In[∘]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
 << KnotTheory`;
]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

pdf

In[∘]:=
```
ω2[v_][p_] := Module[{q = Expand[p], n, c},
   If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
    c v^n + ω2[v][q - c (ω + ω^-1)^n]]];
```

pdf

In[∘]:=
```
θ[x_] /; NumericQ[x] := HeavisideTheta[x]
```

pdf

In[∘]:=
```
sign[ℰ_] := Module[{num, den, v, p, rs, d, k},
  {num, den} = NumeratorDenominator[ℰ]; {num, den} /= ω^(Exponent[num,ω]/2);
  p = Factor[Times @@ (ω2[v] /@ {num, den}) /. v → 4 u^2 - 2];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Return[Sign[p /. u → 0]]];
  rs = Union@(u /. rs);
  Sign[Coefficient[p, u, Exponent[p, u]]] (-1)^Exponent[p,u] + Sum[
    k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
    If[EvenQ[k], 0, 2 Sign[d]] θ[u - r],
    {r, rs}] ]
```

pdf

In[∘]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

In[∘]:=
```
CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_ | η̄_, ∞]},
  Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) :→ Factor[c] Times @@ ηs^ps] ]
```

pdf

In[∘]:=
```
CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs] ]
```

pdf

In[∘]:=
```
(ℰ_)* := ℰ /. {η̄ → η, η → η̄, ω → ω^-1, c_Complex :→ c*};
r_Rule⁺ := {r, r*}
```

In[∘]:=
```
{((2 u - ω + 3 ω^-1) η̄1 η2)^*, (η1 → ω η2)^+}
```
Out[∘]=

$$\left\{\left(2\,u-\frac{1}{\omega}+3\,\omega\right)\eta_1\,\overline{\eta}_2,\ \left\{\eta_1\to\omega\,\eta_2,\ \overline{\eta}_1\to\frac{\overline{\eta}_2}{\omega}\right\}\right\}$$

pdf

In[∘]:=
```
RulesOf[η_i_ + rest_.] := (η_i → -rest)^+;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

In[∘]:=
```
CF[{η1 - η2, η1 - η3}]
```
Out[∘]=

$\{\eta_1-\eta_3,\ \eta_2-\eta_3\}$

In[∘]:=
```
RulesOf[η1 + η2 + η3]
```
Out[∘]=

$\{\eta_1\to-\eta_2-\eta_3,\ \overline{\eta}_1\to-\overline{\eta}_2-\overline{\eta}_3\}$

pdf

In[∘]:=
```
CF[Σ_b_[σ_, pq_]] := Σ_CF[b][σ, CF[pq]]
```

pdf

In[∘]:=
```
Format[Σ_b_B[σ_, PQ[rs_, q_]]] := Module[{ηs},
    ηs = η_# & /@ Join @@ b;
    Join[
     Prepend[""] /@ Table[TraditionalForm[∂_c r], {r, rs}, {c, ηs}],
     {Prepend[TraditionalForm@σ][
       Join @@ (b /. {l_, m___, r_} ⧴ {DisplayForm@RowBox[{"(", l}],
            m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer ⧴ η_i
      ]},
     MapThread[Prepend, {Table[TraditionalForm[∂_r,c q], {r, ηs^*}, {c, ηs}], ηs^*}]
    ]
   ] // MatrixForm;
```

The disjoint union in the world of multi-tangles.

pdf

In[∘]:=
```
Σ_b1_[σ1_, PQ[rs1_, q1_]] ⋃ Σ_b2_[σ2_, PQ[rs2_, q2_]] ^:=
  CF@Σ_Join[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

Gaps are named after the strand that follows them!

FM for Face Merge:

pdf

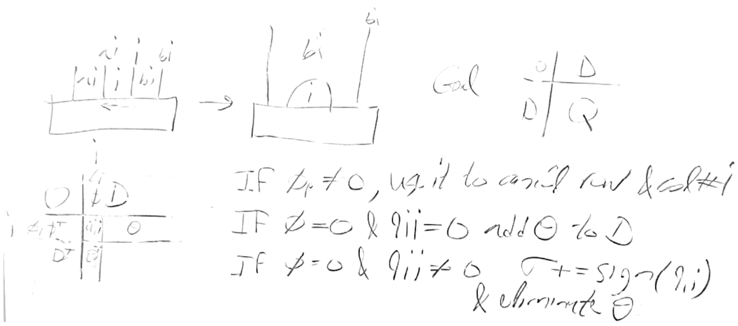
In[∘]:=
```
FM_{i_,j_} @ Σ_B[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[rs_, q_]] :=
  CF @ Σ_B[{ri,li,j,rj,lj,i},bs][σ, PQ[rs ⋃ {η_i - η_j}, q]]
```

In[∘]:=
```
Σ_B[{-1,2}][0, PQ[{}, 0]] ⋃ Σ_B[{-3,4}][0, PQ[{}, 0]] // FM_{-1,4}
```

Out[∘]=

	0	1	0	-1
0	(η_{-3}	η_{-1}	η_2	η_4)
$\bar\eta_{-3}$	0	0	0	0
$\bar\eta_{-1}$	0	0	0	0
$\bar\eta_2$	0	0	0	0
$\bar\eta_4$	0	0	0	0

pdf



pdf

In[∘]:=
```
Cordon_{i_} @ Σ_B[{li___,i_,ri___},bs___][σ_, PQ[rs_, q_]] :=
  Module[{ϕ = ∂_{η_i} rs, nσ = σ, nrs = rs, nq = q, qii, p},
    Which[
      Or @@ ((# =!= 0) & /@ ϕ), ({p} = FirstPosition[(# === 0) & /@ ϕ, False];
        {nrs, nq} = {rs, q} /. (η_i → -rs⟦p⟧ / ϕ⟦p⟧)^+ /. (η_i → 0)^+),
      (qii = ∂_{η̄_i,η_i} q) =!= 0, (nσ += sign[qii];
        nq = q /. (η_i → -(∂_{η̄_i} q) / qii)^+ /. (η_i → 0)^+),
      qii === 0, AppendTo[nrs, ∂_{η̄_i} q]; nq = q /. (η_i → 0)^+];
    CF @ Σ_B[Most@{ri,li},bs][nσ, PQ[nrs, nq] /. (η_{Last@{ri,li}} → η_{First@{ri,li}})^+] ]
```

tex

c for contract:

pdf

In[∘]:=
```
c_{i_,j_} @ t : Σ_B[{li___,i_,ri___},{___,j_,___},___][__] := t // FM_{j,First@{ri,li}} // Cordon_j
```

