In[∘]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
 << KnotTheory`;
]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

In[∘]:=
```
sign[x_?NumberQ] := Sign[Re[x]]
```

pdf

In[∘]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

In[∘]:=
```
CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_ | η̄_, ∞]},
  Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) ⧴ Factor[c] Times @@ ηs^ps] ]
```

pdf

In[∘]:=
```
CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs] ]
```

pdf

In[∘]:=
```
(ℰ_)* := ℰ /. {η̄ → η, η → η̄, ω → ω^-1, c_Complex ⧴ c*};
r_Rule⁺ := {r, r*}
```

In[∘]:= $\left\{\left(\left(2\,u-\omega+3\,\omega^{-1}\right)\bar\eta_1\,\eta_2\right)^*,\ \left(\eta_1\to\omega\,\eta_2\right)^+\right\}$

Out[∘]=

$$\left\{\left(2\,u-\frac{1}{\omega}+3\,\omega\right)\eta_1\,\bar\eta_2,\ \left\{\eta_1\to\omega\,\eta_2,\ \bar\eta_1\to\frac{\bar\eta_2}{\omega}\right\}\right\}$$

pdf

In[∘]:=
```
RulesOf[η_i_ + rest_.] := (η_i → -rest)⁺;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

In[∘]:= CF[{$\eta_1-\eta_2$, $\eta_1-\eta_3$}]

Out[∘]=

$\{\eta_1-\eta_3,\ \eta_2-\eta_3\}$

In[∘]:= RulesOf[$\eta_1+\eta_2+\eta_3$]

Out[∘]=

$\{\eta_1\to-\eta_2-\eta_3,\ \bar\eta_1\to-\bar\eta_2-\bar\eta_3\}$

pdf

```
In[∘]:= CF[Σb_[σ_, pq_]] := ΣCF[b][σ, CF[pq]]
```

pdf

```
In[∘]:= Format[Σb_B[σ_, PQ[rs_, q_]]] := Module[{ηs},
          ηs = η# & /@ Join @@ b;
          Join[
           Prepend[""] /@ Table[∂c r, {r, rs}, {c, ηs}],
           {Prepend[σ][
             Join @@ (b /. {l_, m___, r_} ⧴ {DisplayForm@RowBox[{"(", l}],
                   m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer ⧴ ηi
            ]},
           MapThread[Prepend, {Table[∂r,c q, {r, ηs*}, {c, ηs}], ηs*}]
          ]
         ] // MatrixForm;
```

```
In[∘]:= Kas[X[1, 2, 3, 4]]
Out[∘]=
Kas[X[1, 2, 3, 4]]
```

The disjoint union in the world of multi-tangles.

pdf

```
In[∘]:= Σb1_[σ1_, PQ[rs1_, q1_]] ⋃ Σb2_[σ2_, PQ[rs2_, q2_]] ^:=
         CF@ΣJoin[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

Gaps are named after the strand that follows them!

FM for Face Merge:

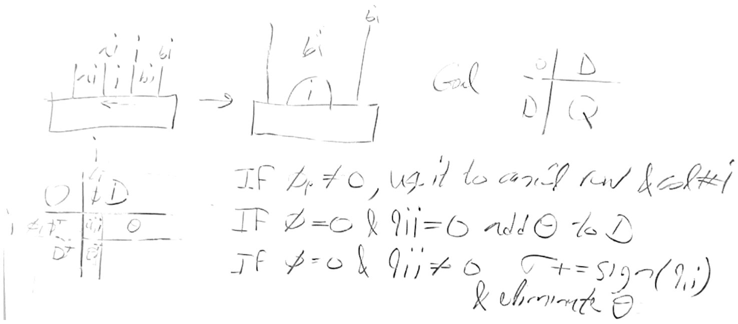
pdf

```
In[∘]:= FMi_,j_@ΣB[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[rs_, q_]] :=
         CF@ΣB[{ri,li,j,rj,lj,i},bs][σ, PQ[rs ⋃ {ηi - ηj}, q]]
```

```
In[∘]:= ΣB[{-1,2}][0, PQ[{}, 0]] ⋃ ΣB[{-3,4}][0, PQ[{}, 0]] // FM-1,4
Out[∘]=
```

	0	1	0	-1
0	(η_{-3}	η_{-1}	η_2	η_4)
$\overline{\eta}_{-3}$	0	0	0	0
$\overline{\eta}_{-1}$	0	0	0	0
$\overline{\eta}_{2}$	0	0	0	0
$\overline{\eta}_{4}$	0	0	0	0

pdf



pdf

In[∘]:=
```
Cordon_{i_}@Σ_{B[{li___,i_,ri___},bs___]}[σ_, PQ[rs_, q_]] :=
 Module[{ϕ = ∂_{η_i} rs, nσ = σ, nrs = rs, nq = q, qii, p},
  Which[
   Or @@ ((# =!= 0) & /@ ϕ), ({p} = FirstPosition[(# === 0) & /@ ϕ, False];
    {nrs, nq} = {rs, q} /. (η_i → -rs⟦p⟧ / ϕ⟦p⟧)^+ /. (η_i → 0)^+),
   (qii = ∂_{η̄_i, η_i} q) =!= 0, (nσ += sign[qii];
    nq = q /. (η_i → -(∂_{η̄_i} q) / qii)^+ /. (η_i → 0)^+),
   qii === 0, AppendTo[nrs, ∂_{η̄_i} q]; nq = q /. (η_i → 0)^+];
  CF@Σ_{B[Most@{ri,li},bs]}[nσ, PQ[nrs, nq] /. (η_{Last@{ri,li}} → η_{First@{ri,li}})^+] ]
```

tex

c for contract:

pdf

In[∘]:=
```
c_{i_,j_}@t : Σ_{B[{li___,i_,ri___},{___,j_,___},___]}[__] := t // FM_{j,First@{ri,li}} // Cordon_j
```

pdf

In[∘]:=
```
c_{i_,j_}@t : Σ_{B[{___,i_,j_,___},___]}[__] := Cordon_j@t
c_{i_,j_}@t : Σ_{B[{j_,___,i_},___]}[__] := Cordon_j@t
c_{i_,j_}@t : Σ_{B[{___,j_,i_,___},___]}[__] := Cordon_i@t
c_{i_,j_}@t : Σ_{B[{i_,___,j_},___]}[__] := Cordon_i@t
```

tex

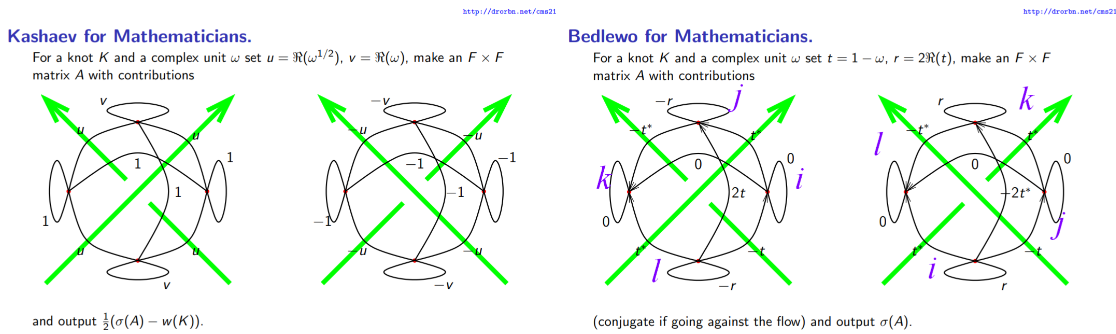
mc for magnetic contract:

pdf

In[∘]:=
```
mc[ℰ_] := ℰ //.
  t : Σ_{B[{___,i_,___},{___,j_,___},___]}[__] | Σ_{B[{___,i_,j_,___},___]}[__] | Σ_{B[{j_,___,i_},___]}[__] /;
    i + j == 0 ⧴ c_{i,j}@t
```

pdf

In[∘]:=
```
Kas[P[i_, j_]] := CF@Σ_{B[{-i,j}]}[0, PQ[{}, 0]];
TL[P[i_, j_]] := CF@Σ_{B[{-i,j}]}[0, PQ[{}, 0]]
```



pdf

In[∘]:=
```
Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x],
    X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
Kas[x : (X₊ | X₋)[__]] := Module[{v = 2 u^2 - 1, p, fs, ηs, m},
  ηs = η# & /@ (fs = List @@ x); p = Head[x] === X₊;
  m = If[p, ( v u 1 u
              u 1 u 1
              1 u v u
              u 1 u 1 ), - ( v u 1 u
                             u 1 u 1
                             1 u v u
                             u 1 u 1 )];
  CF@Σ_B[fs][If[p, -1, 1], PQ[{}, ηs*.m.ηs]]]
```

In[∘]:=
```
Kas /@ {X₊[1, 2, 3, 4], X₋[1, 4, 3, 2]}
```

Out[∘]=

$$\left\{\begin{pmatrix} -1 & (\eta_1 & \eta_2 & \eta_3 & \eta_4) \\ \overline{\eta}_1 & -1+2u^2 & u & 1 & u \\ \overline{\eta}_2 & u & 1 & u & 1 \\ \overline{\eta}_3 & 1 & u & -1+2u^2 & u \\ \overline{\eta}_4 & u & 1 & u & 1 \end{pmatrix}, \begin{pmatrix} 1 & (\eta_1 & \eta_4 & \eta_3 & \eta_2) \\ \overline{\eta}_1 & 1-2u^2 & -u & -1 & -u \\ \overline{\eta}_4 & -u & -1 & -u & -1 \\ \overline{\eta}_3 & -1 & -u & 1-2u^2 & -u \\ \overline{\eta}_2 & -u & -1 & -u & -1 \end{pmatrix}\right\}$$

pdf

In[∘]:=
```
TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x],
    X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
TL[x : (X₊ | X₋)[__]] := Module[{t = 1 - ω, r, p, fs, ηs, m},
  r = t + t*; ηs = η# & /@ (fs = List @@ x); p = Head[x] === X₊;
  m = If[p, ( -r   -t  2 t  t*
              -t*  0   t*   0
              2 t* t   -r   -t*
              t    0   -t   0 ), ( r    -t  -2 t*  t*
                                   -t*  0   t*     0
                                   -2 t t   r      -t*
                                   t    0   -t     0 )];
  CF@Σ_B[fs][0, PQ[{}, ηs*.m.ηs]]]
```

In[∘]:=
```
TL /@ {X[1, 2, 3, 4], X[1, 4, 3, 2]}
```

Out[∘]=

$$\left\{\begin{pmatrix} 0 & (\eta_{-4} & \eta_{-1} & \eta_{2} & \eta_{3}) \\ \overline{\eta}_{-4} & 0 & 1-\omega & 0 & -1+\omega \\ \overline{\eta}_{-1} & \frac{-1+\omega}{\omega} & \frac{(-1+\omega)^2}{\omega} & -1+\omega & -2\,(-1+\omega) \\ \overline{\eta}_{2} & 0 & -\frac{-1+\omega}{\omega} & 0 & \frac{-1+\omega}{\omega} \\ \overline{\eta}_{3} & -\frac{-1+\omega}{\omega} & \frac{2\,(-1+\omega)}{\omega} & 1-\omega & \frac{(-1+\omega)^2}{\omega} \end{pmatrix}, \begin{pmatrix} 0 & (\eta_{-4} & \eta_{3} & \eta_{2} & \eta_{-1}) \\ \overline{\eta}_{-4} & -\frac{(-1+\omega)^2}{\omega} & -1+\omega & -\frac{2\,(-1+\omega)}{\omega} & \frac{-1+\omega}{\omega} \\ \overline{\eta}_{3} & -\frac{-1+\omega}{\omega} & 0 & \frac{-1+\omega}{\omega} & 0 \\ \overline{\eta}_{2} & 2\,(-1+\omega) & 1-\omega & -\frac{(-1+\omega)^2}{\omega} & -\frac{-1+\omega}{\omega} \\ \overline{\eta}_{-1} & 1-\omega & 0 & -1+\omega & 0 \end{pmatrix}\right\}$$

pdf

In[∘]:=
```
Kas[K_] := Fold[mc[#1 ⋃ #2] &, ΣB[][0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
KasSig[K_] := Kas[K]〚1〛 / 2
```

