In[∘]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
 << KnotTheory`;
]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

pdf

In[∘]:=
```
sign[x_?NumberQ] := Sign[Re[x]]
```

pdf

In[∘]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

In[∘]:=
```
CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_ | η̄_, ∞]},
  Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) :→ Factor[c] Times @@ ηs^ps] ]
```

pdf

In[∘]:=
```
CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs] ]
```

pdf

In[∘]:=
```
(ℰ_)* := ℰ /. {η̄ → η, η → η̄, ω → ω^-1};
r_Rule+ := {r, r*}
```

In[∘]:= $\{((2\,u - \omega + 3\,\omega^{-1})\,\bar\eta_1\,\eta_2)^*,\ (\eta_1 \to \omega\,\eta_2)^+\}$

Out[∘]=

$$\left\{\left(2\,u - \frac{1}{\omega} + 3\,\omega\right)\eta_1\,\bar\eta_2,\ \left\{\eta_1 \to \omega\,\eta_2,\ \bar\eta_1 \to \frac{\bar\eta_2}{\omega}\right\}\right\}$$

pdf

In[∘]:=
```
RulesOf[η_i_ + rest_.] := (η_i → -rest)+;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

In[∘]:= $\mathrm{CF}[\{\eta_1 - \eta_2,\ \eta_1 - \eta_3\}]$

Out[∘]=

$$\{\eta_1 - \eta_3,\ \eta_2 - \eta_3\}$$

In[∘]:= $\mathrm{RulesOf}[\eta_1 + \eta_2 + \eta_3]$

Out[∘]=

$$\{\eta_1 \to -\eta_2 - \eta_3,\ \bar\eta_1 \to -\bar\eta_2 - \bar\eta_3\}$$

pdf

```
In[∘]:= CF[TSI_b_[σ_, pq_]] := TSI_CF[b][σ, CF[pq]]
```

The disjoint union in the world of multi-tangles.

pdf

```
In[∘]:= TSI_b1_[σ1_, PQ[rs1_, q1_]] ⋃ TSI_b2_[σ2_, PQ[rs2_, q2_]] ^:=
          CF@TSI_Join[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

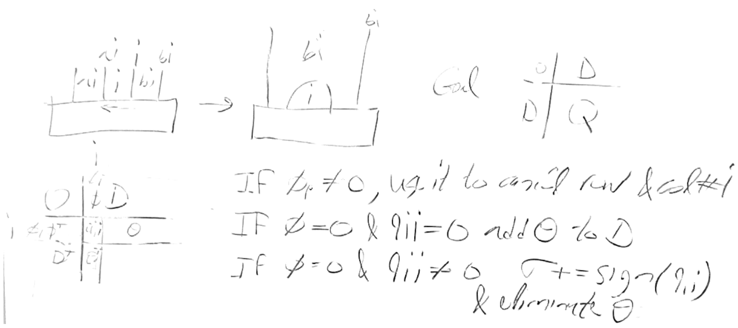
FM for Face Merge:

pdf

```
In[∘]:= FM_i_,j_@TSI_B[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[rs_, q_]] :=
         CF@TSI_B[{ri,li,i,rj,lj,j},bs][σ, PQ[rs ⋃ {η_i - η_j}, q]]
```

```
In[∘]:= TSI_B[{-1,2}][0, PQ[{}, 0]] ⋃ TSI_B[{-3,4}][0, PQ[{}, 0]] // FM_-1,4
Out[∘]=
        TSI_B[{-3,4,2,-1}][0, PQ[{η_-1 - η_4}, 0]]
```

pdf



pdf

```
In[∘]:= Cordon_i_@TSI_B[{li___,i_,ri___},bs___][σ_, PQ[rs_, q_]] :=
         Module[{ϕ = ∂_η_i rs, nσ = σ, nrs = rs, nq = q, qii, p},
          Which[
           Or @@ ((# =!= 0) & /@ ϕ), ({p} = FirstPosition[(# === 0) & /@ ϕ, False];
             {nrs, nq} = {rs, q} /. (η_i → -rs⟦p⟧ / ϕ⟦p⟧)^+ /. (η_i → 0)^+),
           (qii = ∂_η̄_i,η_i q) =!= 0, (nσ += sign[qii];
             nq = q /. (η_i → -(∂_η̄_i q) / qii)^+ /. (η_i → 0)^+),
           qii === 0, AppendTo[nrs, ∂_η̄_i q]; nq = q /. (η_i → 0)^+];
          CF@TSI_B[Rest@{ri,li},bs][nσ, PQ[nrs, nq] /. (η_First@{ri,li} → η_Last@{ri,li})^+] ]
```

tex

c for contract:

pdf

```
In[∘]:= c_i_,j_@t : TSI_B[{li___,i_,ri___},{___,j_,___},___][__] := t // FM_j,Last@{ri,li} // Cordon_j
```

pdf

In[∘]:=
```
c_{i_,j_}@t : TSI_{B[{___,i_,j_,___},___]}[__] := Cordon_i@t
c_{i_,j_}@t : TSI_{B[{j_,___,i_},___]}[__] := Cordon_i@t
c_{i_,j_}@t : TSI_{B[{___,j_,i_,___},___]}[__] := Cordon_j@t
c_{i_,j_}@t : TSI_{B[{i_,___,j_},___]}[__] := Cordon_j@t
```

tex

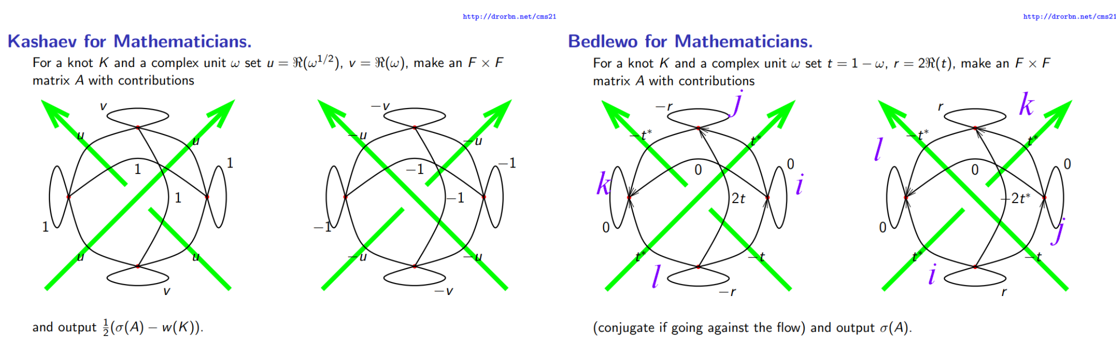
mc for magnetic contract:

pdf

In[∘]:=
```
mc[ℰ_] := ℰ //.
  t : TSI_{B[{___,i_,___},{___,j_,___},___]}[__] |
     TSI_{B[{j_,___,i_},___]}[__] | TSI_{B[{___,i_,j_,___},___]}[__] /; i + j == 0 :> c_{i,j}@t
```

pdf

In[∘]:=
```
Kas[P[i_, j_]] := CF@TSI_{B[{-i,j}]}[0, PQ[{}, 0]];
Bed[P[i_, j_]] := CF@TSI_{B[{-i,j}]}[0, PQ[{}, 0]]
```



pdf

In[∘]:=
```
Kas[x_X] := Module[{v = 2 u^2 - 1, fs, s, m, ηs},
  fs = List @@ x; s = PositiveQ@x;
  fs *= If[s, {-1, 1, 1, -1}, {-1, -1, 1, 1}];
  m = If[s, (1 u 1 u
             u v u 1
             1 u 1 u
             u 1 u v), -(v u 1 u
                         u 1 u 1
                         1 u v u
                         u 1 u 1)];
  ηs = η_# & /@ fs; CF@TSI_{B[fs]}[0, PQ[{}, ηs*.m.ηs]]]
```

In[∘]:=
```
Kas /@ {X[1, 2, 3, 4], X[1, 4, 3, 2]}
```

Out[∘]=

$$\left\{\text{TSI}_{\text{B}[\{-4,-1,2,3\}]}\left[0, \text{PQ}\left[\{\}, \left(-1+2u^2\right)\eta_{-4}\overline{\eta}_{-4} + u\,\eta_{-1}\overline{\eta}_{-4} + \eta_2\overline{\eta}_{-4} + u\,\eta_3\overline{\eta}_{-4} + u\,\eta_{-4}\overline{\eta}_{-1} + \eta_{-1}\overline{\eta}_{-1} + u\,\eta_2\overline{\eta}_{-1} + \eta_3\overline{\eta}_{-1} + \eta_{-4}\overline{\eta}_2 + u\,\eta_{-1}\overline{\eta}_2 + \left(-1+2u^2\right)\eta_2\overline{\eta}_2 + u\,\eta_3\overline{\eta}_2 + u\,\eta_{-4}\overline{\eta}_3 + \eta_{-1}\overline{\eta}_3 + u\,\eta_2\overline{\eta}_3 + \eta_3\overline{\eta}_3\right]\right],\right.$$
$$\left.\text{TSI}_{\text{B}[\{-4,3,2,-1\}]}\left[0, \text{PQ}\left[\{\}, -\eta_{-4}\overline{\eta}_{-4} - u\,\eta_{-1}\overline{\eta}_{-4} - \eta_2\overline{\eta}_{-4} - u\,\eta_3\overline{\eta}_{-4} - u\,\eta_{-4}\overline{\eta}_{-1} + \left(1-2u^2\right)\eta_{-1}\overline{\eta}_{-1} - u\,\eta_2\overline{\eta}_{-1} - \eta_3\overline{\eta}_{-1} - \eta_{-4}\overline{\eta}_2 - u\,\eta_{-1}\overline{\eta}_2 - \eta_2\overline{\eta}_2 - u\,\eta_3\overline{\eta}_2 - u\,\eta_{-4}\overline{\eta}_3 - \eta_{-1}\overline{\eta}_3 - u\,\eta_2\overline{\eta}_3 + \left(1-2u^2\right)\eta_3\overline{\eta}_3\right]\right]\right\}$$

pdf

In[∘]:=

```
Bed[x_X] := Module[{t = 1 - ω, r, fs, s, m, ηs},
  r = t + t*; fs = List @@ x; s = PositiveQ@x;
  fs *= If[s, {-1, 1, 1, -1}, {-1, -1, 1, 1}];
  m = If[s, ( 0   t*   0   -t*
              t   -r  -t*  2 t*
              0   -t   0    t
             -t   2 t  t*  -r ), ( r    -t  -2 t*  t*
                                  -t*   0    t*    0
                                  -2 t  t    r    -t*
                                   t    0   -t     0 )];
  ηs = η_# & /@ fs; CF@TSI_B[fs][0, PQ[{}, ηs*.m.ηs]]]
```

In[∘]:=

```
Bed /@ {X[1, 2, 3, 4], X[1, 4, 3, 2]}
```

Out[∘]=

$$\Big\{\mathrm{TSI}_{B[\{-4,-1,2,3\}]}\Big[0, \mathrm{PQ}\Big[\{\}, \frac{(-1+\omega)^2\,\eta_{-4}\,\overline{\eta}_{-4}}{\omega} + (-1+\omega)\,\eta_{-1}\,\overline{\eta}_{-4} - 2\,(-1+\omega)\,\eta_2\,\overline{\eta}_{-4} + \frac{(-1+\omega)\,\eta_3\,\overline{\eta}_{-4}}{\omega} - \frac{(-1+\omega)\,\eta_{-4}\,\overline{\eta}_{-1}}{\omega} + \frac{(-1+\omega)\,\eta_2\,\overline{\eta}_{-1}}{\omega} + \frac{2\,(-1+\omega)\,\eta_{-4}\,\overline{\eta}_2}{\omega} + (1-\omega)\,\eta_{-1}\,\overline{\eta}_2 + \frac{(-1+\omega)^2\,\eta_2\,\overline{\eta}_2}{\omega} - \frac{(-1+\omega)\,\eta_3\,\overline{\eta}_2}{\omega} + (1-\omega)\,\eta_{-4}\,\overline{\eta}_3 + (-1+\omega)\,\eta_2\,\overline{\eta}_3\Big]\Big],$$