pdf

In[∘]:=
```
c_{i_,j_} @ t : Σ_B[{___,i_,j_,___},___][__] := Cordon_j @ t
c_{i_,j_} @ t : Σ_B[{j_,___,i_},___][__] := Cordon_j @ t
c_{i_,j_} @ t : Σ_B[{___,j_,i_,___},___][__] := Cordon_i @ t
c_{i_,j_} @ t : Σ_B[{i_,___,j_},___][__] := Cordon_i @ t
```

tex

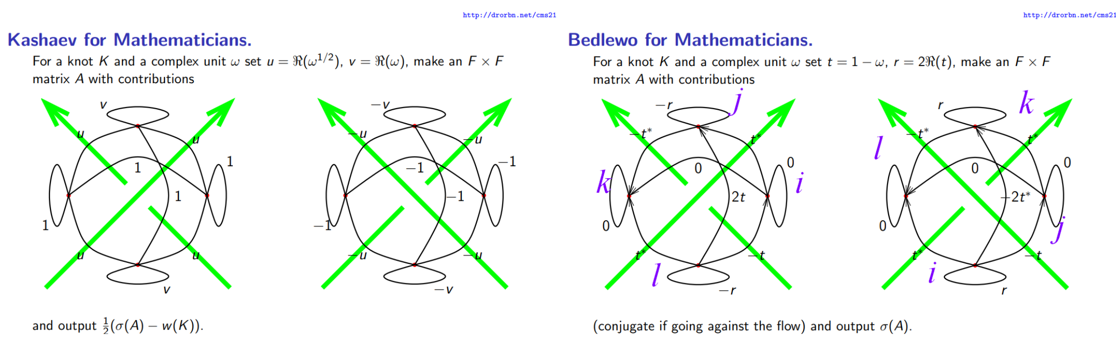
mc for magnetic contract:

pdf

In[∘]:=
```
mc[ℰ_] := ℰ //.
  t : Σ_B[{___,i_,___},{___,j_,___},___][__] | Σ_B[{___,i_,j_,___},___][__] | Σ_B[{j_,___,i_},___][__] /;
    i + j == 0 :→ c_{i,j}@t
```

pdf

In[∘]:=
```
Kas[P[i_, j_]] := CF@Σ_B[{-i,j}][0, PQ[{}, 0]];
TL[P[i_, j_]] := CF@Σ_B[{-i,j}][0, PQ[{}, 0]]
```



pdf

In[∘]:=
```
Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x],
    X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
Kas[x : (X₊ | X₋)[__]] := Module[{v = 2 u^2 - 1, p, fs, ηs, m},
  ηs = η_# & /@ (fs = List @@ x); p = Head[x] === X₊;
  m = If[p, {{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}},
         -{{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}}];
  CF@Σ_B[fs][If[p, -1, 1], PQ[{}, ηs*.m.ηs]]]
```

In[∘]:=
```
Kas /@ {X₊[1, 2, 3, 4], X₋[1, 4, 3, 2]}
```

Out[∘]=

$$\left\{\begin{pmatrix} -1 & (\eta_1 & \eta_2 & \eta_3 & \eta_4) \\ \bar\eta_1 & -1+2u^2 & u & 1 & u \\ \bar\eta_2 & u & 1 & u & 1 \\ \bar\eta_3 & 1 & u & -1+2u^2 & u \\ \bar\eta_4 & u & 1 & u & 1 \end{pmatrix}, \begin{pmatrix} 1 & (\eta_1 & \eta_4 & \eta_3 & \eta_2) \\ \bar\eta_1 & 1-2u^2 & -u & -1 & -u \\ \bar\eta_4 & -u & -1 & -u & -1 \\ \bar\eta_3 & -1 & -u & 1-2u^2 & -u \\ \bar\eta_2 & -u & -1 & -u & -1 \end{pmatrix}\right\}$$

pdf

In[∘]:=
```
TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x],
    X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
TL[x : (X₊ | X₋)[__]] := Module[{t = 1 - ω, r, p, fs, ηs, m},
  r = t + t*; ηs = η# & /@ (fs = List @@ x); p = Head[x] === X₊;
  m = If[p, ( -r   -t   2t   t*
              -t*   0   t*   0
              2t*   t   -r  -t*
               t    0   -t   0 ), ( r    -t   -2t*  t*
                                   -t*    0    t*   0
                                   -2t    t    r   -t*
                                    t     0   -t    0 )];
  CF@Σ_B[fs][0, PQ[{}, ηs*.m.ηs]]]
```

In[∘]:=
```
TL /@ {X[1, 2, 3, 4], X[1, 4, 3, 2]}
```

Out[∘]=

$$\left\{\begin{pmatrix} 0 & (\eta_{-4} & \eta_{-1} & \eta_2 & \eta_3) \\ \overline{\eta}_{-4} & 0 & 1-\omega & 0 & -1+\omega \\ \overline{\eta}_{-1} & \frac{-1+\omega}{\omega} & \frac{(-1+\omega)^2}{\omega} & -1+\omega & -2\,(-1+\omega) \\ \overline{\eta}_2 & 0 & -\frac{-1+\omega}{\omega} & 0 & \frac{-1+\omega}{\omega} \\ \overline{\eta}_3 & -\frac{-1+\omega}{\omega} & \frac{2\,(-1+\omega)}{\omega} & 1-\omega & \frac{(-1+\omega)^2}{\omega} \end{pmatrix}, \begin{pmatrix} 0 & (\eta_{-4} & \eta_3 & \eta_2 & \eta_{-1}) \\ \overline{\eta}_{-4} & -\frac{(-1+\omega)^2}{\omega} & -1+\omega & -\frac{2\,(-1+\omega)}{\omega} & \frac{-1+\omega}{\omega} \\ \overline{\eta}_3 & -\frac{-1+\omega}{\omega} & 0 & \frac{-1+\omega}{\omega} & 0 \\ \overline{\eta}_2 & 2\,(-1+\omega) & 1-\omega & -\frac{(-1+\omega)^2}{\omega} & -\frac{-1+\omega}{\omega} \\ \overline{\eta}_{-1} & 1-\omega & 0 & -1+\omega & 0 \end{pmatrix}\right\}$$

pdf

In[∘]:=
```
Kas[K_] := Fold[mc[#1 ⋃ #2] &, Σ_B[][0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
KasSig[K_] := Expand[Kas[K][[1]] / 2]
```

In[∘]:=
```
Kas[Knot[3, 1]]
```

Out[∘]=

$$\left(-4\,\Theta\left[-\frac{\sqrt{3}}{2}+u\right]+4\,\Theta\left[\frac{\sqrt{3}}{2}+u\right]\right)$$

