In[∘]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
 << KnotTheory`;
]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

pdf

In[∘]:=
```
ω2[v_][p_] := Module[{q = Expand[p], n, c},
   If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
    c v^n + ω2[v][q - c (ω + ω^-1)^n]]];
```

pdf

In[∘]:=
```
θ[x_] /; NumericQ[x] := HeavisideTheta[x]
```

pdf

In[∘]:=
```
sign[ℰ_] := Module[{num, den, v, p, rs, d, k},
  {num, den} = NumeratorDenominator[ℰ]; {num, den} /= ω^(Exponent[num,ω]/2);
  p = Factor[Times @@ (ω2[v] /@ {num, den}) /. v → 4 u^2 - 2];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Return[Sign[p /. u → 0]]];
  rs = Union@(u /. rs);
  Sign[Coefficient[p, u, Exponent[p, u]]] (-1)^Exponent[p,u] + Sum[
    k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
    If[EvenQ[k], 0, 2 Sign[d]] θ[u - r],
    {r, rs}] ]
```

pdf

In[∘]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

In[∘]:=
```
CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_ | η̄_, ∞]},
  Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) :> Factor[c] Times @@ ηs^ps] ]
```

pdf

In[∘]:=
```
CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs] ]
```

pdf

In[∘]:=
```
(ℰ_)* := ℰ /. {η̄ → η, η → η̄, ω → ω^-1, c_Complex :> c*};
r_Rule⁺ := {r, r*}
```

pdf

In[∘]:=
```
RulesOf[η_i_ + rest_.] := (η_i → -rest)^+;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

pdf

In[∘]:=
```
CF[Σ_b_[σ_, pq_]] := Σ_CF[b][σ, CF[pq]]
```

pdf

In[∘]:=
```
Format[Σ_b_B[σ_, PQ[rs_, q_]]] := TableForm[Module[{ηs},
    ηs = η_# & /@ Join @@ b;
    Join[
     Prepend[""] /@ Table[TraditionalForm[∂_c r], {r, rs}, {c, ηs}],
     {Prepend[TraditionalForm@σ][
       Join @@ (b /. {l_, m___, r_} ↦ {DisplayForm@RowBox[{"(", l}],
             m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer ↦ η_i
      ]},
     MapThread[Prepend, {Table[TraditionalForm[∂_r,c q], {r, ηs*}, {c, ηs}], ηs*}]
    ]
   ], TableAlignments → Center];
```

pdf

In[∘]:=
```
Format[Σ_b_B[σ_, PQ[rs_, q_]]] := Module[{ηs},
   ηs = η_# & /@ Join @@ b;
   Column[{TraditionalForm@σ,
     TableForm[Join[
       Prepend[""] /@ Table[TraditionalForm[∂_c r], {r, rs}, {c, ηs}],
       {Prepend[""][
         Join @@ (b /. {l_, m___, r_} ↦ {DisplayForm@RowBox[{"(", l}],
               m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer ↦ η_i
        ]},
       MapThread[Prepend, {Table[TraditionalForm[∂_r,c q], {r, ηs*}, {c, ηs}], ηs*}]
      ], TableAlignments → Center]
    }, Center]
  ];
```

pdf

In[∘]:=
```
Σ_b1_[σ1_, PQ[rs1_, q1_]] ⋃ Σ_b2_[σ2_, PQ[rs2_, q2_]] ^:=
  CF@Σ_Join[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

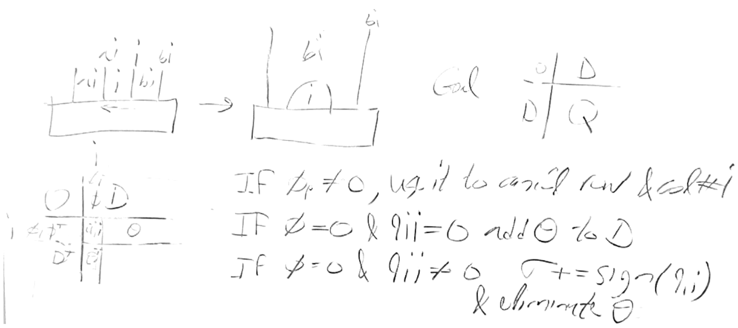
\par Gaps are named after the strand that follows them!