$$\mathrm{TSI}_{B[\{-4,3,2,-1\}]}\Big[0, \mathrm{PQ}\Big[\{\}, -\frac{(-1+\omega)\,\eta_{-1}\,\overline{\eta}_{-4}}{\omega} + \frac{(-1+\omega)\,\eta_3\,\overline{\eta}_{-4}}{\omega} + (-1+\omega)\,\eta_{-4}\,\overline{\eta}_{-1} - \frac{(-1+\omega)^2\,\eta_{-1}\,\overline{\eta}_{-1}}{\omega} + \frac{(-1+\omega)\,\eta_2\,\overline{\eta}_{-1}}{\omega} - \frac{2\,(-1+\omega)\,\eta_3\,\overline{\eta}_{-1}}{\omega} + (1-\omega)\,\eta_{-1}\,\overline{\eta}_2 + (-1+\omega)\,\eta_3\,\overline{\eta}_2 + (1-\omega)\,\eta_{-4}\,\overline{\eta}_3 + 2\,(-1+\omega)\,\eta_{-1}\,\overline{\eta}_3 - \frac{(-1+\omega)\,\eta_2\,\overline{\eta}_3}{\omega} - \frac{(-1+\omega)^2\,\eta_3\,\overline{\eta}_3}{\omega}\Big]\Big]\Big\}$$

pdf

In[∘]:=

```
Kas[K_] := Fold[mc[#1 ⋃ #2] &, TSI_B[][0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
Writhe[K_] := Plus @@ (If[PositiveQ@#, 1, -1] & /@ PD[K]);
KasSig[K_] := (Kas[K][[1]] - Writhe[K]) / 2
```

In[∘]:=

```
Kas[Knot[3, 1]]
```

Out[∘]=

$$\mathrm{TSI}_{B[]}\Big[\mathrm{sign}\Big[\frac{1}{2}\,(3-4\,u^2)\Big] + \mathrm{sign}\big[-2\,(-1+2\,u^2)\big] + \mathrm{sign}\Big[-\frac{-3+4\,u^2}{-1+2\,u^2}\Big], \mathrm{PQ}[\{\}, 0]\Big]$$

In[∘]:=

```
KasSig[Knot[3, 1]]
```

Out[∘]=

$$3 + \mathrm{sign}\Big[\frac{1}{2}\,(3-4\,u^2)\Big] + \mathrm{sign}\big[-2\,(-1+2\,u^2)\big] + \mathrm{sign}\Big[-\frac{-3+4\,u^2}{-1+2\,u^2}\Big]$$

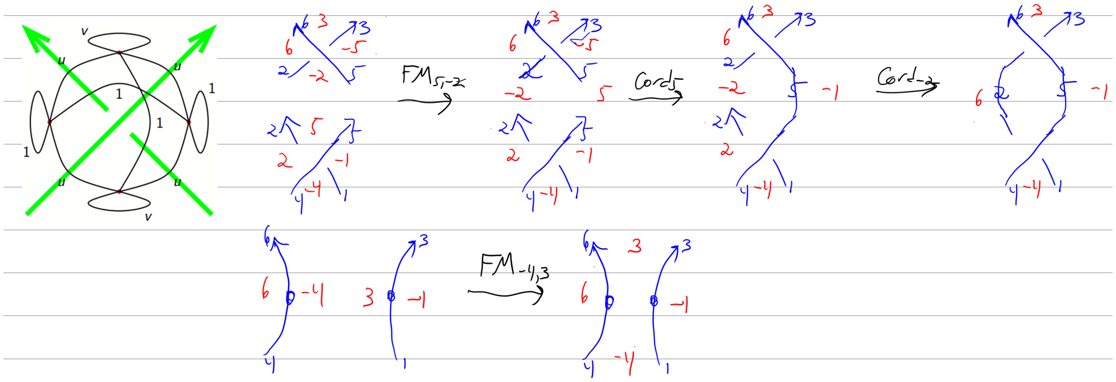
pdf

In[∘]:=

```
Bed[K_] := Fold[mc[#1 ⋃ #2] &, TSI_B[][0, PQ[{}, 0]], List @@ (Bed /@ PD@K)];
BedSig[K_] := Bed[K][[1]]
```

pdf

Reidemeister 2



In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]]**

Out[∘]=

$$\mathrm{TSI}_{B[\{-5,3,6,-2\},\{-4,-1,5,2\}]}\Big[0,\ \mathrm{PQ}\Big[\{\},\ -\eta_{-5}\bar\eta_{-5} - u\,\eta_{-2}\bar\eta_{-5} - u\,\eta_3\bar\eta_{-5} - \eta_6\bar\eta_{-5} + (-1+2u^2)\,\eta_{-4}\bar\eta_{-4} + u\,\eta_{-1}\bar\eta_{-4} + u\,\eta_2\bar\eta_{-4} + \eta_5\bar\eta_{-4} - u\,\eta_{-5}\bar\eta_{-2} + (1-2u^2)\,\eta_{-2}\bar\eta_{-2} - \eta_3\bar\eta_{-2} - u\,\eta_6\bar\eta_{-2} + u\,\eta_{-4}\bar\eta_{-1} + \eta_{-1}\bar\eta_{-1} + \eta_2\bar\eta_{-1} + u\,\eta_5\bar\eta_{-1} + u\,\eta_{-4}\bar\eta_2 + \eta_{-1}\bar\eta_2 + \eta_2\bar\eta_2 + u\,\eta_5\bar\eta_2 - u\,\eta_{-5}\bar\eta_3 - \eta_{-2}\bar\eta_3 + (1-2u^2)\,\eta_3\bar\eta_3 - u\,\eta_6\bar\eta_3 + \eta_{-4}\bar\eta_5 + u\,\eta_{-1}\bar\eta_5 + u\,\eta_2\bar\eta_5 + (-1+2u^2)\,\eta_5\bar\eta_5 - \eta_{-5}\bar\eta_6 - u\,\eta_{-2}\bar\eta_6 - u\,\eta_3\bar\eta_6 - \eta_6\bar\eta_6\Big]\Big]$$