In[∘]:=
```
KasSig[Knot[3, 1]]
```

Out[∘]=

$$-2\,\Theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\Theta\left[\frac{\sqrt{3}}{2}+u\right]$$

pdf

In[∘]:=
```
TL[K_] := Fold[mc[#1 ⋃ #2] &, Σ_B[][0, PQ[{}, 0]], List @@ (TL /@ PD@K)] /.
   θ[c_ + u] /; Abs[c] ≥ 1 :→ θ[c];
TLSig[K_] := TL[K][[1]]
```

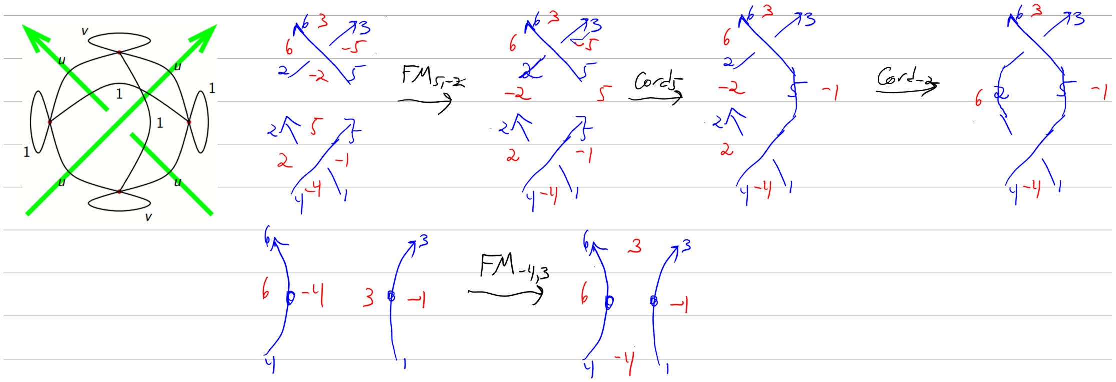
In[∘]:=
```
TLSig[Knot[3, 1]]
```

Out[∘]=

$$-2\,\Theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\Theta\left[\frac{\sqrt{3}}{2}+u\right]$$

pdf

Reidemeister 2



In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]]**

Out[∘]=

0	$(\eta_{-5}$	η_3	η_6	$\eta_{-2})$	$(\eta_{-4}$	η_{-1}	η_5	$\eta_2)$
$\overline{\eta}_{-5}$	$1-2u^2$	$-u$	-1	$-u$	0	0	0	0
$\overline{\eta}_3$	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_6$	-1	$-u$	$1-2u^2$	$-u$	0	0	0	0
$\overline{\eta}_{-2}$	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_{-4}$	0	0	0	0	1	u	1	u
$\overline{\eta}_{-1}$	0	0	0	0	u	$-1+2u^2$	u	1
$\overline{\eta}_5$	0	0	0	0	1	u	1	u
$\overline{\eta}_2$	0	0	0	0	u	1	u	$-1+2u^2$

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM$_{5,-2}$**

Out[∘]=

	0	0	0	-1	0	0	0	1
0	$(\eta_{-5}$	η_3	η_6	η_5	η_2	η_{-4}	η_{-1}	$\eta_{-2})$
$\overline{\eta}_{-5}$	$1-2u^2$	$-u$	-1	$-u$	0	0	0	0
$\overline{\eta}_3$	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_6$	-1	$-u$	$1-2u^2$	$-u$	0	0	0	0
$\overline{\eta}_5$	$-u$	-1	$-u$	0	u	1	u	0
$\overline{\eta}_2$	0	0	0	u	$-1+2u^2$	u	1	0
$\overline{\eta}_{-4}$	0	0	0	1	u	1	u	0
$\overline{\eta}_{-1}$	0	0	0	u	1	u	$-1+2u^2$	0
$\overline{\eta}_{-2}$	0	0	0	0	0	0	0	0

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM$_{5,-2}$ // Cordon$_{-2}$**

Out[∘]=

0	$(\eta_{-5}$	η_3	η_6	η_5	η_2	$\eta_{-4})$
$\overline{\eta}_{-5}$	0	$-u$	-1	0	1	u
$\overline{\eta}_3$	$-u$	-1	$-u$	-1	0	0
$\overline{\eta}_6$	-1	$-u$	$1-2u^2$	$-u$	0	0
$\overline{\eta}_5$	0	-1	$-u$	0	u	1
$\overline{\eta}_2$	1	0	0	u	$-1+2u^2$	u
$\overline{\eta}_{-4}$	u	0	0	1	u	1

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // $FM_{-2,5}$ // $Cordon_5$ // $Cordon_{-2}$**

Out[∘]=

$$\begin{pmatrix} & 0 & -1 & 0 & 1 \\ 0 & (\eta_{-5} & \eta_3 & \eta_2 & \eta_{-4}) \\ \overline{\eta}_{-5} & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_2 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \end{pmatrix}$$

pdf

In[∘]:= **{Kas[P[1, 3]] ⋃ Kas[P[4, 6]] // $FM_{-4,3}$, Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // mc}**

Out[∘]=
pdf

$$\left\{\begin{pmatrix} & 1 & 0 & -1 & 0 \\ 0 & (\eta_{-4} & \eta_6 & \eta_3 & \eta_{-1}) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} & 0 & 1 & 0 & -1 \\ 0 & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

pdf

In[∘]:= **{TL[P[1, 3]] ⋃ TL[P[4, 6]] // $FM_{-4,3}$, TL[X[1, 5, 2, 4]] ⋃ TL[X[2, 5, 3, 6]] // mc}**

Out[∘]=
pdf

$$\left\{\begin{pmatrix} & 1 & 0 & -1 & 0 \\ 0 & (\eta_{-4} & \eta_6 & \eta_3 & \eta_{-1}) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} & 0 & 1 & 0 & -1 \\ 0 & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

pdf

Reidemeister 3

pdf

In[∘]:=
```
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // mc;
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // mc
{lhs⟦1⟧, rhs⟦1⟧}
Simplify[lhs⟦2, 2⟧ == rhs⟦2, 2⟧]
```

Out[∘]=
pdf

$-2+2\Theta\left[-\frac{1}{2}+u\right]-2\Theta\left[\frac{1}{2}+u\right]$	η_{-7}	η_3	η_6	η_9	η_{-1}
$\bar\eta_{-7}$	$\frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)}$
$\bar\eta_3$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2(-1+2u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)}$
$\bar\eta_6$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)}$
$\bar\eta_9$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)}$	u (−3+4 / (−1+2 u) (
$\bar\eta_{-1}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	2 (−1+2 / (−1+2 u) (
$\bar\eta_{-4}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	u (−3+4 / (−1+2 u) (