\par FM for Face Merge:

pdf

```
In[∘]:= FM_{i_,j_}@Σ_{B[{li___,i_,ri___},{lj___,j_,rj___},bs___]}[σ_, PQ[rs_, q_]] :=
   CF@Σ_{B[{ri,li,j,rj,lj,i},bs]}[σ, PQ[rs ⋃ {η_i - η_j}, q]]
```

pdf




pdf

```
In[∘]:= Cordon_{i_}@Σ_{B[{li___,i_,ri___},bs___]}[σ_, PQ[rs_, q_]] :=
   Module[{ϕ = ∂_{η_i} rs, nσ = σ, nrs = rs, nq = q, qii, p},
    Which[
     Or @@ ((# =!= 0) & /@ ϕ), ({p} = FirstPosition[(# === 0) & /@ ϕ, False];
      {nrs, nq} = {rs, q} /. (η_i → -rs⟦p⟧ / ϕ⟦p⟧)^+ /. (η_i → 0)^+),
     (qii = ∂_{η̄_i,η_i} q) =!= 0, (nσ += sign[qii];
      nq = q /. (η_i → -(∂_{η̄_i} q) / qii)^+ /. (η_i → 0)^+),
     qii === 0, AppendTo[nrs, ∂_{η̄_i} q]; nq = q /. (η_i → 0)^+];
    CF@Σ_{B[Most@{ri,li},bs]}[nσ, PQ[nrs, nq] /. (η_{Last@{ri,li}} → η_{First@{ri,li}})^+] ]
```

tex

\par c for contract:

pdf

```
In[∘]:= c_{i_,j_}@t : Σ_{B[{li___,i_,ri___},{___,j_,___},___]}[__] := t // FM_{j,First@{ri,li}} // Cordon_j
```

pdf

```
In[∘]:= c_{i_,j_}@t : Σ_{B[{___,i_,j_,___},___]}[__] := Cordon_j@t
        c_{i_,j_}@t : Σ_{B[{j_,___,i_},___]}[__] := Cordon_j@t
        c_{i_,j_}@t : Σ_{B[{___,j_,i_,___},___]}[__] := Cordon_i@t
        c_{i_,j_}@t : Σ_{B[{i_,___,j_},___]}[__] := Cordon_i@t
```

tex

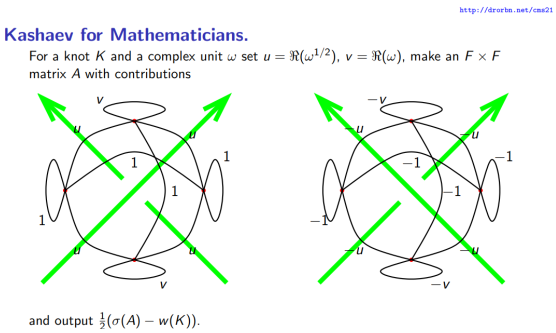
\par mc for magnetic contract:

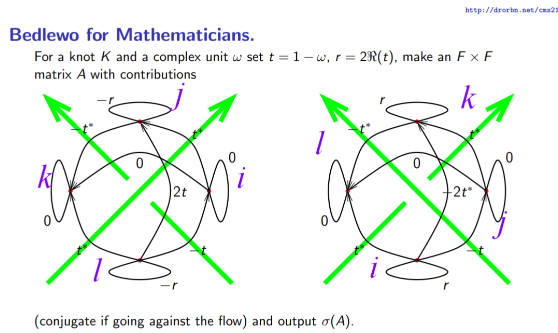
pdf

```
In[∘]:= mc[ℰ_] := ℰ //.
   t : Σ_{B[{___,i_,___},{___,j_,___},___]}[__] | Σ_{B[{___,i_,j_,___},___]}[__] | Σ_{B[{j_,___,i_},___]}[__] /;
     i + j == 0 :→ c_{i,j}@t
```