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM$_{-2,5}$**

Out[∘]=

$$\mathrm{TSI}_{B[\{-5,3,6,-2,2,-4,-1,5\}]}\Big[0,\ \mathrm{PQ}\Big[\{\eta_{-2}-\eta_5\},\ -\eta_{-5}\bar\eta_{-5} - u\,\eta_3\bar\eta_{-5} - u\,\eta_5\bar\eta_{-5} - \eta_6\bar\eta_{-5} + (-1+2u^2)\,\eta_{-4}\bar\eta_{-4} + u\,\eta_{-1}\bar\eta_{-4} + u\,\eta_2\bar\eta_{-4} + \eta_5\bar\eta_{-4} + u\,\eta_{-4}\bar\eta_{-1} + \eta_{-1}\bar\eta_{-1} + \eta_2\bar\eta_{-1} + u\,\eta_5\bar\eta_{-1} + u\,\eta_{-4}\bar\eta_2 + \eta_{-1}\bar\eta_2 + \eta_2\bar\eta_2 + u\,\eta_5\bar\eta_2 - u\,\eta_{-5}\bar\eta_3 + (1-2u^2)\,\eta_3\bar\eta_3 - \eta_5\bar\eta_3 - u\,\eta_6\bar\eta_3 - u\,\eta_{-5}\bar\eta_5 + \eta_{-4}\bar\eta_5 + u\,\eta_{-1}\bar\eta_5 + u\,\eta_2\bar\eta_5 - \eta_3\bar\eta_5 - u\,\eta_6\bar\eta_5 - \eta_{-5}\bar\eta_6 - u\,\eta_3\bar\eta_6 - u\,\eta_5\bar\eta_6 - \eta_6\bar\eta_6\Big]\Big]$$

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM$_{-2,5}$ // Cordon$_5$**

Out[∘]=

$$\mathrm{TSI}_{B[\{-4,-1,3,6,-2,2\}]}\Big[0,\ \mathrm{PQ}\Big[\{\},\ (-1+2u^2)\,\eta_{-4}\bar\eta_{-4} + \eta_{-2}\bar\eta_{-4} + u\,\eta_{-1}\bar\eta_{-4} + u\,\eta_2\bar\eta_{-4} + \eta_{-4}\bar\eta_{-2} + u\,\eta_2\bar\eta_{-2} - \eta_3\bar\eta_{-2} - u\,\eta_6\bar\eta_{-2} + u\,\eta_{-4}\bar\eta_{-1} + \eta_2\bar\eta_{-1} - u\,\eta_3\bar\eta_{-1} - \eta_6\bar\eta_{-1} + u\,\eta_{-4}\bar\eta_2 + u\,\eta_{-2}\bar\eta_2 + \eta_{-1}\bar\eta_2 + \eta_2\bar\eta_2 - \eta_{-2}\bar\eta_3 - u\,\eta_{-1}\bar\eta_3 + (1-2u^2)\,\eta_3\bar\eta_3 - u\,\eta_6\bar\eta_3 - u\,\eta_{-2}\bar\eta_6 - \eta_{-1}\bar\eta_6 - u\,\eta_3\bar\eta_6 - \eta_6\bar\eta_6\Big]\Big]$$

In[∘]:= **Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM$_{-2,5}$ // Cordon$_5$ // Cordon$_{-2}$**

Out[∘]=

$$\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]]$$

pdf

In[∘]:= **{Kas[P[1, 3]] ⋃ Kas[P[4, 6]] // FM$_{-4,3}$, Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // mc}**

Out[∘]=
pdf

$$\{\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]],\ \mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]]\}$$

pdf

In[∘]:=
```
{Bed[P[1, 3]] ⋃ Bed[P[4, 6]] // FM-4,3, Bed[X[1, 5, 2, 4]] ⋃ Bed[X[2, 5, 3, 6]] // mc}
```

Out[∘]=
pdf

$$\{\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0, \mathrm{PQ}[\{\eta_{-4}-\eta_3\}, 0]], \mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0, \mathrm{PQ}[\{\eta_{-4}-\eta_3\}, 0]]\}$$

pdf

Reidemeister 3

In[∘]:=
```
{u = 7 / 29};
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // c2,-2 // c5,-5 // c8,-8
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // c2,-2 // c5,-5 // c8,-8
Clear[u]
```

Out[∘]=

$$\mathrm{TSI}_{B[\{-7,3,6,9,-1,-4\}]}\Big[-1, \mathrm{PQ}\Big[\{\}, \frac{1486}{645}\eta_{-7}\bar\eta_{-7}+\frac{16289\,\eta_{-4}\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_{-1}\bar\eta_{-7}+\frac{16289\,\eta_3\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_6\bar\eta_{-7}+\frac{406}{645}\eta_9\bar\eta_{-7}+\frac{16289\,\eta_{-7}\bar\eta_{-4}}{18705}+\frac{228046\,\eta_{-4}\bar\eta_{-4}}{542445}+\frac{16289\,\eta_{-1}\bar\eta_{-4}}{18705}+\frac{841}{645}\eta_3\bar\eta_{-4}+\frac{406}{645}\eta_6\bar\eta_{-4}+\frac{841}{645}\eta_9\bar\eta_{-4}+\frac{841}{645}\eta_{-7}\bar\eta_{-1}+\frac{16289\,\eta_{-4}\bar\eta_{-1}}{18705}+\frac{228046\,\eta_{-1}\bar\eta_{-1}}{542445}+\frac{406}{645}\eta_3\bar\eta_{-1}+\frac{841}{645}\eta_6\bar\eta_{-1}+\frac{16289\,\eta_9\bar\eta_{-1}}{18705}+\frac{16289\,\eta_{-7}\bar\eta_3}{18705}+\frac{841}{645}\eta_{-4}\bar\eta_3+\frac{406}{645}\eta_{-1}\bar\eta_3+\frac{228046\,\eta_3\bar\eta_3}{542445}+\frac{16289\,\eta_6\bar\eta_3}{18705}+\frac{841}{645}\eta_9\bar\eta_3+\frac{841}{645}\eta_{-7}\bar\eta_6+\frac{406}{645}\eta_{-4}\bar\eta_6+\frac{841}{645}\eta_{-1}\bar\eta_6+\frac{16289\,\eta_3\bar\eta_6}{18705}+\frac{228046\,\eta_6\bar\eta_6}{542445}+\frac{16289\,\eta_9\bar\eta_6}{18705}+\frac{406}{645}\eta_{-7}\bar\eta_9+\frac{841}{645}\eta_{-4}\bar\eta_9+\frac{16289\,\eta_{-1}\bar\eta_9}{18705}+\frac{841}{645}\eta_3\bar\eta_9+\frac{16289\,\eta_6\bar\eta_9}{18705}+\frac{1486}{645}\eta_9\bar\eta_9\Big]\Big]$$