Out[∘]=
pdf

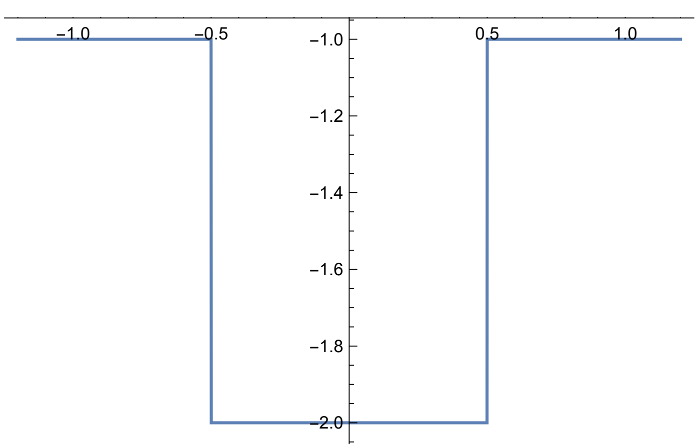
$$\left\{-2+2\Theta\left[-\frac{1}{2}+u\right]-2\Theta\left[\frac{1}{2}+u\right],\ -2+2\Theta\left[-\frac{1}{2}+u\right]-2\Theta\left[\frac{1}{2}+u\right]\right\}$$

Out[∘]=
pdf

True

In[∘]:=
```
f = KasSig@PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]];
Plot[f, {u, -1.2, 1.2}]
```

Out[∘]=



pdf

In[ ]:=
```
lhs = TL[X[4, 2, 5, 1]] ⋃ TL[X[7, 3, 8, 2]] ⋃ TL[X[8, 6, 9, 5]] // mc;
rhs = TL[X[7, 5, 8, 4]] ⋃ TL[X[8, 2, 9, 1]] ⋃ TL[X[5, 3, 6, 2]] // mc
{lhs⟦1⟧, rhs⟦1⟧}
lhs⟦2, 2⟧ == rhs⟦2, 2⟧
```

Out[ ]=
pdf

$1+2\theta[-1+u]-2\theta[1+u]$	η_{-7}	η_3	η_6	η_9	η_{-1}	η_{-4}
$\overline{\eta}_{-7}$	$\frac{1+\omega^2}{\omega}$	$-1+\omega$	-2ω	2	0	$-\frac{1+\omega}{\omega}$
$\overline{\eta}_{3}$	$-\frac{-1+\omega}{\omega}$	0	$\frac{-1+\omega}{\omega}$	0	0	0
$\overline{\eta}_{6}$	$-\frac{2}{\omega}$	$1-\omega$	$\frac{1+\omega^2}{\omega}$	$-\frac{1+\omega}{\omega}$	0	$\frac{2}{\omega}$
$\overline{\eta}_{9}$	2	0	$-1-\omega$	$\frac{1+\omega^2}{\omega}$	$-\frac{-1+\omega}{\omega}$	$-\frac{2}{\omega}$
$\overline{\eta}_{-1}$	0	0	0	$-1+\omega$	0	$1-\omega$
$\overline{\eta}_{-4}$	$-1-\omega$	0	2ω	-2ω	$\frac{-1+\omega}{\omega}$	$\frac{1+\omega^2}{\omega}$

Out[ ]=
pdf

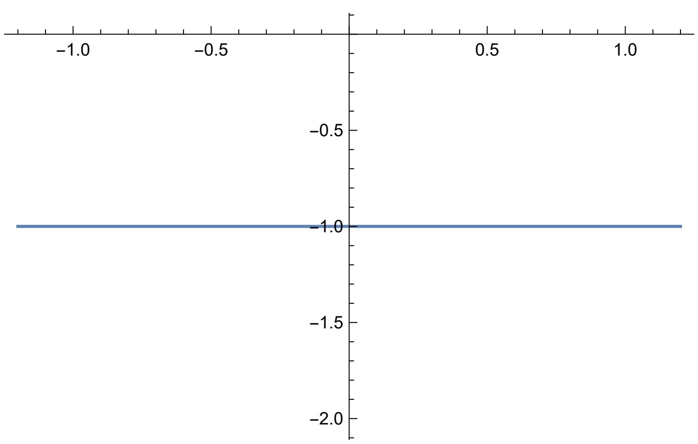
$\{1+2\theta[-1+u]-2\theta[1+u],\ 1+2\theta[-1+u]-2\theta[1+u]\}$

Out[ ]=
pdf

True

In[ ]:=
```
f = TLSig@PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]];
Plot[f, {u, -1.2, 1.2}]
```

Out[ ]=



Kashaev for Knots

In[ ]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

Out[ ]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[•]:=
```
(*u=0;*)
Kas[Knot[3, 1]]
Clear[u]
```

Out[•]=

$$\left(3+\text{sign}\left[\frac{1}{2}\left(3-4u^2\right)\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right]\right)$$

In[•]:=

$$\Sigma_{B[]}\left[\text{sign}\left[\frac{1}{2}\left(3-4u^2\right)\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right],\ PQ[\{\},0]\right]$$

Out[•]=

$$\left(\text{sign}\left[\frac{1}{2}\left(3-4u^2\right)\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right]\right)$$

In[•]:=
```
Table[K → 2 KasSig[K], {K, AllKnots[{3, 7}]}] // Column
```

Out[•]=

$\text{Knot}[3,1]\to 2\left(-2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]\right)$

$\text{Knot}[4,1]\to 0$

$\text{Knot}[5,1]\to 2\left(2\,\theta[u-(-0.951\ldots)]+2\,\theta[u-(-0.588\ldots)]-2\,\theta[u-0.588\ldots]-2\,\theta[u-0.951\ldots]\right)$

$\text{Knot}[5,2]\to 2\left(-2\,\theta\left[-\frac{\sqrt{\frac{7}{2}}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{\frac{7}{2}}}{2}+u\right]\right)$

$\text{Knot}[6,1]\to 0$

$\text{Knot}[6,2]\to 2\left(2\,\theta[u-(-0.772\ldots)]-2\,\theta[u-0.772\ldots]\right)$

$\text{Knot}[6,3]\to 0$

$\text{Knot}[7,1]\to 2\left(2\,\theta[u-(-0.975\ldots)]+2\,\theta[u-(-0.782\ldots)]+2\,\theta[u-(-0.434\ldots)]-2\,\theta[u-0.434\ldots]-2\,\theta[u-0.782\ldots]-2\,\theta[u-0.975\ldots]\right)$

$\text{Knot}[7,2]\to 2\left(-2\,\theta\left[-\frac{\sqrt{\frac{11}{3}}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{\frac{11}{3}}}{2}+u\right]\right)$