pdf

In[∘]:=
```
Kas[P[i_, j_]] := CF@Σ_B[{-i,j}][0, PQ[{}, 0]];
TL[P[i_, j_]] := CF@Σ_B[{-i,j}][0, PQ[{}, 0]]
```




pdf

In[∘]:=
```
Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x],
    X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
Kas[x : (X₊ | X₋)[__]] := Module[{v = 2 u^2 - 1, p, fs, ηs, m},
  ηs = η# & /@ (fs = List @@ x); p = Head[x] === X₊;
  m = If[p, ( v u 1 u
              u 1 u 1
              1 u v u
              u 1 u 1 ), -( v u 1 u
                            u 1 u 1
                            1 u v u
                            u 1 u 1 )];
  CF@Σ_B[fs][If[p, -1, 1], PQ[{}, ηs*.m.ηs]]]
```

pdf

In[∘]:=
```
TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x],
    X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
TL[x : (X₊ | X₋)[__]] := Module[{t = 1 - ω, r, p, fs, ηs, m},
  r = t + t*; ηs = η# & /@ (fs = List @@ x); p = Head[x] === X₊;
  m = If[p, ( -r   -t  2t   t*
              -t*   0   t*  0
              2t*   t  -r  -t*
               t    0  -t   0 ), (  r   -t  -2t*  t*
                                    -t*   0   t*   0
                                    -2t   t   r   -t*
                                     t    0  -t    0 )];
  CF@Σ_B[fs][0, PQ[{}, ηs*.m.ηs]]]
```

pdf

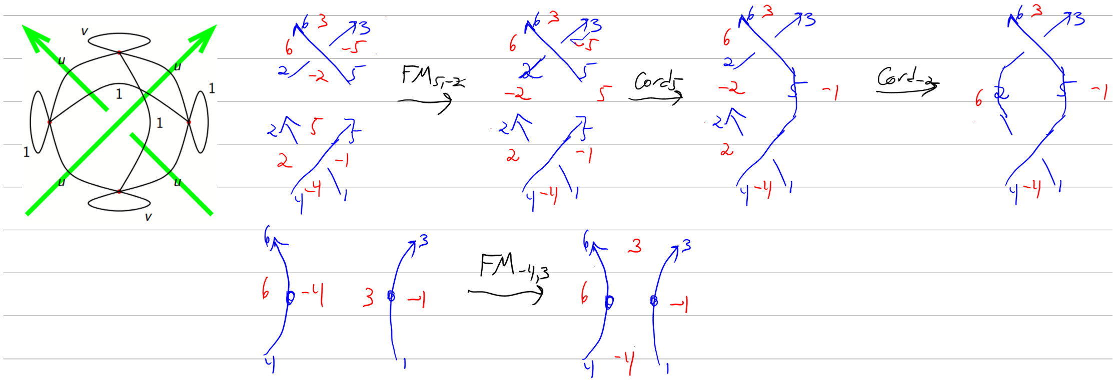
In[∘]:=
```
Kas[K_] := Fold[mc[#1 ⋃ #2] &, Σ_B[][0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
KasSig[K_] := Expand[Kas[K]〚1〛 / 2]
```

pdf

In[∘]:=
```
TL[K_] := Fold[mc[#1 ⋃ #2] &, Σ_B[][0, PQ[{}, 0]], List @@ (TL /@ PD@K)] /.
   θ[c_ + u] /; Abs[c] ≥ 1 :→ θ[c];
TLSig[K_] := TL[K]〚1〛
```

pdf

## Reidemeister 2



pdf

In[∘]:=
```
{FM6,-1[TL[P[1, 3]] ⋃ TL[P[4, 6]]] == mc[TL[X[1, 5, 2, 4]] ⋃ TL[X[2, 5, 3, 6]]],
 FM6,-1[Kas[P[1, 3]] ⋃ Kas[P[4, 6]]] == mc[Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]]]}
```