Out[∘]=

$$\mathrm{TSI}_{B[\{-7,3,6,9,-1,-4\}]}\Big[-1, \mathrm{PQ}\Big[\{\}, \frac{1486}{645}\eta_{-7}\bar\eta_{-7}+\frac{16289\,\eta_{-4}\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_{-1}\bar\eta_{-7}+\frac{16289\,\eta_3\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_6\bar\eta_{-7}+\frac{406}{645}\eta_9\bar\eta_{-7}+\frac{16289\,\eta_{-7}\bar\eta_{-4}}{18705}+\frac{228046\,\eta_{-4}\bar\eta_{-4}}{542445}+\frac{16289\,\eta_{-1}\bar\eta_{-4}}{18705}+\frac{841}{645}\eta_3\bar\eta_{-4}+\frac{406}{645}\eta_6\bar\eta_{-4}+\frac{841}{645}\eta_9\bar\eta_{-4}+\frac{841}{645}\eta_{-7}\bar\eta_{-1}+\frac{16289\,\eta_{-4}\bar\eta_{-1}}{18705}+\frac{228046\,\eta_{-1}\bar\eta_{-1}}{542445}+\frac{406}{645}\eta_3\bar\eta_{-1}+\frac{841}{645}\eta_6\bar\eta_{-1}+\frac{16289\,\eta_9\bar\eta_{-1}}{18705}+\frac{16289\,\eta_{-7}\bar\eta_3}{18705}+\frac{841}{645}\eta_{-4}\bar\eta_3+\frac{406}{645}\eta_{-1}\bar\eta_3+\frac{228046\,\eta_3\bar\eta_3}{542445}+\frac{16289\,\eta_6\bar\eta_3}{18705}+\frac{841}{645}\eta_9\bar\eta_3+\frac{841}{645}\eta_{-7}\bar\eta_6+\frac{406}{645}\eta_{-4}\bar\eta_6+\frac{841}{645}\eta_{-1}\bar\eta_6+\frac{16289\,\eta_3\bar\eta_6}{18705}+\frac{228046\,\eta_6\bar\eta_6}{542445}+\frac{16289\,\eta_9\bar\eta_6}{18705}+\frac{406}{645}\eta_{-7}\bar\eta_9+\frac{841}{645}\eta_{-4}\bar\eta_9+\frac{16289\,\eta_{-1}\bar\eta_9}{18705}+\frac{841}{645}\eta_3\bar\eta_9+\frac{16289\,\eta_6\bar\eta_9}{18705}+\frac{1486}{645}\eta_9\bar\eta_9\Big]\Big]$$

pdf

In[∘]:=
```
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // mc;
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // mc;
{lhs[[1]], rhs[[1]]}
Simplify[lhs[[2, 2]] == rhs[[2, 2]]]
```

Out[∘]=
pdf

{sign[(−1 + 2 u) (1 + 2 u)], sign[(−1 + 2 u) (1 + 2 u)]}

Out[∘]=
pdf

True

In[∘]:= lhs[[2, 2]]

Out[∘]=

$$\frac{2\left(-1+2u^2\right)\eta_{-7}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-4}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}-\frac{\eta_{-1}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{3}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{6}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{9}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-7}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}+\frac{2u^2\left(-3+4u^2\right)\eta_{-4}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}+$$
$$\frac{u\left(-3+4u^2\right)\eta_{-1}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-\frac{\eta_{3}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{6}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-\frac{\eta_{9}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{-7}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-4}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{2u^2\left(-3+4u^2\right)\eta_{-1}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{3}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{6}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{9}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-7}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-\frac{\eta_{-4}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-$$
$$\frac{2u\,\eta_{-1}\overline{\eta}_{3}}{(-1+2u)(1+2u)}+\frac{2u^2\left(-3+4u^2\right)\eta_{3}\overline{\eta}_{3}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{6}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-\frac{\eta_{9}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{-7}\overline{\eta}_{6}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{-4}\overline{\eta}_{6}}{(-1+2u)(1+2u)}-\frac{\eta_{-1}\overline{\eta}_{6}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{3}\overline{\eta}_{6}}{(-1+2u)(1+2u)}+$$
$$\frac{2u^2\left(-3+4u^2\right)\eta_{6}\overline{\eta}_{6}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{9}\overline{\eta}_{6}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{-7}\overline{\eta}_{9}}{(-1+2u)(1+2u)}-\frac{\eta_{-4}\overline{\eta}_{9}}{(-1+2u)(1+2u)}+$$
$$\frac{u\left(-3+4u^2\right)\eta_{-1}\overline{\eta}_{9}}{(-1+2u)(1+2u)}-\frac{\eta_{3}\overline{\eta}_{9}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{6}\overline{\eta}_{9}}{(-1+2u)(1+2u)}+\frac{2\left(-1+2u^2\right)\eta_{9}\overline{\eta}_{9}}{(-1+2u)(1+2u)}$$

pdf

In[∘]:=
```
lhs = Bed[X[4, 2, 5, 1]] ⋃ Bed[X[7, 3, 8, 2]] ⋃ Bed[X[8, 6, 9, 5]] // mc;
rhs = Bed[X[7, 5, 8, 4]] ⋃ Bed[X[8, 2, 9, 1]] ⋃ Bed[X[5, 3, 6, 2]] // mc;
{lhs⟦1⟧, rhs⟦1⟧}
lhs⟦2, 2⟧ == rhs⟦2, 2⟧
```