In[∘]:=
```
Kas[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

$$\left(3+\text{sign}\left[\frac{1}{2}\left(3-4\,u^2\right)\right]+\text{sign}\left[-2\left(-1+2\,u^2\right)\right]+\text{sign}\left[-\frac{-3+4\,u^2}{-1+2\,u^2}\right]\right)$$

In[∘]:=
```
KasSig[Knot[3, 1]]
```

Out[∘]=

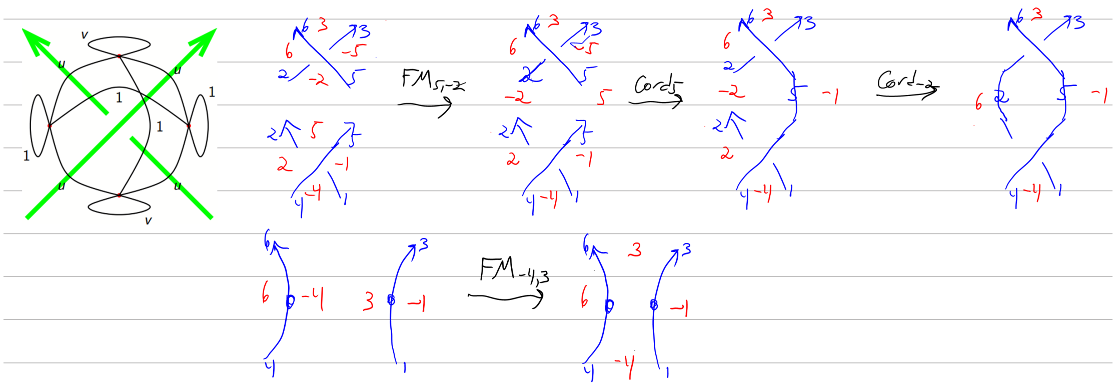
$$\frac{1}{2}\left(3+\text{sign}\left[\frac{1}{2}\left(3-4\,u^2\right)\right]+\text{sign}\left[-2\left(-1+2\,u^2\right)\right]+\text{sign}\left[-\frac{-3+4\,u^2}{-1+2\,u^2}\right]\right)$$

pdf

In[∘]:=
```
TL[K_] := Fold[mc[#1 ⋃ #2] &, ΣB[][0, PQ[{}, 0]], List @@ (TL /@ PD@K)];
TLSig[K_] := TL[K]〚1〛
```

pdf

Reidemeister 2



In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]]**

Out[∘]=

0	$(\eta_{-5}$	η_3	η_6	$\eta_{-2})$	$(\eta_{-4}$	η_{-1}	η_5	$\eta_2)$
$\overline{\eta}_{-5}$	$1-2u^2$	$-u$	-1	$-u$	0	0	0	0
$\overline{\eta}_3$	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_6$	-1	$-u$	$1-2u^2$	$-u$	0	0	0	0
$\overline{\eta}_{-2}$	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_{-4}$	0	0	0	0	1	u	1	u
$\overline{\eta}_{-1}$	0	0	0	0	u	$-1+2u^2$	u	1
$\overline{\eta}_5$	0	0	0	0	1	u	1	u
$\overline{\eta}_2$	0	0	0	0	u	1	u	$-1+2u^2$

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // $FM_{5,-2}$**

Out[∘]=

	0	0	0	-1	0	0	0	1
0	$(\eta_{-5}$	η_3	η_6	η_5	η_2	η_{-4}	η_{-1}	$\eta_{-2})$
$\overline{\eta}_{-5}$	$1-2u^2$	$-u$	-1	$-u$	0	0	0	0
$\overline{\eta}_3$	$-u$	-1	$-u$	-1	0	0	0	0
$\overline{\eta}_6$	-1	$-u$	$1-2u^2$	$-u$	0	0	0	0
$\overline{\eta}_5$	$-u$	-1	$-u$	0	u	1	u	0
$\overline{\eta}_2$	0	0	0	u	$-1+2u^2$	u	1	0
$\overline{\eta}_{-4}$	0	0	0	1	u	1	u	0
$\overline{\eta}_{-1}$	0	0	0	u	1	u	$-1+2u^2$	0
$\overline{\eta}_{-2}$	0	0	0	0	0	0	0	0

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // $FM_{5,-2}$ // $Cordon_{-2}$**

Out[∘]=

0	$(\eta_{-5}$	η_3	η_6	η_5	η_2	$\eta_{-4})$
$\overline{\eta}_{-5}$	0	$-u$	-1	0	1	u
$\overline{\eta}_3$	$-u$	-1	$-u$	-1	0	0
$\overline{\eta}_6$	-1	$-u$	$1-2u^2$	$-u$	0	0
$\overline{\eta}_5$	0	-1	$-u$	0	u	1
$\overline{\eta}_2$	1	0	0	u	$-1+2u^2$	u
$\overline{\eta}_{-4}$	u	0	0	1	u	1

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // $FM_{-2,5}$ // $Cordon_5$ // $Cordon_{-2}$**

Out[∘]=

	0	-1	0	1
0	$(\eta_{-5}$	η_3	η_2	$\eta_{-4})$
$\overline{\eta}_{-5}$	0	0	0	0
$\overline{\eta}_3$	0	0	0	0
$\overline{\eta}_2$	0	0	0	0
$\overline{\eta}_{-4}$	0	0	0	0

pdf

In[•]:= {Kas[P[1, 3]] ⋃ Kas[P[4, 6]] // $FM_{-4,3}$, Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // mc}

Out[•]=
pdf

$$\left\{\begin{pmatrix} & 1 & 0 & -1 & 0 \\ 0 & (\eta_{-4} & \eta_6 & \eta_3 & \eta_{-1}) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} & 0 & 1 & 0 & -1 \\ 0 & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

pdf

In[•]:= {TL[P[1, 3]] ⋃ TL[P[4, 6]] // $FM_{-4,3}$, TL[X[1, 5, 2, 4]] ⋃ TL[X[2, 5, 3, 6]] // mc}

Out[•]=
pdf

$$\left\{\begin{pmatrix} & 1 & 0 & -1 & 0 \\ 0 & (\eta_{-4} & \eta_6 & \eta_3 & \eta_{-1}) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} & 0 & 1 & 0 & -1 \\ 0 & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \overline{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \overline{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \overline{\eta}_3 & 0 & 0 & 0 & 0 \\ \overline{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

pdf

Reidemeister 3

In[•]:=
```
{u = 7 / 29};
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // c2,-2 // c5,-5 // c8,-8
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // c2,-2 // c5,-5 // c8,-8
Clear[u]
```

Out[•]=

$$\begin{pmatrix} -4 & (\eta_{-7} & \eta_{3} & \eta_{6} & \eta_{9} & \eta_{-1} & \eta_{-4}) \\ \overline{\eta}_{-7} & \frac{228\,046}{542\,445} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} \\ \overline{\eta}_{3} & \frac{16\,289}{18\,705} & \frac{1486}{645} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} \\ \overline{\eta}_{6} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{228\,046}{542\,445} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} \\ \overline{\eta}_{9} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{228\,046}{542\,445} & \frac{16\,289}{18\,705} & \frac{841}{645} \\ \overline{\eta}_{-1} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{1486}{645} & \frac{16\,289}{18\,705} \\ \overline{\eta}_{-4} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{228\,046}{542\,445} \end{pmatrix}$$

Out[•]=

$$\begin{pmatrix} -4 & (\eta_{-7} & \eta_{3} & \eta_{6} & \eta_{9} & \eta_{-1} & \eta_{-4}) \\ \overline{\eta}_{-7} & \frac{228\,046}{542\,445} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} \\ \overline{\eta}_{3} & \frac{16\,289}{18\,705} & \frac{1486}{645} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} \\ \overline{\eta}_{6} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{228\,046}{542\,445} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} \\ \overline{\eta}_{9} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{228\,046}{542\,445} & \frac{16\,289}{18\,705} & \frac{841}{645} \\ \overline{\eta}_{-1} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{1486}{645} & \frac{16\,289}{18\,705} \\ \overline{\eta}_{-4} & \frac{16\,289}{18\,705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16\,289}{18\,705} & \frac{228\,046}{542\,445} \end{pmatrix}$$

pdf

In[∘]:=
```
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // mc;
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // mc
{lhs⟦1⟧, rhs⟦1⟧}
Simplify[lhs⟦2, 2⟧ == rhs⟦2, 2⟧]
```

Out[∘]=
pdf

$-3+\text{sign}[(-1+2u)(1+2u)]$	η_{-7}	η_3	η_6	η_9	η
$\bar\eta_{-7}$	$\frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{}{(-1+2}$
$\bar\eta_3$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2(-1+2u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{}{(-1+2}$
$\bar\eta_6$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{}{(-1+2}$
$\bar\eta_9$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{u(-3}{(-1+2u}$
$\bar\eta_{-1}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$\frac{2(-1}{(-1+2u}$
$\bar\eta_{-4}$	$\frac{u(-3+4u^2)}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$-\frac{2u}{(-1+2u)(1+2u)}$	$-\frac{1}{(-1+2u)(1+2u)}$	$\frac{u(-3}{(-1+2u}$

Out[∘]=
pdf

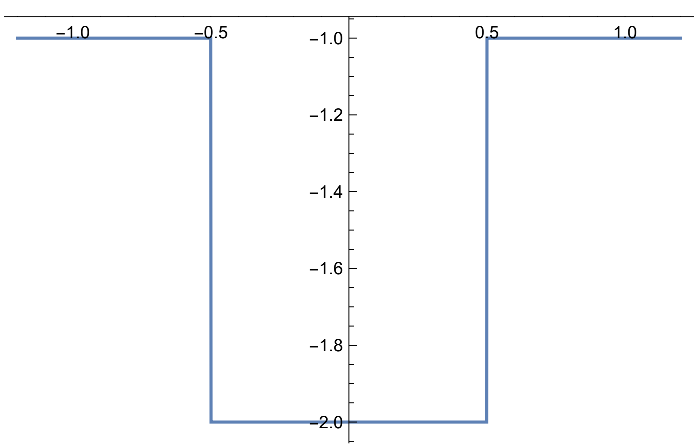
{−3 + sign[(−1 + 2 u) (1 + 2 u)], −3 + sign[(−1 + 2 u) (1 + 2 u)]}

Out[∘]=
pdf

True

In[∘]:=
```
f = KasSig@PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]];
Plot[f, {u, -1.2, 1.2}]
```

Out[∘]=



pdf

In[∘]:=
```
lhs = TL[X[4, 2, 5, 1]] ⋃ TL[X[7, 3, 8, 2]] ⋃ TL[X[8, 6, 9, 5]] // mc;
rhs = TL[X[7, 5, 8, 4]] ⋃ TL[X[8, 2, 9, 1]] ⋃ TL[X[5, 3, 6, 2]] // mc
{lhs⟦1⟧, rhs⟦1⟧}
lhs⟦2, 2⟧ == rhs⟦2, 2⟧
```

Out[∘]=
pdf

$$\begin{pmatrix} \text{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right] & (\eta_{-7} & \eta_3 & \eta_6 & \eta_9 & \eta_{-1} & \eta_{-4}) \\ \overline{\eta}_{-7} & \frac{1+\omega^2}{\omega} & -1+\omega & -2\omega & 2 & 0 & -\frac{1+\omega}{\omega} \\ \overline{\eta}_3 & -\frac{-1+\omega}{\omega} & 0 & \frac{-1+\omega}{\omega} & 0 & 0 & 0 \\ \overline{\eta}_6 & -\frac{2}{\omega} & 1-\omega & \frac{1+\omega^2}{\omega} & -\frac{1+\omega}{\omega} & 0 & \frac{2}{\omega} \\ \overline{\eta}_9 & 2 & 0 & -1-\omega & \frac{1+\omega^2}{\omega} & -\frac{-1+\omega}{\omega} & -\frac{2}{\omega} \\ \overline{\eta}_{-1} & 0 & 0 & 0 & -1+\omega & 0 & 1-\omega \\ \overline{\eta}_{-4} & -1-\omega & 0 & 2\omega & -2\omega & \frac{-1+\omega}{\omega} & \frac{1+\omega^2}{\omega} \end{pmatrix}$$

Out[∘]=
pdf

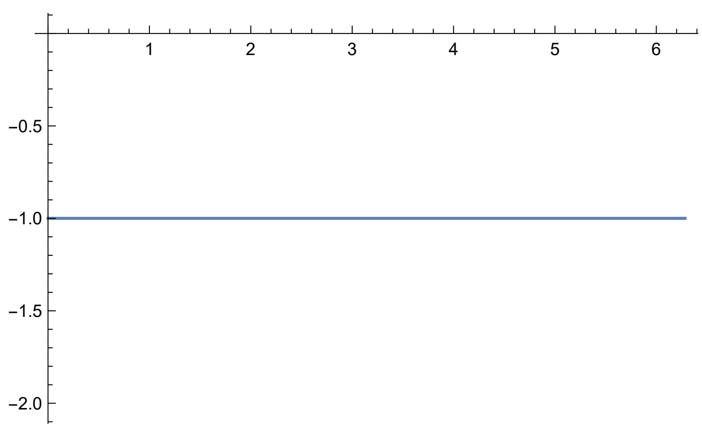
$$\left\{\text{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right], \text{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right]\right\}$$

Out[∘]=
pdf

True

In[∘]:=
```
f = TLSig@PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]] /. ω → e^(i t);
Plot[f, {t, 0, 2 π}]
```

Out[∘]=



Kashaev for Knots

In[∘]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
(*u=0;*)
Kas[Knot[3, 1]]
Clear[u]
```