Out[∘]=
pdf

{True, True}

pdf

## Reidemeister 3

pdf

In[∘]:=
```
R3L = PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]];
R3R = PD[X[7, 5, 8, 4], X[8, 2, 9, 1], X[5, 3, 6, 2]];
{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R}
```

Out[∘]=
pdf

{True, True}

pdf

In[∘]:=
```
Kas@R3L
```

Out[∘]=
pdf

$$2\,\Theta\left(u-\frac{1}{2}\right)-2\,\Theta\left(u+\frac{1}{2}\right)-2$$

| | ( $\eta_{-7}$ | $\eta_3$ | $\eta_6$ | $\eta_9$ | $\eta_{-1}$ | $\eta_{-4}$ |
|---|---|---|---|---|---|---|
| $\overline{\eta}_{-7}$ | $\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $-\frac{2u}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $\frac{u\,(4u\ldots}{(2u-1)\ldots}$ |
| $\overline{\eta}_3$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{2(2u^2-1)}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $-\frac{2u}{(2u-1)(2u+1)}$ | $-\frac{\ldots}{(2u-1)\ldots}$ |
| $\overline{\eta}_6$ | $-\frac{1}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $-\frac{2\ldots}{(2u-1)\ldots}$ |
| $\overline{\eta}_9$ | $-\frac{2u}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $-\frac{\ldots}{(2u-1)\ldots}$ |
| $\overline{\eta}_{-1}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $-\frac{2u}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{2(2u^2-1)}{(2u-1)(2u+1)}$ | $\frac{u\,(4u\ldots}{(2u-1)\ldots}$ |
| $\overline{\eta}_{-4}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $-\frac{2u}{(2u-1)(2u+1)}$ | $-\frac{1}{(2u-1)(2u+1)}$ | $\frac{u(4u^2-3)}{(2u-1)(2u+1)}$ | $\frac{2u^2\,(4\ldots}{(2u-1)\ldots}$ |

pdf

# A Knot

pdf

In[◦]:= 
```
f = TLSig[Knot[8, 5]]
```

Out[◦]=
pdf

$$2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[u-\boxed{-0.630\ldots}\right]+2\,\theta\left[u-\boxed{0.630\ldots}\right]$$

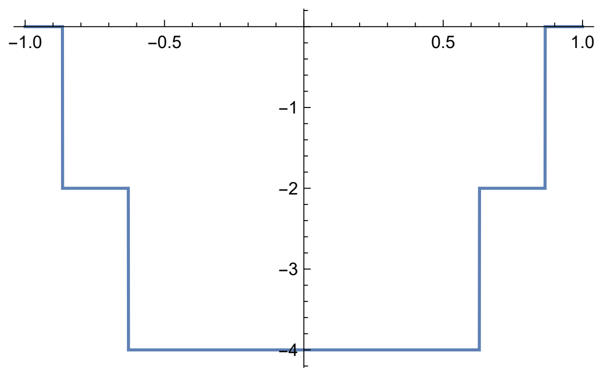
tex

{
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfOutput#1{\hfill\includegraphics[width=0.5\linewidth]{#1}}