Out[∘]=
pdf

$$\left\{\mathrm{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right], \mathrm{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right]\right\}$$

Out[∘]=
pdf

True

In[∘]:=
```
lhs⟦2, 2⟧
```

Out[∘]=

$$-\frac{(-1+\omega)\,\eta_{-4}\,\overline{\eta}_{-7}}{\omega}+\frac{(-1+\omega)\,\eta_3\,\overline{\eta}_{-7}}{\omega}+(-1+\omega)\,\eta_{-7}\,\overline{\eta}_{-4}+\frac{(1+\omega^2)\,\eta_{-4}\,\overline{\eta}_{-4}}{\omega}-\frac{(1+\omega)\,\eta_{-1}\,\overline{\eta}_{-4}}{\omega}-2\,\omega\,\eta_3\,\overline{\eta}_{-4}+2\,\eta_6\,\overline{\eta}_{-4}+(-1-\omega)\,\eta_{-4}\,\overline{\eta}_{-1}+\frac{(1+\omega^2)\,\eta_{-1}\,\overline{\eta}_{-1}}{\omega}+2\,\omega\,\eta_3\,\overline{\eta}_{-1}-2\,\omega\,\eta_6\,\overline{\eta}_{-1}+\frac{(-1+\omega)\,\eta_9\,\overline{\eta}_{-1}}{\omega}+(1-\omega)\,\eta_{-7}\,\overline{\eta}_3-\frac{2\,\eta_{-4}\,\overline{\eta}_3}{\omega}+\frac{2\,\eta_{-1}\,\overline{\eta}_3}{\omega}+\frac{(1+\omega^2)\,\eta_3\,\overline{\eta}_3}{\omega}-\frac{(1+\omega)\,\eta_6\,\overline{\eta}_3}{\omega}+2\,\eta_{-4}\,\overline{\eta}_6-\frac{2\,\eta_{-1}\,\overline{\eta}_6}{\omega}+(-1-\omega)\,\eta_3\,\overline{\eta}_6+\frac{(1+\omega^2)\,\eta_6\,\overline{\eta}_6}{\omega}-\frac{(-1+\omega)\,\eta_9\,\overline{\eta}_6}{\omega}+(1-\omega)\,\eta_{-1}\,\overline{\eta}_9+(-1+\omega)\,\eta_6\,\overline{\eta}_9$$