Out[∘]=

$$\left(3 + \text{sign}\left[\frac{1}{2}\left(3 - 4u^2\right)\right] + \text{sign}\left[-2\left(-1 + 2u^2\right)\right] + \text{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right]\right)$$

In[∘]:=

$$\Sigma_{B[]}\left[\text{sign}\left[\frac{1}{2}\left(3 - 4u^2\right)\right] + \text{sign}\left[-2\left(-1 + 2u^2\right)\right] + \text{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right], \text{PQ}[\{\}, 0]\right]$$

Out[∘]=

$$\left(\text{sign}\left[\frac{1}{2}\left(3 - 4u^2\right)\right] + \text{sign}\left[-2\left(-1 + 2u^2\right)\right] + \text{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right]\right)$$

In[◦]:= **Table[K → 2 KasSig[K], {K, AllKnots[{3, 7}]}] // Column**

Out[◦]=

$$\text{Knot}[3,1]\to 3+\text{sign}\left[\tfrac{1}{2}\left(3-4u^2\right)\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\tfrac{-3+4u^2}{-1+2u^2}\right]$$

$$\text{Knot}[4,1]\to 1+\text{sign}\left[-3+2u^2\right]+\text{sign}\left[-\tfrac{-5+4u^2}{-3+4u^2}\right]+\text{sign}\left[-\tfrac{\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(-3+2u^2\right)}\right]$$

$$\text{Knot}[5,1]\to 5+2\,\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\tfrac{1-8u^2+8u^4}{-1+2u^2}\right]+\text{sign}\left[-\tfrac{5-20u^2+16u^4}{4\left(-1+2u^2\right)}\right]+\text{sign}\left[-\tfrac{5-20u^2+16u^4}{1-8u^2+8u^4}\right]$$

$$\text{Knot}[5,2]\to 4+\text{sign}\left[\tfrac{1}{4}\left(7-8u^2\right)\right]+\text{sign}\left[-2\left(-2+3u^2\right)\right]+\text{sign}\left[-\tfrac{-3+4u^2}{-2+3u^2}\right]+\text{sign}\left[-\tfrac{-7+8u^2}{-3+4u^2}\right]$$

$$\text{Knot}[6,1]\to 3+\text{sign}\left[\tfrac{1}{2}\left(9-8u^2\right)\right]+\text{sign}\left[-3+2u^2\right]+\text{sign}\left[-\tfrac{-5+4u^2}{-3+2u^2}\right]+\text{sign}\left[-\tfrac{-7+6u^2}{-5+4u^2}\right]+\text{sign}\left[-\tfrac{-9+8u^2}{-7+6u^2}\right]$$

$$\text{Knot}[6,2]\to 3+\text{sign}\left[-3+2u^2\right]+\text{sign}\left[-\tfrac{\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(-3+2u^2\right)}\right]+\text{sign}\left[-\tfrac{8(-1+u)(1+u)\left(3-12u^2+8u^4\right)}{\left(-5+4u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\tfrac{11-28u^2+16u^4}{8(-1+u)(1+u)}\right]+\text{sign}\left[-\tfrac{11-28u^2+16u^4}{3-12u^2+8u^4}\right]$$