pdf

In[◦]:= 
```
Plot[f, {u, -1, 1}]
```

Out[◦]=
pdf




tex

}

tex

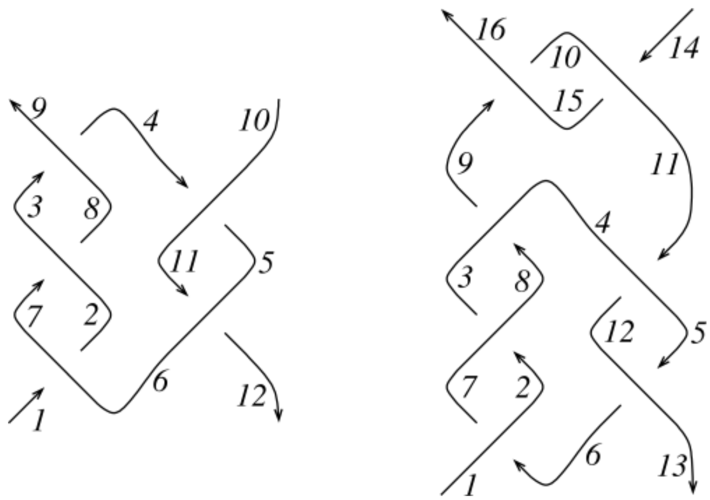
\par\parpic[r]{\includegraphics[width=2in]{CKT.pdf}}

pdf

# Some Tangles

tex

\par The  Conway-Kinoshita-Terasaka Tangles:




pdf

In[∘]:=
```
T1 = PD[X[1, 6, 2, 7],
   X[7, 2, 8, 3],
   X[3, 8, 4, 9],
   X[11, 6, 12, 5], X[4, 11, 5, 10]];
T2 = PD[X[6, 2, 7, 1], X[2, 8, 3, 7], X[8, 4, 9, 3],
   X[5, 12, 6, 13], X[11, 4, 12, 5], X[14, 10, 15, 11], X[9, 15, 10, 16]];
```

pdf

In[∘]:=
```
Column@{TL[T1], Kas[T1]}
```

Out[∘]=
pdf

$$-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

| | $(\eta_{-10}$ | $\eta_9$ | $\eta_{-1}$ | $\eta_{12})$ |
|---|---|---|---|---|
| $\overline{\eta}_{-10}$ | $0$ | $1-\omega$ | $0$ | $\omega-1$ |
| $\overline{\eta}_9$ | $\frac{\omega-1}{\omega}$ | $\frac{2\omega}{\omega^2-\omega+1}$ | $-\frac{\omega-1}{\omega}$ | $-\frac{2\omega}{\omega^2-\omega+1}$ |
| $\overline{\eta}_{-1}$ | $0$ | $\omega-1$ | $0$ | $1-\omega$ |
| $\overline{\eta}_{12}$ | $-\frac{\omega-1}{\omega}$ | $-\frac{2\omega}{\omega^2-\omega+1}$ | $\frac{\omega-1}{\omega}$ | $\frac{2\omega}{\omega^2-\omega+1}$ |

$$-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

| | $(\eta_{-10}$ | $\eta_9$ | $\eta_{-1}$ | $\eta_{12})$ |
|---|---|---|---|---|
| $\overline{\eta}_{-10}$ | $2\,(u-1)\,(u+1)\,(4u^2-3)$ | $0$ | $-2\,(u-1)\,(u+1)\,(4u^2-3)$ | $0$ |
| $\overline{\eta}_9$ | $0$ | $\frac{1}{2\,(4u^2-3)}$ | $0$ | $-\frac{1}{2\,(4u^2-3)}$ |
| $\overline{\eta}_{-1}$ | $-2\,(u-1)\,(u+1)\,(4u^2-3)$ | $0$ | $2\,(u-1)\,(u+1)\,(4u^2-3)$ | $0$ |
| $\overline{\eta}_{12}$ | $0$ | $-\frac{1}{2\,(4u^2-3)}$ | $0$ | $\frac{1}{2\,(4u^2-3)}$ |

pdf

In[∘]:=
```
Column@{TL[T2], Kas[T2]}
```

Out[∘]=
pdf

0

| | $(\eta_{-14}$ | $\eta_{16}$ | $\eta_{-1}$ | $\eta_{13})$ |
|---|---|---|---|---|
| $\overline{\eta}_{-14}$ | 0 | $1-\omega$ | 0 | $\omega-1$ |
| $\overline{\eta}_{16}$ | $\frac{\omega-1}{\omega}$ | $-\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$ | $-\frac{\omega-1}{\omega}$ | $\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$ |
| $\overline{\eta}_{-1}$ | 0 | $\omega-1$ | 0 | $1-\omega$ |
| $\overline{\eta}_{13}$ | $-\frac{\omega-1}{\omega}$ | $\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$ | $\frac{\omega-1}{\omega}$ | $-\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$ |