Kashaev for Knots

In[∘]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4, 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
(*u=0;*)
Kas[Knot[3, 1]]
Clear[u]
```

Out[∘]=

$$\mathrm{TSI}_{B[]}\left[\mathrm{sign}\left[\frac{1}{2}\left(3-4u^2\right)\right]+\mathrm{sign}\left[-2\left(-1+2u^2\right)\right]+\mathrm{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right], \mathrm{PQ}[\{\}, 0]\right]$$

In[∘]:=
```
(*u=0;*)
KasSig /@ AllKnots[{3, 7}]
Clear[u]
```

Out[∘]=

$$\left\{\frac{1}{2}\left(3+\mathrm{sign}\left[\frac{1}{2}\left(3-4u^2\right)\right]+\mathrm{sign}\left[-2\left(-1+2u^2\right)\right]+\mathrm{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right]\right),\right.$$

$$\frac{1}{2}\left(1+\text{sign}\left[-3+2\,u^2\right]+\text{sign}\left[-\frac{-5+4\,u^2}{-3+4\,u^2}\right]+\text{sign}\left[-\frac{\left(-5+4\,u^2\right)\,\left(-3+4\,u^2\right)}{2\,\left(-3+2\,u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(5+2\,\text{sign}\left[-2\,\left(-1+2\,u^2\right)\right]+\text{sign}\left[-\frac{1-8\,u^2+8\,u^4}{-1+2\,u^2}\right]+\text{sign}\left[-\frac{5-20\,u^2+16\,u^4}{4\,\left(-1+2\,u^2\right)}\right]+\text{sign}\left[-\frac{5-20\,u^2+16\,u^4}{1-8\,u^2+8\,u^4}\right]\right),$$

$$\frac{1}{2}\left(4+\text{sign}\left[\frac{1}{4}\,\left(7-8\,u^2\right)\right]+\text{sign}\left[-2\,\left(-2+3\,u^2\right)\right]+\text{sign}\left[-\frac{-3+4\,u^2}{-2+3\,u^2}\right]+\text{sign}\left[-\frac{-7+8\,u^2}{-3+4\,u^2}\right]\right),$$

$$\frac{1}{2}\left(3+\text{sign}\left[\frac{1}{2}\,\left(9-8\,u^2\right)\right]+\text{sign}\left[-3+2\,u^2\right]+\text{sign}\left[-\frac{-5+4\,u^2}{-3+2\,u^2}\right]+\text{sign}\left[-\frac{-7+6\,u^2}{-5+4\,u^2}\right]+\text{sign}\left[-\frac{-9+8\,u^2}{-7+6\,u^2}\right]\right),\ \frac{1}{2}\left(3+\text{sign}\left[-3+2\,u^2\right]+\text{sign}\left[-\frac{\left(-5+4\,u^2\right)\,\left(-3+4\,u^2\right)}{2\,\left(-3+2\,u^2\right)}\right]+\text{sign}\left[-\frac{8\,(-1+u)\,(1+u)\,\left(3-12\,u^2+8\,u^4\right)}{\left(-5+4\,u^2\right)\,\left(-3+4\,u^2\right)}\right]+\text{sign}\left[-\frac{11-28\,u^2+16\,u^4}{8\,(-1+u)\,(1+u)}\right]+\text{sign}\left[-\frac{11-28\,u^2+16\,u^4}{3-12\,u^2+8\,u^4}\right]\right),$$

$$\frac{1}{2}\left(\text{sign}\left[3-4\,u^2\right]+\text{sign}\left[-3+4\,u^2\right]+\text{sign}\left[\frac{13-28\,u^2+16\,u^4}{8\,(-1+u)\,(1+u)}\right]+\text{sign}\left[\frac{13-28\,u^2+16\,u^4}{5-12\,u^2+8\,u^4}\right]+\text{sign}\left[-\frac{8\,(-1+u)\,(1+u)\,\left(5-12\,u^2+8\,u^4\right)}{\left(-3+4\,u^2\right)\,\left(5-20\,u^2+16\,u^4\right)}\right]+\text{sign}\left[-\frac{5-20\,u^2+16\,u^4}{-3+4\,u^2}\right]\right),$$

$$\frac{1}{2}\left(7+3\,\text{sign}\left[-2\,\left(-1+2\,u^2\right)\right]+\text{sign}\left[-\frac{1-8\,u^2+8\,u^4}{-1+2\,u^2}\right]+\text{sign}\left[-\frac{(-1+2\,u)\,(1+2\,u)\,\left(-3+4\,u^2\right)\,\left(1-16\,u^2+16\,u^4\right)}{4\,\left(-1+2\,u^2\right)\,\left(1-8\,u^2+8\,u^4\right)}\right]+\text{sign}\left[-\frac{-7+56\,u^2-112\,u^4+64\,u^6}{2\,(-1+2\,u)\,(1+2\,u)\,\left(-3+4\,u^2\right)}\right]+\text{sign}\left[-\frac{-7+56\,u^2-112\,u^4+64\,u^6}{\left(-1+2\,u^2\right)\,\left(1-16\,u^2+16\,u^4\right)}\right]\right),$$

$$\frac{1}{2}\left(5+\text{sign}\left[\frac{1}{6}\,\left(11-12\,u^2\right)\right]+\text{sign}\left[-2\,\left(-3+4\,u^2\right)\right]+\text{sign}\left[-\frac{3\,\left(-5+6\,u^2\right)}{2\,\left(-9+11\,u^2\right)}\right]+\text{sign}\left[-\frac{-9+11\,u^2}{2\,\left(-3+4\,u^2\right)}\right]+\text{sign}\left[-\frac{-11+12\,u^2}{-5+6\,u^2}\right]\right),$$

$$\frac{1}{2}\left(-5+\text{sign}\left[2\,\left(-1+2\,u^2\right)\right]+\text{sign}\left[\frac{4}{3}\,\left(-3+4\,u^2\right)\right]+\text{sign}\left[\frac{\left(-5+8\,u^2\right)\,\left(3-18\,u^2+16\,u^4\right)}{4\,\left(-1+2\,u^2\right)\,\left(-3+4\,u^2\right)}\right]+\text{sign}\left[\frac{13-44\,u^2+32\,u^4}{2\,\left(-5+8\,u^2\right)}\right]+\text{sign}\left[\frac{13-44\,u^2+32\,u^4}{3-18\,u^2+16\,u^4}\right]\right),$$

$$\frac{1}{2}\left(-6+\text{sign}\left[2\left(-2+3u^2\right)\right]+\text{sign}\left[\frac{-3+4u^2}{-2+3u^2}\right]+\text{sign}\left[\frac{-7+8u^2}{-3+4u^2}\right]+\text{sign}\left[\frac{-11+12u^2}{-7+8u^2}\right]+\text{sign}\left[\frac{1}{4}\left(-15+16u^2\right)\right]+\text{sign}\left[\frac{-15+16u^2}{-11+12u^2}\right]\right),$$