$\text{Knot}[7,3]\to 2\left(-2\,\theta[u-(-0.972\ldots)]-2\,\theta[u-(-0.656\ldots)]+2\,\theta[u-0.656\ldots]+2\,\theta[u-0.972\ldots]\right)$

$\text{Knot}[7,4]\to 2\left(2\,\theta\left[-\frac{\sqrt{15}}{4}+u\right]-2\,\theta\left[\frac{\sqrt{15}}{4}+u\right]\right)$

$\text{Knot}[7,5]\to 2\left(2\,\theta[u-(-0.963\ldots)]+2\,\theta[u-(-0.757\ldots)]-2\,\theta[u-0.757\ldots]-2\,\theta[u-0.963\ldots]\right)$

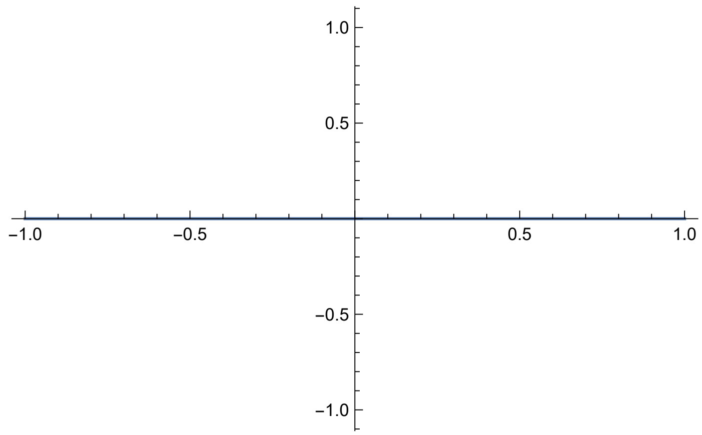
$\text{Knot}[7,6]\to 2\left(2\,\theta[u-(-0.920\ldots)]-2\,\theta[u-0.920\ldots]\right)$

$\text{Knot}[7,7]\to 0$

In[◦]:=
```
f = KasSig[Knot[10, 1]]
Plot[f, {u, -1, 1}]
```
Out[◦]=

0

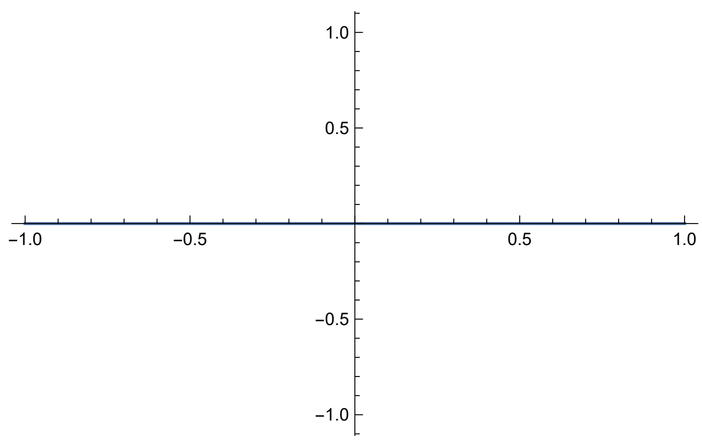
Out[◦]=



In[◦]:=
```
f = TLSig[Knot[10, 1]]
Plot[f, {u, -1, 1}]
```
Out[◦]=

0

Out[◦]=



In[◦]:=
```
(KasSig /@ AllKnots[{3, 8}]) /. u → 1 / 2
```
Out[◦]=

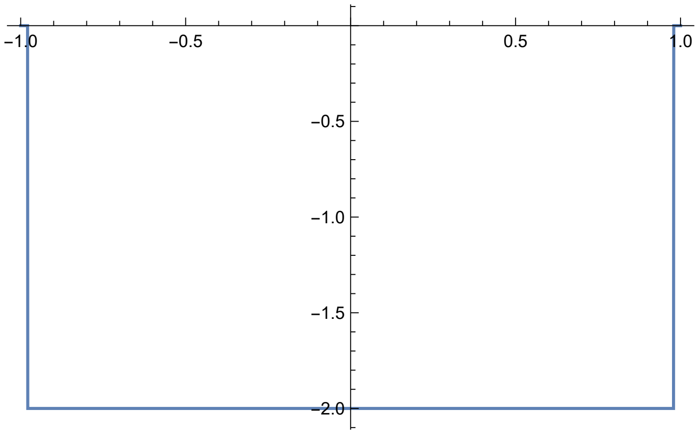
{2, 0, 4, 2, 0, 2, 0, 4, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -4, 0, 2}

In[∘]:=
```
f = KasSig[Knot[9, 5]]
Plot[f, {u, -1, 1}]
```

Out[∘]=

$$2\,\theta\left[-\frac{\sqrt{\frac{23}{6}}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{\frac{23}{6}}}{2}+u\right]$$

Out[∘]=

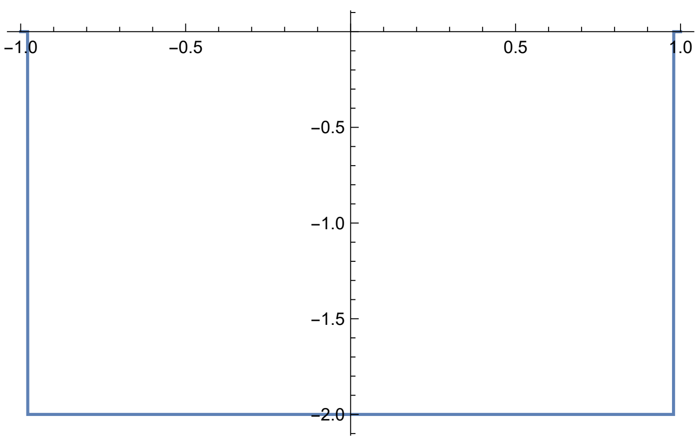


In[∘]:=
```
f = TLSig[Knot[9, 5]]
Plot[f, {u, -1, 1}]
```

Out[∘]=

$$2\,\theta\left[-\frac{\sqrt{\frac{23}{6}}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{\frac{23}{6}}}{2}+u\right]$$