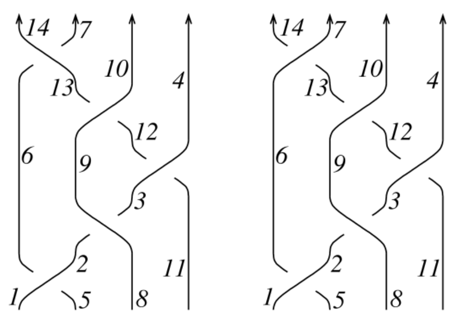
1

| | $(\eta_{-14}$ | $\eta_{16}$ | $\eta_{-1}$ | $\eta_{13})$ |
|---|---|---|---|---|
| $\overline{\eta}_{-14}$ | $\frac{1}{2}\left(-16u^4+28u^2-13\right)$ | 0 | $\frac{1}{2}\left(16u^4-28u^2+13\right)$ | 0 |
| $\overline{\eta}_{16}$ | 0 | $-\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$ | 0 | $\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$ |
| $\overline{\eta}_{-1}$ | $\frac{1}{2}\left(16u^4-28u^2+13\right)$ | 0 | $\frac{1}{2}\left(-16u^4+28u^2-13\right)$ | 0 |
| $\overline{\eta}_{13}$ | 0 | $\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$ | 0 | $-\frac{2(u-1)(u+1)}{16u^4-28u^2+13}$ |

tex

\parpic[r]{\includegraphics[width=2in]{B1B2.pdf}}

Examples with non-trivial codimension:




pdf

In[∘]:=
```
B1 = PD[X[5, 2, 6, 1],
   X[2, 8, 3, 9],
   X[11, 4, 12, 3],
   X[12, 10, 13, 9],
   X[6, 13, 7, 14]];
B2 = PD[X[5, 2, 6, 1], X[2, 8, 3, 9], X[11, 4, 12, 3], X[12, 10, 13, 9], X[13, 7, 14, 6]];
```

*pdf*

*In[*∘*]:=* **Column@{TL[B1], Kas[B1]}**

*Out[*∘*]=*
*pdf*

0

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0 | −1 | 0 | $\frac{1}{\omega}$ | 0 | $-\frac{1}{\omega}$ | 0 |
| | 0 | 0 | 0 | −1 | $\frac{1}{\omega}$ | 0 | $-\frac{1}{\omega}$ | 1 |
| | ($\eta_{-11}$ | $\eta_4$ | $\eta_{10}$ | $\eta_7$ | $\eta_{14}$ | $\eta_{-1}$ | $\eta_{-5}$ | $\eta_{-8}$) |
| $\overline{\eta}_{-11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{\eta}_4$ | 0 | 0 | 0 | 0 | $\frac{\omega-1}{\omega^2}$ | 0 | $-\frac{\omega-1}{\omega^2}$ | 0 |
| $\overline{\eta}_{10}$ | 0 | 0 | 0 | 0 | $-\frac{\omega-1}{\omega}$ | 0 | $\frac{\omega-1}{\omega}$ | 0 |
| $\overline{\eta}_7$ | 0 | 0 | 0 | 0 | $\frac{(\omega-1)^2}{\omega^2}$ | 0 | $-\frac{(\omega-1)^2}{\omega^2}$ | 0 |
| $\overline{\eta}_{14}$ | 0 | $-((\omega-1)\ \omega)$ | $\omega-1$ | $(\omega-1)^2$ | 0 | $-\frac{\omega-1}{\omega}$ | $\frac{\omega-1}{\omega}$ | 0 |
| $\overline{\eta}_{-1}$ | 0 | 0 | 0 | 0 | $\omega-1$ | 0 | $1-\omega$ | 0 |
| $\overline{\eta}_{-5}$ | 0 | $(\omega-1)\ \omega$ | $1-\omega$ | $-(\omega-1)^2$ | $1-\omega$ | $\frac{\omega-1}{\omega}$ | $\frac{(\omega-1)^2}{\omega}$ | 0 |
| $\overline{\eta}_{-8}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