$$\text{Knot}[6,3]\to \text{sign}\left[3-4u^2\right]+\text{sign}\left[-3+4u^2\right]+\text{sign}\left[\tfrac{13-28u^2+16u^4}{8(-1+u)(1+u)}\right]+\text{sign}\left[\tfrac{13-28u^2+16u^4}{5-12u^2+8u^4}\right]+\text{sign}\left[-\tfrac{8(-1+u)(1+u)\left(5-12u^2+8u^4\right)}{\left(-3+4u^2\right)\left(5-20u^2+16u^4\right)}\right]+\text{sign}\left[-\tfrac{5-20u^2+16u^4}{-3+4u^2}\right]$$

$$\text{Knot}[7,1]\to 7+3\,\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\tfrac{1-8u^2+8u^4}{-1+2u^2}\right]+\text{sign}\left[-\tfrac{(-1+2u)(1+2u)\left(-3+4u^2\right)\left(1-16u^2+16u^4\right)}{4\left(-1+2u^2\right)\left(1-8u^2+8u^4\right)}\right]+\text{sign}\left[-\tfrac{-7+56u^2-112u^4+64u^6}{2(-1+2u)(1+2u)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\tfrac{-7+56u^2-112u^4+64u^6}{\left(-1+2u^2\right)\left(1-16u^2+16u^4\right)}\right]$$

$$\text{Knot}[7,2]\to 5+\text{sign}\left[\tfrac{1}{6}\left(11-12u^2\right)\right]+\text{sign}\left[-2\left(-3+4u^2\right)\right]+\text{sign}\left[-\tfrac{3\left(-5+6u^2\right)}{2\left(-9+11u^2\right)}\right]+\text{sign}\left[-\tfrac{-9+11u^2}{2\left(-3+4u^2\right)}\right]+\text{sign}\left[-\tfrac{-11+12u^2}{-5+6u^2}\right]$$

$$\text{Knot}[7,3]\to -5+\text{sign}\left[2\left(-1+2u^2\right)\right]+\text{sign}\left[\tfrac{4}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[\tfrac{\left(-5+8u^2\right)\left(3-18u^2+16u^4\right)}{4\left(-1+2u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[\tfrac{13-44u^2+32u^4}{2\left(-5+8u^2\right)}\right]+\text{sign}\left[\tfrac{13-44u^2+32u^4}{3-18u^2+16u^4}\right]$$

$$\text{Knot}[7,4]\to -6+\text{sign}\left[2\left(-2+3u^2\right)\right]+\text{sign}\left[\tfrac{-3+4u^2}{-2+3u^2}\right]+\text{sign}\left[\tfrac{-7+8u^2}{-3+4u^2}\right]+\text{sign}\left[\tfrac{-11+12u^2}{-7+8u^2}\right]+\text{sign}\left[\tfrac{1}{4}\left(-15+16u^2\right)\right]+\text{sign}\left[\tfrac{-15+16u^2}{-11+12u^2}\right]$$

$$\text{Knot}[7,5]\to 7+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\tfrac{2\left(-2+3u^2\right)}{-1+2u^2}\right]+\text{sign}\left[-\tfrac{\left(-1+2u^2\right)\left(-3+4u^2\right)}{-2+3u^2}\right]+\text{sign}\left[-\tfrac{2\left(-1+2u^2\right)\left(-5+6u^2\right)}{-3+4u^2}\right]+\text{sign}\left[-\tfrac{\left(-7+8u^2\right)\left(-5+8u^2\right)}{4\left(-1+2u^2\right)\left(-5+6u^2\right)}\right]+\text{sign}\left[-\tfrac{17-48u^2+32u^4}{\left(-1+2u^2\right)\left(-7+8u^2\right)}\right]+\text{sign}\left[-\tfrac{17-48u^2+32u^4}{2\left(-5+8u^2\right)}\right]$$

$$\text{Knot}[7,6]\to 3+\text{sign}\left[4(-1+u)(1+u)\right]+\text{sign}\left[3-4u^2\right]+\text{sign}\left[-\tfrac{\left(-2-u+2u^2\right)\left(-2+u+2u^2\right)\left(-3+4u^2\right)}{5-10u^2+4u^4}\right]+\text{sign}\left[\tfrac{5-10u^2+4u^4}{(-1+u)(1+u)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\tfrac{\left(-3+2u^2\right)\left(7-16u^2+8u^4\right)}{\left(-2-u+2u^2\right)\left(-2+u+2u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\tfrac{19-36u^2+16u^4}{4\left(-3+2u^2\right)}\right]+\text{sign}\left[-\tfrac{19-36u^2+16u^4}{7-16u^2+8u^4}\right]$$

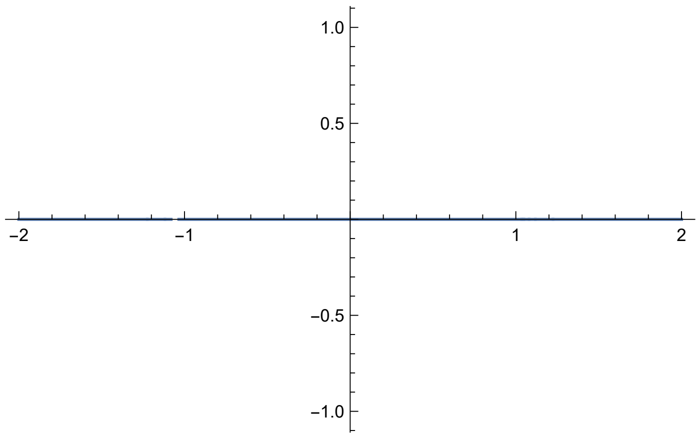
$$\text{Knot}[7,7]\to \text{sign}\left[-3+2u^2\right]+\text{sign}\left[-\tfrac{8(-1+u)(1+u)}{-3+4u^2}\right]+\text{sign}\left[-\tfrac{\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(-3+2u^2\right)}\right]+\text{sign}\left[-\tfrac{13-20u^2+8u^4}{-5+4u^2}\right]+\text{sign}\left[\tfrac{21-36u^2+16u^4}{13-28u^2+16u^4}\right]+\text{sign}\left[\tfrac{\left(21-36u^2+16u^4\right)\left(13-28u^2+16u^4\right)}{8(-1+u)(1+u)\left(13-20u^2+8u^4\right)}\right]$$

In[*]:=
```
f = KasSig[Knot[10, 1]]
Plot[f, {u, -2, 2}]
```

Out[*]=

$$\frac{1}{2}\left(5+\text{sign}\left[\frac{1}{2}\left(17-16\,u^2\right)\right]+\text{sign}\left[-3+2\,u^2\right]+\text{sign}\left[-\frac{-5+4\,u^2}{-3+2\,u^2}\right]+\text{sign}\left[-\frac{-7+6\,u^2}{-5+4\,u^2}\right]+\text{sign}\left[-\frac{-11+10\,u^2}{2\left(-7+6\,u^2\right)}\right]+\text{sign}\left[-\frac{-15+14\,u^2}{2\left(-11+10\,u^2\right)}\right]+\text{sign}\left[-\frac{-17+16\,u^2}{-15+14\,u^2}\right]\right)$$