Out[∘]=

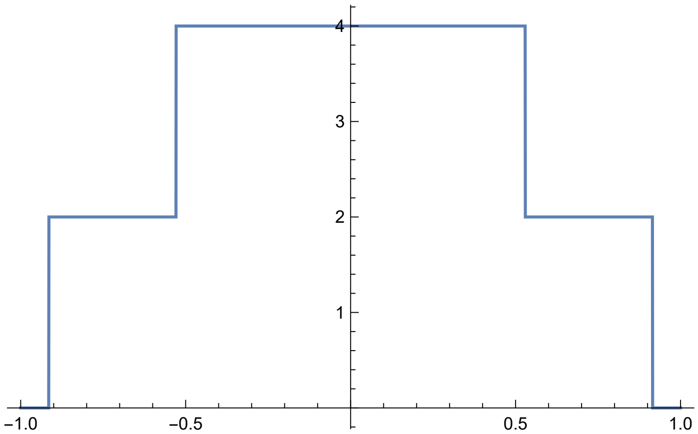


In[∘]:=
```
f = KasSig[Knot[8, 2]]
Plot[f, {u, -1, 1}]
```

Out[∘]=

$2\,\theta[u - (-0.915\ldots)] + 2\,\theta[u - (-0.529\ldots)] - 2\,\theta[u - 0.529\ldots] - 2\,\theta[u - 0.915\ldots]$

Out[∘]=

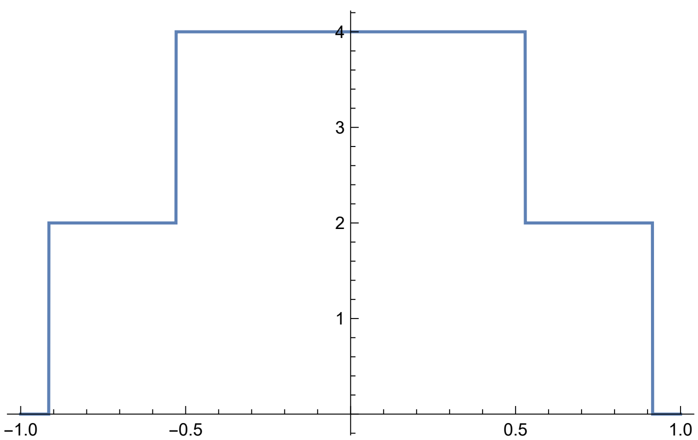


In[∘]:=
```
f = TLSig[Knot[8, 2]]
Plot[f, {u, -1, 1}]
```

Out[∘]=

$2\,\theta[u - (-0.915\ldots)] + 2\,\theta[u - (-0.529\ldots)] - 2\,\theta[u - 0.529\ldots] - 2\,\theta[u - 0.915\ldots]$

Out[∘]=

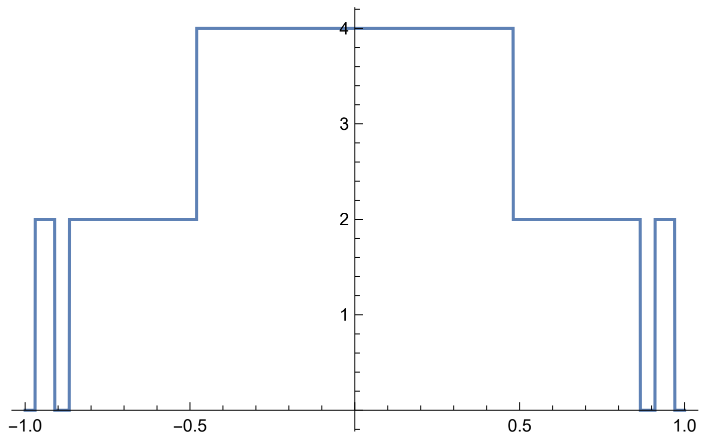


In[∘]:=
```
f = KasSig[Knot[12, Alternating, 422]]
Plot[f, {u, -1, 1}]
```

Out[∘]=

$$-2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]+2\,\theta[u-(-0.970\ldots)]-2\,\theta[u-(-0.910\ldots)]+2\,\theta[u-(-0.480\ldots)]-2\,\theta[u-0.480\ldots]+2\,\theta[u-0.910\ldots]-2\,\theta[u-0.970\ldots]$$

Out[∘]=

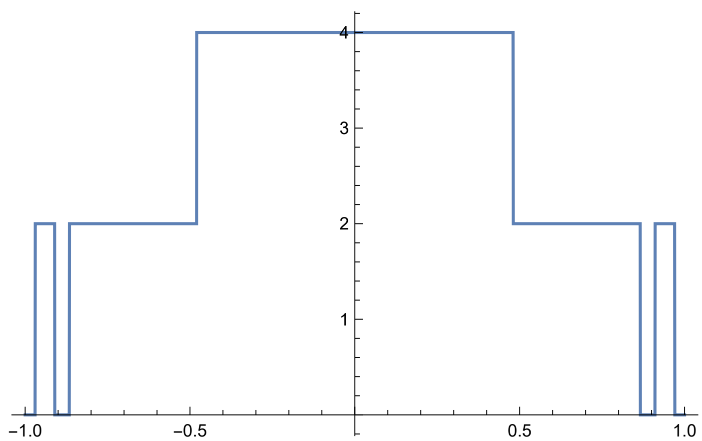


In[∘]:=
```
f = TLSig[Knot[12, Alternating, 422]]
Plot[f, {u, -1, 1}]
```

Out[∘]=

$$-2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]+2\,\theta[u-(-0.970\ldots)]-2\,\theta[u-(-0.910\ldots)]+2\,\theta[u-(-0.480\ldots)]-2\,\theta[u-0.480\ldots]+2\,\theta[u-0.910\ldots]-2\,\theta[u-0.970\ldots]$$

Out[∘]=



Tristram-Levine for Knots

In[∘]:= `-KnotSignature /@ AllKnots[{3, 8}]`

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:= `TL[Knot[3, 1]]`

Out[∘]=

$$\left(-2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]\right)$$

In[∘]:= `TLSig /@ AllKnots[{3, 8}] /. u → 0`

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:= `TLSig /@ AllKnots[{3, 8}] /. u → 0.9999`

Out[∘]=

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[∘]:= `Table[K → TLSig[K], {K, AllKnots[{3, 8}]}] // Column`

Out[∘]=

Knot[3, 1] → $-2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]$

Knot[4, 1] → 0

Knot[5, 1] →
$2\,\theta[u-(-0.951\ldots)]+2\,\theta[u-(-0.588\ldots)]-2\,\theta[u-0.588\ldots]-2\,\theta[u-0.951\ldots]$

Knot[5, 2] → $-2\,\theta\left[-\frac{\sqrt{\frac{7}{2}}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{\frac{7}{2}}}{2}+u\right]$

Knot[6, 1] → 0

Knot[6, 2] → $2\,\theta[u-(-0.772\ldots)]-2\,\theta[u-0.772\ldots]$

Knot[6, 3] → 0

Knot[7, 1] → $2\,\theta[u-(-0.975\ldots)]+2\,\theta[u-(-0.782\ldots)]+$
$2\,\theta[u-(-0.434\ldots)]-2\,\theta[u-0.434\ldots]-2\,\theta[u-0.782\ldots]-2\,\theta[u-0.975\ldots]$