0

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0 | −1 | 0 | 1 | 0 | −1 | 0 |
| | ($\eta_{-11}$ | $\eta_4$ | $\eta_{10}$ | $\eta_7$ | $\eta_{14}$ | $\eta_{-1}$ | $\eta_{-5}$ | $\eta_{-8}$) |
| $\overline{\eta}_{-11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{\eta}_4$ | 0 | 0 | 0 | −1 | $-u$ | 0 | $u$ | 1 |
| $\overline{\eta}_{10}$ | 0 | 0 | 0 | $-u$ | $1-2u^2$ | 0 | $2u^2-1$ | $u$ |
| $\overline{\eta}_7$ | 0 | −1 | $-u$ | $2u^2-3$ | $-u$ | −1 | 0 | 1 |
| $\overline{\eta}_{14}$ | 0 | $-u$ | $1-2u^2$ | $-u$ | −1 | $-u$ | $-2\ (u-1)\ (u+1)$ | $u$ |
| $\overline{\eta}_{-1}$ | 0 | 0 | 0 | −1 | $-u$ | 0 | $u$ | 1 |
| $\overline{\eta}_{-5}$ | 0 | $u$ | $2u^2-1$ | 0 | $-2\ (u-1)\ (u+1)$ | $u$ | $4u^2-3$ | 0 |
| $\overline{\eta}_{-8}$ | 0 | 1 | $u$ | 1 | $u$ | 1 | 0 | $1-2$… |

pdf

In[∘]:=
```
Column@{TL[B2], Kas[B2]}
```

Out[∘]=
pdf

$$-1$$

| | 1 | 0 | −1 | 1 | 0 | 0 | 0 | −1 |
|---|---|---|---|---|---|---|---|---|
| | $(\eta_{-11}$ | $\eta_4$ | $\eta_{10}$ | $\eta_7$ | $\eta_{14}$ | $\eta_{-1}$ | $\eta_{-5}$ | $\eta_{-8})$ |
| $\overline{\eta}_{-11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{\eta}_{4}$ | 0 | 0 | 0 | $\frac{\omega-1}{\omega}$ | 0 | 0 | 0 | $-\frac{\omega-1}{\omega}$ |
| $\overline{\eta}_{10}$ | 0 | 0 | 0 | $1-\omega$ | 0 | 0 | 0 | $\omega-1$ |
| $\overline{\eta}_{7}$ | 0 | $1-\omega$ | $\frac{\omega-1}{\omega}$ | $\frac{2\left(\omega^2-\omega+1\right)}{\omega}$ | $-\frac{\omega+1}{\omega}$ | 0 | $\frac{2}{\omega}$ | $-\frac{\omega^2-\omega+2}{\omega}$ |
| $\overline{\eta}_{14}$ | 0 | 0 | 0 | $-\omega-1$ | $\frac{\omega^2+1}{\omega}$ | $-\frac{\omega-1}{\omega}$ | $-\frac{2}{\omega}$ | 2 |
| $\overline{\eta}_{-1}$ | 0 | 0 | 0 | 0 | $\omega-1$ | 0 | $1-\omega$ | 0 |
| $\overline{\eta}_{-5}$ | 0 | 0 | 0 | $2\,\omega$ | $-2\,\omega$ | $\frac{\omega-1}{\omega}$ | $\frac{\omega^2+1}{\omega}$ | $-\omega-1$ |
| $\overline{\eta}_{-8}$ | 0 | $\omega-1$ | $-\frac{\omega-1}{\omega}$ | $-\frac{2\,\omega^2-\omega+1}{\omega}$ | 2 | 0 | $-\frac{\omega+1}{\omega}$ | $\frac{\omega^2+1}{\omega}$ |