Out[*]=

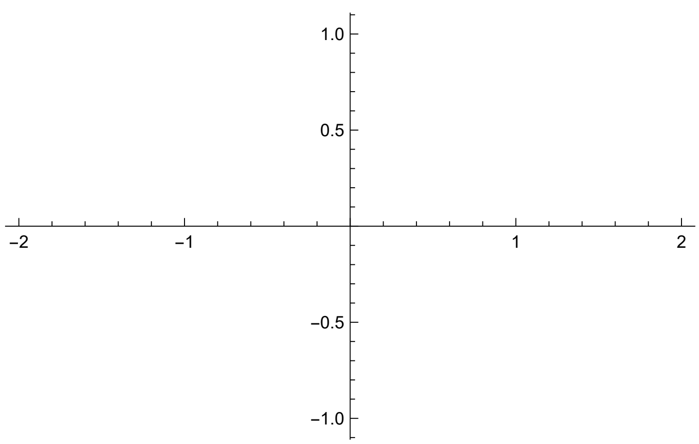


In[*]:=
```
f = TLSig[Knot[10, 1]]
Plot[f, {u, -2, 2}]
```

Out[*]=

$$\text{sign}\left[\frac{2\,(-1+\omega)^2}{\omega}\right]+\text{sign}\left[-\frac{2\left(4-9\,\omega+4\,\omega^2\right)}{\omega}\right]$$

Out[*]=



In[*]:=
```
u = 1 / 2;
KasSig /@ AllKnots[{3, 8}]
Clear[u]
```

Out[*]=

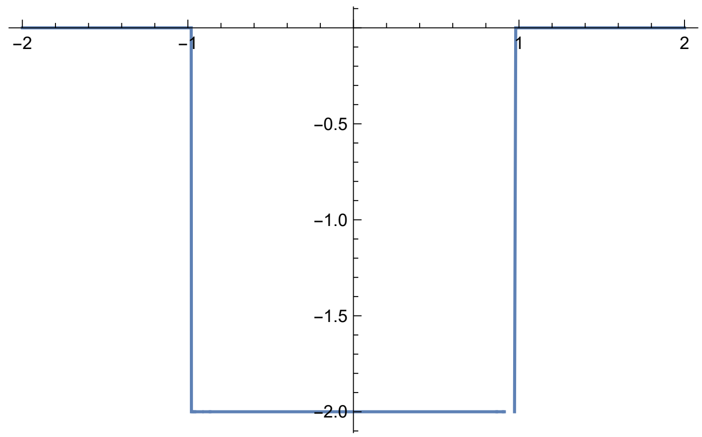
{2, 0, 4, 2, 0, 2, 0, 4, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -4, 0, 2}

In[ ]:=
```
f = KasSig[Knot[9, 5]]
Plot[f, {u, -2, 2}]
```

Out[ ]=

$$\frac{1}{2}\left(-7+\text{sign}\left[2\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{3\left(-5+6u^2\right)}{2\left(-9+11u^2\right)}\right]+\text{sign}\left[\frac{-9+11u^2}{2\left(-3+4u^2\right)}\right]+\text{sign}\left[\frac{-11+12u^2}{-5+6u^2}\right]+\text{sign}\left[\frac{-17+18u^2}{-11+12u^2}\right]+\text{sign}\left[\frac{1}{6}\left(-23+24u^2\right)\right]+\text{sign}\left[\frac{-23+24u^2}{-17+18u^2}\right]\right)$$

Out[ ]=

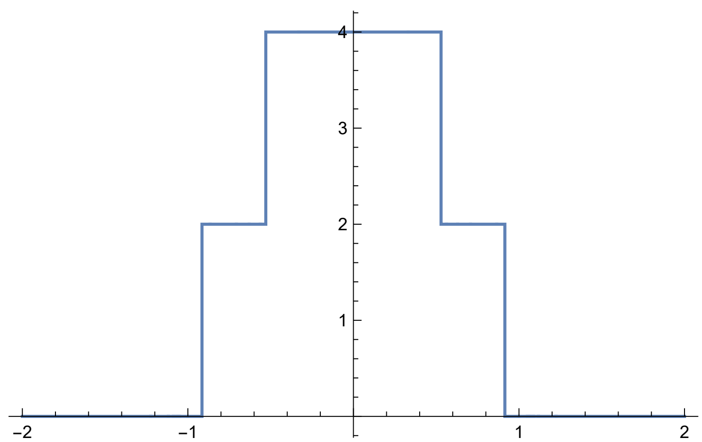


In[∘]:=
```
f = KasSig[Knot[8, 2]]
Plot[f, {u, -2, 2}, PlotPoints → 1000]
```

Out[∘]=

$$\frac{1}{2}\left(5+\text{sign}\left[-3+2u^2\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(-3+2u^2\right)}\right]+\text{sign}\left[-\frac{8(-1+u)(1+u)\left(3-12u^2+8u^4\right)}{\left(-5+4u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(1-10u^2+8u^4\right)\left(7-24u^2+16u^4\right)}{8(-1+u)(1+u)\left(-1+2u^2\right)\left(3-12u^2+8u^4\right)}\right]+\text{sign}\left[-\frac{-17+96u^2-144u^4+64u^6}{\left(-3+4u^2\right)\left(1-10u^2+8u^4\right)}\right]+\text{sign}\left[-\frac{-17+96u^2-144u^4+64u^6}{2\left(7-24u^2+16u^4\right)}\right]\right)$$

Out[∘]=



In[∘]:=
```
f = KasSig[Knot[12, Alternating, 422]]
Plot[f, {u, -1, 1}, PlotPoints → 1000]
```

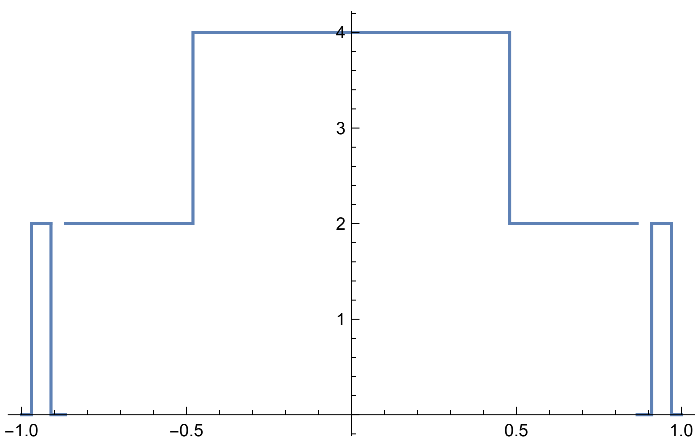
KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[∘]=

$$\frac{1}{2}\left(4+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[\frac{2}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{-7+8u^2}{2\left(-3+4u^2\right)}\right]+\right.$$
$$\text{sign}\left[-\frac{2\left(8-23u^2+16u^4\right)}{-7+8u^2}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)^2\left(11-28u^2+16u^4\right)}{-44+155u^2-176u^4+64u^6}\right]+$$
$$\text{sign}\left[\frac{-44+155u^2-176u^4+64u^6}{8-23u^2+16u^4}\right]+\text{sign}\left[-\frac{-11+76u^2-128u^4+64u^6}{11-28u^2+16u^4}\right]+$$
$$\text{sign}\left[-\frac{\left(-29+160u^2-256u^4+128u^6\right)\left(11-170u^2+544u^4-640u^6+256u^8\right)}{4\left(-1+2u^2\right)\left(-3+4u^2\right)^2\left(-11+76u^2-128u^4+64u^6\right)}\right]+$$
$$\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)}{11-228u^2+864u^4-1152u^6+512u^8}\right]+$$
$$\left.\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)\left(11-228u^2+864u^4-1152u^6+512u^8\right)}{2\left(-29+160u^2-256u^4+128u^6\right)\left(11-170u^2+544u^4-640u^6+256u^8\right)}\right]\right)$$