Knot[7, 2] → $-2\,\theta\left[-\frac{\sqrt{\frac{11}{3}}}{2}+u\right]+2\,\theta\left[\frac{\sqrt{\frac{11}{3}}}{2}+u\right]$

Knot[7, 3] →
$-2\,\theta[u-(-0.972\ldots)]-2\,\theta[u-(-0.656\ldots)]+2\,\theta[u-0.656\ldots]+2\,\theta[u-0.972\ldots]$

Knot[7, 4] → $2\,\theta\left[-\frac{\sqrt{15}}{4}+u\right]-2\,\theta\left[\frac{\sqrt{15}}{4}+u\right]$

Knot[7, 5] →
$2\,\theta[u-(-0.963\ldots)]+2\,\theta[u-(-0.757\ldots)]-2\,\theta[u-0.757\ldots]-2\,\theta[u-0.963\ldots]$

Knot[7, 6] → $2\,\theta[u-(-0.920\ldots)]-2\,\theta[u-0.920\ldots]$

Knot[7, 7] → 0
Knot[8, 1] → 0
Knot[8, 2] →
$2\theta[u - (-0.915\ldots)] + 2\theta[u - (-0.529\ldots)] - 2\theta[u - 0.529\ldots] - 2\theta[u - 0.915\ldots]$
Knot[8, 3] → 0
Knot[8, 4] → $2\theta[u - (-0.745\ldots)] - 2\theta[u - 0.745\ldots]$
Knot[8, 5] → $2\theta[-\frac{\sqrt{3}}{2} + u] - 2\theta[\frac{\sqrt{3}}{2} + u] - 2\theta[u - (-0.630\ldots)] + 2\theta[u - 0.630\ldots]$
Knot[8, 6] → $2\theta[u - (-0.811\ldots)] - 2\theta[u - 0.811\ldots]$
Knot[8, 7] → $-2\theta[u - (-0.647\ldots)] + 2\theta[u - 0.647\ldots]$
Knot[8, 8] → 0
Knot[8, 9] → 0
Knot[8, 10] → $2\theta[-\frac{\sqrt{3}}{2} + u] - 2\theta[\frac{\sqrt{3}}{2} + u]$
Knot[8, 11] → $-2\theta[-\frac{\sqrt{3}}{2} + u] + 2\theta[\frac{\sqrt{3}}{2} + u]$
Knot[8, 12] → 0
Knot[8, 13] → 0
Knot[8, 14] → $2\theta[u - (-0.907\ldots)] - 2\theta[u - 0.907\ldots]$
Knot[8, 15] → $-2\theta[-\frac{\sqrt{3}}{2} + u] + 2\theta[\frac{\sqrt{3}}{2} + u] - 2\theta[-\frac{\sqrt{\frac{11}{3}}}{2} + u] + 2\theta[\frac{\sqrt{\frac{11}{3}}}{2} + u]$
Knot[8, 16] → $2\theta[u - (-0.749\ldots)] - 2\theta[u - 0.749\ldots]$
Knot[8, 17] → 0
Knot[8, 18] → 0
Knot[8, 19] → $2\theta[-\frac{\sqrt{3}}{2} + u] - 2\theta[\frac{\sqrt{3}}{2} + u] - 2\theta[u - (-0.966\ldots)] -$
$2\theta[u - (-0.259\ldots)] + 2\theta[u - 0.259\ldots] + 2\theta[u - 0.966\ldots]$
Knot[8, 20] → 0
Knot[8, 21] → $-2\theta[-\frac{\sqrt{3}}{2} + u] + 2\theta[\frac{\sqrt{3}}{2} + u]$

pdf

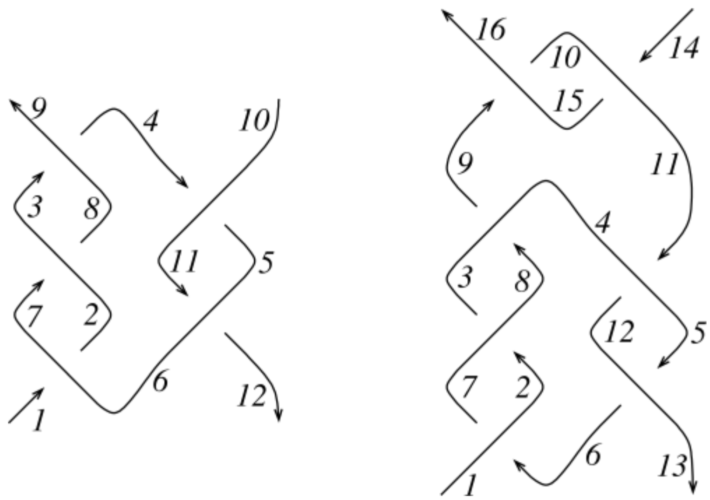
Some Tangles

tex

The ``first'' tangle T_1.

tex

The Conway-Kinoshita-Terasaka Tangles.



pdf

```
In[◦]:= T2 = PD[X[1, 6, 2, 7], X[7, 2, 8, 3], X[3, 8, 4, 9], X[11, 6, 12, 5], X[4, 11, 5, 10]];
        T3 = PD[X[6, 2, 7, 1], X[2, 8, 3, 7], X[8, 4, 9, 3],
           X[5, 12, 6, 13], X[11, 4, 12, 5], X[14, 10, 15, 11], X[9, 15, 10, 16]];
```

pdf

```
In[◦]:= {TL[T2], Kas[T2]}
```

Out[◦]=
pdf

$$\left\{\begin{pmatrix} -2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1 & (\eta_{-10} & \eta_{9} & \eta_{-1} & \eta_{12}) \\ \bar{\eta}_{-10} & 0 & 1-\omega & 0 & \omega-1 \\ \bar{\eta}_{9} & \frac{\omega-1}{\omega} & \frac{2\,\omega}{\omega^2-\omega+1} & -\frac{\omega-1}{\omega} & -\frac{2\,\omega}{\omega^2-\omega+1} \\ \bar{\eta}_{-1} & 0 & \omega-1 & 0 & 1-\omega \\ \bar{\eta}_{12} & -\frac{\omega-1}{\omega} & -\frac{2\,\omega}{\omega^2-\omega+1} & \frac{\omega-1}{\omega} & \frac{2\,\omega}{\omega^2-\omega+1} \end{pmatrix}, \begin{pmatrix} -2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1 & \\ \bar{\eta}_{-10} & 2 \\ \bar{\eta}_{9} & \\ \bar{\eta}_{-1} & - \\ \bar{\eta}_{12} & \end{pmatrix}\right.$$

