline{\eta}_{14}$	0	$-u$	$1-2u^2$	$-u$	−1	$-u$	$-2(u-1)(u+1)$	u
$\overline{\eta}_{-1}$	0	0	0	−1	$-u$	0	u	1
$\overline{\eta}_{-5}$	0	u	$2u^2-1$	0	$-2(u-1)(u+1)$	u	$4u^2-3$	0
$\overline{\eta}_{-8}$	0	1	u	1	u	1	0	1 − 2 ι

pdf

In[∘]:=
```
Column@{TL[B2], Kas[B2]}
```

Out[∘]=
pdf

-1

	1	0	-1	1	0	0	0	-1
	$(\eta_{-11}$	η_4	η_{10}	η_7	η_{14}	η_{-1}	η_{-5}	$\eta_{-8})$
$\overline{\eta}_{-11}$	0	0	0	0	0	0	0	0
$\overline{\eta}_{4}$	0	0	0	$\frac{\omega-1}{\omega}$	0	0	0	$-\frac{\omega-1}{\omega}$
$\overline{\eta}_{10}$	0	0	0	$1-\omega$	0	0	0	$\omega-1$
$\overline{\eta}_{7}$	0	$1-\omega$	$\frac{\omega-1}{\omega}$	$\frac{2(\omega^2-\omega+1)}{\omega}$	$-\frac{\omega+1}{\omega}$	0	$\frac{2}{\omega}$	$-\frac{\omega^2-\omega+2}{\omega}$
$\overline{\eta}_{14}$	0	0	0	$-\omega-1$	$\frac{\omega^2+1}{\omega}$	$-\frac{\omega-1}{\omega}$	$-\frac{2}{\omega}$	2
$\overline{\eta}_{-1}$	0	0	0	0	$\omega-1$	0	$1-\omega$	0
$\overline{\eta}_{-5}$	0	0	0	2ω	-2ω	$\frac{\omega-1}{\omega}$	$\frac{\omega^2+1}{\omega}$	$-\omega-1$
$\overline{\eta}_{-8}$	0	$\omega-1$	$-\frac{\omega-1}{\omega}$	$-\frac{2\omega^2-\omega+1}{\omega}$	2	0	$-\frac{\omega+1}{\omega}$	$\frac{\omega^2+1}{\omega}$

$$2\,\theta\left(u-\frac{1}{\sqrt{2}}\right)-2\,\theta\left(u+\frac{1}{\sqrt{2}}\right)-1$$

	$(\eta_{-11}$	η_4	η_{10}	η_7	η_{14}	η_{-1}	η_{-5}
$\overline{\eta}_{-11}$	$\frac{8u^4-12u^2+3}{2(2u^2-1)}$	u	$\frac{(2u-1)(2u+1)}{2(2u^2-1)}$	$\frac{u}{2u^2-1}$	$\frac{1}{2(2u^2-1)}$	$\frac{u}{2u^2-1}$	$\frac{1}{2\,(2\,u^2}$
$\overline{\eta}_{4}$	u	0	$-u$	-1	0	0	0
$\overline{\eta}_{10}$	$\frac{(2u-1)(2u+1)}{2(2u^2-1)}$	$-u$	$-\frac{8u^4-4u^2+1}{2(2u^2-1)}$	$-\frac{2u^3}{2u^2-1}$	$-\frac{1}{2(2u^2-1)}$	$-\frac{u}{2u^2-1}$	$-\frac{1}{2\,(2\,u}$
$\overline{\eta}_{7}$	$\frac{u}{2u^2-1}$	-1	$-\frac{2u^3}{2u^2-1}$	$\frac{4u^4-6u^2+1}{2u^2-1}$	$\frac{2(u-1)u(u+1)}{2u^2-1}$	$-\frac{1}{2u^2-1}$	$-\frac{u}{2\,u^2}$
$\overline{\eta}_{14}$	$\frac{1}{2(2u^2-1)}$	0	$-\frac{1}{2(2u^2-1)}$	$\frac{2(u-1)u(u+1)}{2u^2-1}$	$\frac{8u^4-8u^2+1}{2(2u^2-1)}$	$\frac{2(u-1)u(u+1)}{2u^2-1}$	$-\frac{1}{2\,(2\,u}$
$\overline{\eta}_{-1}$	$\frac{u}{2u^2-1}$	0	$-\frac{u}{2u^2-1}$	$-\frac{1}{2u^2-1}$	$\frac{2(u-1)u(u+1)}{2u^2-1}$	$\frac{2(u-1)(u+1)}{2u^2-1}$	$\frac{2\,(u-1)\,u}{2\,u^2-}$
$\overline{\eta}_{-5}$	$\frac{1}{2(2u^2-1)}$	0	$-\frac{1}{2(2u^2-1)}$	$-\frac{u}{2u^2-1}$	$-\frac{1}{2(2u^2-1)}$	$\frac{2(u-1)u(u+1)}{2u^2-1}$	$\frac{8\,u^4-8}{2\,(2\,u^2}$
$\overline{\eta}_{-8}$	$-u$	1	$2u$	1	0	1	u