Out[∘]=



Tristram-Levine for Knots

In[∘]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

Out[∘]=
```
{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}
```

In[∘]:=
```
TL[Knot[3, 1]]
```
Out[∘]=

$$\left(\text{sign}\left[-\frac{2(-1+\omega)^2}{\omega}\right]+\text{sign}\left[-\frac{2(1-\omega+\omega^2)}{\omega}\right]\right)$$

In[∘]:=
```
ω = -1;
TLSig /@ AllKnots[{3, 8}]
Clear[ω]
```
Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4, 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
sign[c_ * ℰ_] /; NumberQ[c] := Sign[c] sign[ℰ]
```

In[∘]:=
```
Table[K → TLSig[K], {K, AllKnots[{3, 8}]}] // Column
```
Out[∘]=

Knot[3, 1] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]$

Knot[4, 1] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]$

Knot[5, 1] → $-2\,\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{1+\omega^2}{\omega}\right]-\text{sign}\left[\frac{1-\omega+\omega^2-\omega^3+\omega^4}{\omega(1+\omega^2)}\right]$

Knot[5, 2] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{2-3\omega+2\omega^2}{\omega}\right]$

Knot[6, 1] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-2+\omega)(-1+2\omega)}{\omega}\right]$

Knot[6, 2] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$

Knot[6, 3] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{1-3\omega+5\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$

Knot[7, 1] → $-3\,\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{1+\omega^2}{\omega}\right]-\text{sign}\left[\frac{(1-\omega+\omega^2)(1+\omega+\omega^2)}{\omega(1+\omega^2)}\right]-\text{sign}\left[\frac{1-\omega+\omega^2-\omega^3+\omega^4-\omega^5+\omega^6}{\omega(1-\omega+\omega^2)(1+\omega+\omega^2)}\right]$

Knot[7, 2] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{3-5\omega+3\omega^2}{\omega}\right]$

Knot[7, 3] → $2\,\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]+\text{sign}\left[\frac{2-\omega+2\omega^2}{\omega}\right]+\text{sign}\left[\frac{2-3\omega+3\omega^2-3\omega^3+2\omega^4}{\omega(2-\omega+2\omega^2)}\right]$

Knot[7, 4] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]+\text{sign}\left[\frac{4-7\omega+4\omega^2}{\omega}\right]$

Knot[7, 5] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^2(2-\omega+2\omega^2)}{\omega(1-\omega+\omega^2)}\right]-\text{sign}\left[\frac{2-4\omega+5\omega^2-4\omega^3+2\omega^4}{\omega(2-\omega+2\omega^2)}\right]$

Knot[7, 6] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^2(1-4\omega+\omega^2)}{\omega(1-3\omega+\omega^2)}\right]+\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{1-5\omega+7\omega^2-5\omega^3+\omega^4}{\omega(1-4\omega+\omega^2)}\right]$

Knot[7, 7] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{1-5\omega+9\omega^2-5\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$

Knot[8, 1] → $\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{3-7\omega+3\omega^2}{\omega}\right]$

Knot[8, 2] → $-\text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{1-2\omega+\omega^2-2\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]-\text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+3\omega^4-3\omega^5+\omega^6}{\omega(1-2\omega+\omega^2-2\omega^3+\omega^4)}\right]$

Knot[8, 3] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]+\text{sign}\left[\frac{4-9\omega+4\omega^2}{\omega}\right]$

Knot[8, 4] → $-\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^2(1+\omega^2)}{\omega(1-\omega+\omega^2)}\right]-\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{2-5\omega+5\omega^2-5\omega^3+2\omega^4}{\omega(1+\omega^2)}\right]$

Knot[8, 5] →
$\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{(-1+\omega)^4(1+\omega^2)}{\omega(1-3\omega+3\omega^2-3\omega^3+\omega^4)}\right]+\text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{\omega(1-\omega+\omega^2)}\right]+\text{sign}\left[\frac{(1-\omega+\omega^2)(1-2\omega+\omega^2-2\omega^3+\omega^4)}{(-1+\omega)^2\omega(1+\omega^2)}\right]$

$$\text{Knot}[8,6]\to -\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{2-6\omega+7\omega^2-6\omega^3+2\omega^4}{\omega\left(1-\omega+\omega^2\right)}\right]$$

$$\text{Knot}[8,7]\to$$
$$\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{\omega\left(1-3\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{(-1+\omega)^2\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)}{\omega\left(1-3\omega+3\omega^2-3\omega^3+\omega^4\right)}\right]-\text{sign}\left[\frac{1-3\omega+5\omega^2-5\omega^3+5\omega^4-3\omega^5+\omega^6}{\omega\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)}\right]$$

$$\text{Knot}[8,8]\to \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{\left(2-2\omega+\omega^2\right)\left(1-2\omega+2\omega^2\right)}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8,9]\to \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(1-3\omega+\omega^2\right)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+$$
$$\text{sign}\left[\frac{\left(-1+\omega-2\omega^2+\omega^3\right)\left(-1+2\omega-\omega^2+\omega^3\right)}{\omega\left(1-\omega+\omega^2\right)^2}\right]+\text{sign}\left[\frac{(-1+\omega)^2\left(1-\omega+\omega^2\right)^2}{\omega\left(1-3\omega+3\omega^2-3\omega^3+\omega^4\right)}\right]-\text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8,10]\to$$
$$\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(1-\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{(-1+\omega)^2\left(1+\omega^2\right)}{\omega\left(1-\omega+\omega^2\right)}\right]+2\,\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{\left(1-\omega+\omega^2\right)^3}{(-1+\omega)^2\omega\left(1+\omega^2\right)}\right]$$

$$\text{Knot}[8,11]\to \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(1-3\omega+\omega^2\right)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{(-2+\omega)(-1+2\omega)\left(1-\omega+\omega^2\right)}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8,12]\to -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^2\left(1-4\omega+\omega^2\right)}{\omega\left(1-3\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{1-7\omega+13\omega^2-7\omega^3+\omega^4}{\omega\left(1-4\omega+\omega^2\right)}\right]$$

$$\text{Knot}[8,13]\to -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]+\text{sign}\left[\frac{(-2+\omega)(-1+\omega)^2(-1+2\omega)}{\omega\left(1-3\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{2-7\omega+11\omega^2-7\omega^3+2\omega^4}{(-2+\omega)\,\omega\,(-1+2\omega)}\right]$$

$$\text{Knot}[8,14]\to \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-2+\omega)(-1+\omega)^2(-1+2\omega)}{\omega\left(1-3\omega+\omega^2\right)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{2-8\omega+11\omega^2-8\omega^3+2\omega^4}{(-2+\omega)\,\omega\,(-1+2\omega)}\right]$$

$$\text{Knot}[8,15]\to -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{2-3\omega+2\omega^2}{\omega}\right]-\text{sign}\left[\frac{\left(1-\omega+\omega^2\right)\left(3-5\omega+3\omega^2\right)}{\omega\left(3-4\omega+3\omega^2\right)}\right]-\text{sign}\left[\frac{(-1+\omega)^2\left(3-4\omega+3\omega^2\right)}{\omega\left(2-3\omega+2\omega^2\right)}\right]$$

$$\text{Knot}[8,16]\to$$
$$-2\,\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{1+\omega^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(1-3\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{1-4\omega+8\omega^2-9\omega^3+8\omega^4-4\omega^5+\omega^6}{(-1+\omega)^2\omega\left(1+\omega^2\right)}\right]$$

$$\text{Knot}[8,17]\to -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(1-3\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]+$$
$$\text{sign}\left[\frac{(-1+\omega)^4\left(1-\omega+\omega^2\right)}{\omega\left(1-3\omega+3\omega^2-3\omega^3+\omega^4\right)}\right]-\text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]+\text{sign}\left[\frac{1-4\omega+8\omega^2-11\omega^3+8\omega^4-4\omega^5+\omega^6}{(-1+\omega)^2\omega\left(1-\omega+\omega^2\right)}\right]$$

$$\text{Knot}[8,18]\to$$
$$2\,\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(1-3\omega+\omega^2\right)}\right]-\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]-\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{\left(1-3\omega+\omega^2\right)\left(1-\omega+\omega^2\right)}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8,19]\to \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]+\text{sign}\left[\frac{(-1+\omega)^2\left(1+\omega^2\right)}{\omega\left(1-\omega+\omega^2\right)}\right]+$$
$$\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]+\text{sign}\left[\frac{(-1+\omega)^2\left(1+\omega+\omega^2\right)}{\omega\left(1+\omega^2\right)}\right]+\text{sign}\left[\frac{1+\omega^4}{\omega\left(1+\omega+\omega^2\right)}\right]+\text{sign}\left[\frac{\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)}{\omega\left(1+\omega^4\right)}\right]$$

$$\text{Knot}[8,20]\to \text{sign}\left[\frac{(-1+\omega)^4}{\omega^2}\right]+\text{sign}\left[\frac{\left(1-\omega+\omega^2\right)^2}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8,21]\to -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right]-\text{sign}\left[\frac{\left(1-3\omega+\omega^2\right)\left(1-\omega+\omega^2\right)}{(-1+\omega)^2\omega}\right]+\text{sign}\left[\frac{(-1+\omega)^4}{\omega\left(2-3\omega+2\omega^2\right)}\right]-\text{sign}\left[\frac{2-3\omega+2\omega^2}{\omega}\right]$$

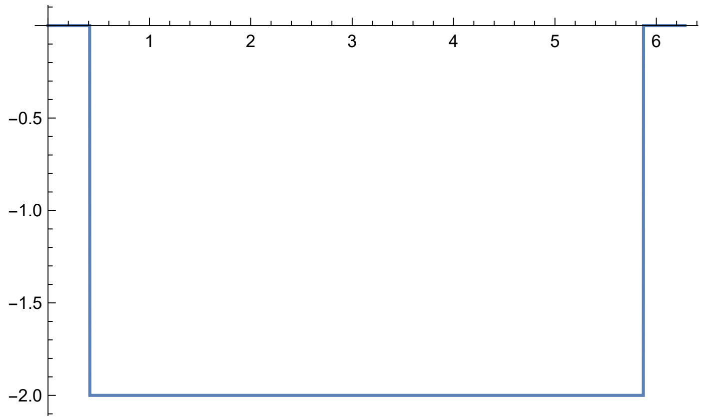
In[∘]:=