pdf

```
In[◦]:= {TL[T3], Kas[T3]}
```

Out[◦]=
pdf

$$\left\{\begin{pmatrix} 0 & (\eta_{-14} & \eta_{16} & \eta_{-1} & \eta_{13}) \\ \bar{\eta}_{-14} & 0 & 1-\omega & 0 & \omega-1 \\ \bar{\eta}_{16} & \frac{\omega-1}{\omega} & -\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} & -\frac{\omega-1}{\omega} & \frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} \\ \bar{\eta}_{-1} & 0 & \omega-1 & 0 & 1-\omega \\ \bar{\eta}_{13} & -\frac{\omega-1}{\omega} & \frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} & \frac{\omega-1}{\omega} & -\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 1 & (\eta_{-14} & \eta_{16} & \eta_{-1} & \eta_{13}) \\ \bar{\eta}_{-14} & \frac{1}{2}\left(-16\,u^4+28\,u^2-13\right) & 0 & \frac{1}{2}\left(16\,u^4-28\,u^2+13\right) & 0 \\ \bar{\eta}_{16} & 0 & -\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} & 0 & \frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} \\ \bar{\eta}_{-1} & \frac{1}{2}\left(16\,u^4-28\,u^2+13\right) & 0 & \frac{1}{2}\left(-16\,u^4+28\,u^2-13\right) & 0 \\ \bar{\eta}_{13} & 0 & \frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} & 0 & -\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} \end{pmatrix}\right\}$$

tex

Non-trivial codimension examples.

pdf

```
{B1, B2} = {PD[X[5, 2, 6, 1], X[2, 8, 3, 9], X[11, 4, 12, 3], X[12, 10, 13, 9], X[6, 13, 7, 14]],
   PD[X[5, 2, 6, 1], X[2, 8, 3, 9], X[11, 4, 12, 3], X[12, 10, 13, 9], X[13, 7, 14, 6]]};
```

pdf

In[∘]:=
```
{TL[B1], Kas[B1]}
```

Out[∘]=
pdf

$$\left\{\left(\begin{array}{ccccccccc}
 & 1 & 0 & -1 & 0 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 0 \\
 & 0 & 0 & 0 & -1 & \frac{1}{\omega} & 0 & -\frac{1}{\omega} & 1 \\
0 & (\eta_{-11} & \eta_4 & \eta_{10} & \eta_7 & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8}) \\
\overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\eta}_{4} & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega^2} & 0 & -\frac{\omega-1}{\omega^2} & 0 \\
\overline{\eta}_{10} & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\
\overline{\eta}_{7} & 0 & 0 & 0 & 0 & \frac{(\omega-1)^2}{\omega^2} & 0 & -\frac{(\omega-1)^2}{\omega^2} & 0 \\
\overline{\eta}_{14} & 0 & -((\omega-1)\,\omega) & \omega-1 & (\omega-1)^2 & 0 & -\frac{\omega-1}{\omega} & \frac{\omega-1}{\omega} & 0 \\
\overline{\eta}_{-1} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 \\
\overline{\eta}_{-5} & 0 & (\omega-1)\,\omega & 1-\omega & -(\omega-1)^2 & 1-\omega & \frac{\omega-1}{\omega} & \frac{(\omega-1)^2}{\omega} & 0 \\
\overline{\eta}_{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),\right.$$

$$\left.\left(\begin{array}{ccccccccc}
 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\
0 & (\eta_{-11} & \eta_4 & \eta_{10} & \eta_7 & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8}) \\
\overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\eta}_{4} & 0 & 0 & 0 & -1 & -u & 0 & u & 1 \\
\overline{\eta}_{10} & 0 & 0 & 0 & -u & 1-2u^2 & 0 & 2u^2-1 & u \\
\overline{\eta}_{7} & 0 & -1 & -u & 2u^2-3 & -u & -1 & 0 & 1 \\
\overline{\eta}_{14} & 0 & -u & 1-2u^2 & -u & -1 & -u & -2\,(u-1)\,(u+1) & u \\
\overline{\eta}_{-1} & 0 & 0 & 0 & -1 & -u & 0 & u & 1 \\
\overline{\eta}_{-5} & 0 & u & 2u^2-1 & 0 & -2\,(u-1)\,(u+1) & u & 4u^2-3 & 0 \\
\overline{\eta}_{-8} & 0 & 1 & u & 1 & u & 1 & 0 & 1-2u^2
\end{array}\right)\right\}$$

pdf

In[∘]:= **{TL[B2], Kas[B2]}**

Out[∘]=
pdf

$$\left\{\left(\begin{array}{ccccccccc}
 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & -1 \\
-1 & (\eta_{-11} & \eta_{4} & \eta_{10} & \eta_{7} & \eta_{14} & \eta_{-1} & \eta_{-5} & \eta_{-8}) \\
\overline{\eta}_{-11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\eta}_{4} & 0 & 0 & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & -\frac{\omega-1}{\omega} \\
\overline{\eta}_{10} & 0 & 0 & 0 & 1-\omega & 0 & 0 & 0 & \omega-1 \\
\overline{\eta}_{7} & 0 & 1-\omega & \frac{\omega-1}{\omega} & \frac{2\left(\omega^2-\omega+1\right)}{\omega} & -\frac{\omega+1}{\omega} & 0 & \frac{2}{\omega} & -\frac{\omega^2-\omega+2}{\omega} \\
\overline{\eta}_{14} & 0 & 0 & 0 & -\omega-1 & \frac{\omega^2+1}{\omega} & -\frac{\omega-1}{\omega} & -\frac{2}{\omega} & 2 \\
\overline{\eta}_{-1} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 \\
\overline{\eta}_{-5} & 0 & 0 & 0 & 2\omega & -2\omega & \frac{\omega-1}{\omega} & \frac{\omega^2+1}{\omega} & -\omega-1 \\
\overline{\eta}_{-8} & 0 & \omega-1 & -\frac{\omega-1}{\omega} & -\frac{2\omega^2-\omega+1}{\omega} & 2 & 0 & -\frac{\omega+1}{\omega} & \frac{\omega^2+1}{\omega}
\end{array}\right),\ \left(\begin{array}{cc}
2\,\Theta\left(u-\frac{1}{\sqrt{2}}\right)-2\,\Theta\left(u+\frac{1}{\sqrt{2}}\right)-1 & (\\
\overline{\eta}_{-11} & \frac{8u^4}{2(} \\
\overline{\eta}_{4} & \\
\overline{\eta}_{10} & \frac{(2u-}{2(} \\
\overline{\eta}_{7} & \\
\overline{\eta}_{14} & \frac{}{2(} \\
\overline{\eta}_{-1} & \\
\overline{\eta}_{-5} & \frac{}{2(} \\
\overline{\eta}_{-8} &
\end{array}\right.$$