$$2\,\theta\left(u-\frac{1}{\sqrt{2}}\right)-2\,\theta\left(u+\frac{1}{\sqrt{2}}\right)-1$$

| | $(\eta_{-11}$ | $\eta_4$ | $\eta_{10}$ | $\eta_7$ | $\eta_{14}$ | $\eta_{-1}$ | $\eta_{-5}$ … |
|---|---|---|---|---|---|---|---|
| $\overline{\eta}_{-11}$ | $\frac{8u^4-12u^2+3}{2\left(2u^2-1\right)}$ | $u$ | $\frac{(2u-1)(2u+1)}{2\left(2u^2-1\right)}$ | $\frac{u}{2u^2-1}$ | $\frac{1}{2\left(2u^2-1\right)}$ | $\frac{u}{2u^2-1}$ | $\frac{1}{2\,(2\,u^2 \ldots}$ |
| $\overline{\eta}_{4}$ | $u$ | 0 | $-u$ | $-1$ | 0 | 0 | 0 |
| $\overline{\eta}_{10}$ | $\frac{(2u-1)(2u+1)}{2\left(2u^2-1\right)}$ | $-u$ | $-\frac{8u^4-4u^2+1}{2\left(2u^2-1\right)}$ | $-\frac{2u^3}{2u^2-1}$ | $-\frac{1}{2\left(2u^2-1\right)}$ | $-\frac{u}{2u^2-1}$ | $-\frac{1}{2\,(2\,u \ldots}$ |
| $\overline{\eta}_{7}$ | $\frac{u}{2u^2-1}$ | $-1$ | $-\frac{2u^3}{2u^2-1}$ | $\frac{4u^4-6u^2+1}{2u^2-1}$ | $\frac{2(u-1)u(u+1)}{2u^2-1}$ | $-\frac{1}{2u^2-1}$ | $-\frac{u}{2\,u^2 \ldots}$ |
| $\overline{\eta}_{14}$ | $\frac{1}{2\left(2u^2-1\right)}$ | 0 | $-\frac{1}{2\left(2u^2-1\right)}$ | $\frac{2(u-1)u(u+1)}{2u^2-1}$ | $\frac{8u^4-8u^2+1}{2\left(2u^2-1\right)}$ | $\frac{2(u-1)u(u+1)}{2u^2-1}$ | $-\frac{1}{2\,(2\,u \ldots}$ |
| $\overline{\eta}_{-1}$ | $\frac{u}{2u^2-1}$ | 0 | $-\frac{u}{2u^2-1}$ | $-\frac{1}{2u^2-1}$ | $\frac{2(u-1)u(u+1)}{2u^2-1}$ | $\frac{2(u-1)(u+1)}{2u^2-1}$ | $\frac{2\,(u-1)\,u \ldots}{2\,u^2 \ldots}$ |
| $\overline{\eta}_{-5}$ | $\frac{1}{2\left(2u^2-1\right)}$ | 0 | $-\frac{1}{2\left(2u^2-1\right)}$ | $-\frac{u}{2u^2-1}$ | $-\frac{1}{2\left(2u^2-1\right)}$ | $\frac{2(u-1)u(u+1)}{2u^2-1}$ | $\frac{8\,u^4-8 \ldots}{2\,(2\,u^2 \ldots}$ |
| $\overline{\eta}_{-8}$ | $-u$ | 1 | $2u$ | 1 | 0 | 1 | $u$ |