```
f = TLSig[Knot[9, 5]] /. ω → ⅇ^(ⅈ t)
Plot[f, {t, 0, 2 π}]
```

Out[∘]=

$$\text{sign}\left[e^{-i t}\left(-1+e^{i t}\right)^2\right]+\text{sign}\left[e^{-i t}\left(6-11\,e^{i t}+6\,e^{2 i t}\right)\right]$$

Out[∘]=

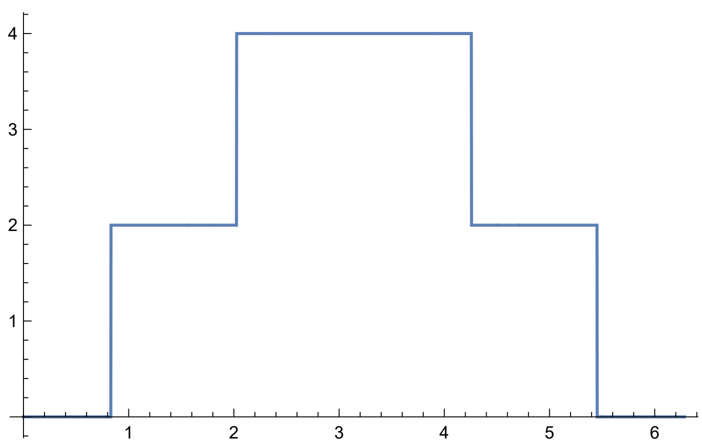


In[∘]:=
```
f = TLSig[Knot[8, 2]] /. ω → ⅇ^(ⅈ t)
Plot[f, {t, 0, 2 π}]
```

Out[∘]=

$$-\text{sign}\left[\frac{e^{-i t}\left(-1+e^{i t}\right)^4}{1-3 e^{i t}+e^{2 i t}}\right]-\text{sign}\left[e^{-i t}\left(1-3 e^{i t}+e^{2 i t}\right)\right]-$$
$$\text{sign}\left[\frac{e^{-i t}\left(1-2 e^{i t}+e^{2 i t}-2 e^{3 i t}+e^{4 i t}\right)}{\left(-1+e^{i t}\right)^2}\right]-$$
$$\text{sign}\left[\frac{e^{-i t}\left(1-3 e^{i t}+3 e^{2 i t}-3 e^{3 i t}+3 e^{4 i t}-3 e^{5 i t}+e^{6 i t}\right)}{1-2 e^{i t}+e^{2 i t}-2 e^{3 i t}+e^{4 i t}}\right]$$

Out[∘]=



In[ ]:=
```
f = TLSig[Knot[12, Alternating, 422]] /. ω → e^(i t)
Plot[f, {t, 0, 2 π}, PlotPoints → 1000]
```

Out[ ]=

$$-\operatorname{sign}\left[e^{-it}\left(-2+e^{it}\right)\left(-1+2e^{it}\right)\right]-\operatorname{sign}\left[\frac{e^{-it}\left(1-e^{it}+e^{2it}\right)^2}{\left(-2+e^{it}\right)\left(-1+2e^{it}\right)}\right]-$$
$$\operatorname{sign}\left[\frac{e^{-it}\left(1-e^{it}+e^{2it}\right)\left(2-4e^{it}+4e^{2it}-3e^{3it}+4e^{4it}-4e^{5it}+2e^{6it}\right)}{2-4e^{it}+6e^{2it}-5e^{3it}+6e^{4it}-4e^{5it}+2e^{6it}}\right]-$$
$$\operatorname{sign}\left[\frac{e^{-it}\left(2-4e^{it}+6e^{2it}-5e^{3it}+6e^{4it}-4e^{5it}+2e^{6it}\right)}{\left(1-e^{it}+e^{2it}\right)^2}\right]$$

Out[ ]=