$$\frac{1}{2}\left(7+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{2\left(-2+3u^2\right)}{-1+2u^2}\right]+\text{sign}\left[-\frac{\left(-1+2u^2\right)\left(-3+4u^2\right)}{-2+3u^2}\right]+\text{sign}\left[-\frac{2\left(-1+2u^2\right)\left(-5+6u^2\right)}{-3+4u^2}\right]+\text{sign}\left[-\frac{\left(-7+8u^2\right)\left(-5+8u^2\right)}{4\left(-1+2u^2\right)\left(-5+6u^2\right)}\right]+\text{sign}\left[-\frac{17-48u^2+32u^4}{\left(-1+2u^2\right)\left(-7+8u^2\right)}\right]+\text{sign}\left[-\frac{17-48u^2+32u^4}{2\left(-5+8u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(3+\text{sign}\left[4(-1+u)(1+u)\right]+\text{sign}\left[3-4u^2\right]+\text{sign}\left[-\frac{\left(-2-u+2u^2\right)\left(-2+u+2u^2\right)\left(-3+4u^2\right)}{5-10u^2+4u^4}\right]+\text{sign}\left[\frac{5-10u^2+4u^4}{(-1+u)(1+u)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{\left(-3+2u^2\right)\left(7-16u^2+8u^4\right)}{\left(-2-u+2u^2\right)\left(-2+u+2u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{19-36u^2+16u^4}{4\left(-3+2u^2\right)}\right]+\text{sign}\left[-\frac{19-36u^2+16u^4}{7-16u^2+8u^4}\right]\right),$$

$$\frac{1}{2}\left(\text{sign}\left[-3+2u^2\right]+\text{sign}\left[-\frac{8(-1+u)(1+u)}{-3+4u^2}\right]+\text{sign}\left[-\frac{\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(-3+2u^2\right)}\right]+\text{sign}\left[-\frac{13-20u^2+8u^4}{-5+4u^2}\right]+\text{sign}\left[\frac{21-36u^2+16u^4}{13-28u^2+16u^4}\right]+\text{sign}\left[\frac{\left(21-36u^2+16u^4\right)\left(13-28u^2+16u^4\right)}{8(-1+u)(1+u)\left(13-20u^2+8u^4\right)}\right]\right)\}$$

In[∘]:=
```
u = 1 / 2;
KasSig /@ AllKnots[{3, 8}]
Clear[u]
```

Out[∘]=

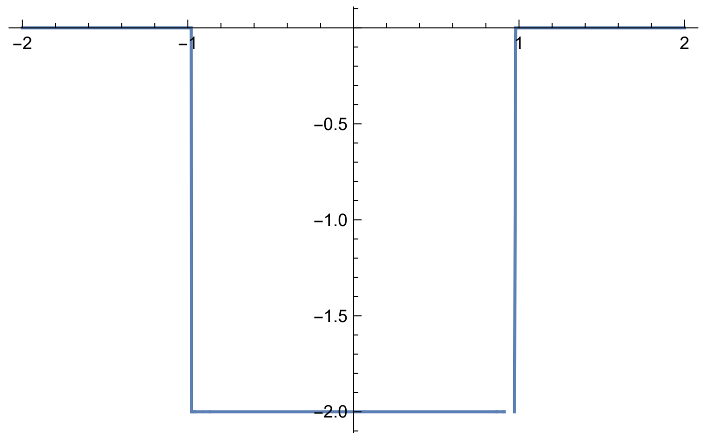
{2, 0, 4, 2, 0, 2, 0, 4, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -4, 0, 2}

In[]:=
```
f = KasSig[Knot[9, 5]]
Plot[f, {u, -2, 2}]
```

Out[]=

$$\frac{1}{2}\left(-7+\text{sign}\left[2\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{3\left(-5+6u^2\right)}{2\left(-9+11u^2\right)}\right]+\text{sign}\left[\frac{-9+11u^2}{2\left(-3+4u^2\right)}\right]+\text{sign}\left[\frac{-11+12u^2}{-5+6u^2}\right]+\text{sign}\left[\frac{-17+18u^2}{-11+12u^2}\right]+\text{sign}\left[\frac{1}{6}\left(-23+24u^2\right)\right]+\text{sign}\left[\frac{-23+24u^2}{-17+18u^2}\right]\right)$$

Out[]=

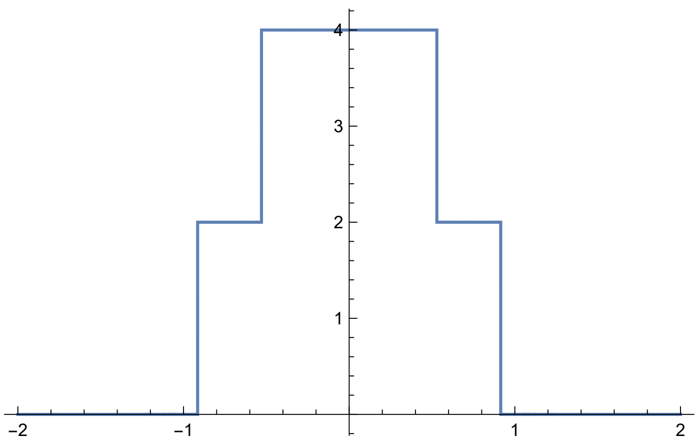


In[ ]:=
```
f = KasSig[Knot[8, 2]]
Plot[f, {u, -2, 2}, PlotPoints → 1000]
```

Out[ ]=

$$\frac{1}{2}\left(5+\text{sign}\left[-3+2u^2\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(-3+2u^2\right)}\right]+\text{sign}\left[-\frac{8(-1+u)(1+u)\left(3-12u^2+8u^4\right)}{\left(-5+4u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(1-10u^2+8u^4\right)\left(7-24u^2+16u^4\right)}{8(-1+u)(1+u)\left(-1+2u^2\right)\left(3-12u^2+8u^4\right)}\right]+\text{sign}\left[-\frac{-17+96u^2-144u^4+64u^6}{\left(-3+4u^2\right)\left(1-10u^2+8u^4\right)}\right]+\text{sign}\left[-\frac{-17+96u^2-144u^4+64u^6}{2\left(7-24u^2+16u^4\right)}\right]\right)$$

Out[ ]=

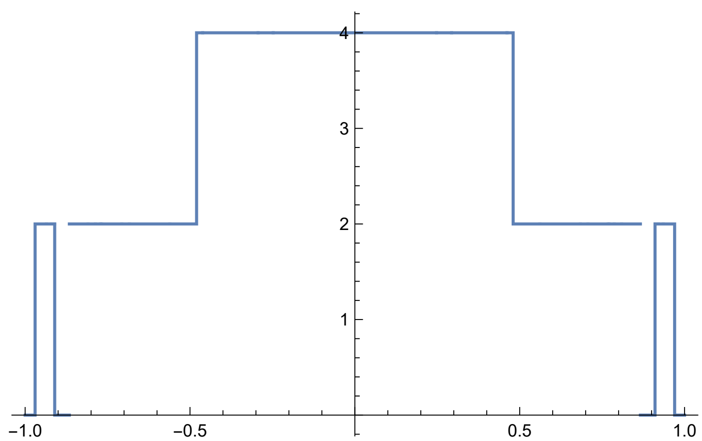


In[∘]:=
```
f = KasSig[Knot[12, Alternating, 422]]
Plot[f, {u, -1, 1}, PlotPoints → 1000]
```

Out[∘]=

$$\frac{1}{2}\left(4+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[\frac{2}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{-7+8u^2}{2\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{2\left(8-23u^2+16u^4\right)}{-7+8u^2}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)^2\left(11-28u^2+16u^4\right)}{-44+155u^2-176u^4+64u^6}\right]+\text{sign}\left[\frac{-44+155u^2-176u^4+64u^6}{8-23u^2+16u^4}\right]+\text{sign}\left[-\frac{-11+76u^2-128u^4+64u^6}{11-28u^2+16u^4}\right]+\text{sign}\left[-\frac{\left(-29+160u^2-256u^4+128u^6\right)\left(11-170u^2+544u^4-640u^6+256u^8\right)}{4\left(-1+2u^2\right)\left(-3+4u^2\right)^2\left(-11+76u^2-128u^4+64u^6\right)}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)}{11-228u^2+864u^4-1152u^6+512u^8}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)\left(11-228u^2+864u^4-1152u^6+512u^8\right)}{2\left(-29+160u^2-256u^4+128u^6\right)\left(11-170u^2+544u^4-640u^6+256u^8\right)}\right]\right)$$

Out[∘]=



Bedlewo for Knots

In[∘]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
Bed[Knot[3, 1]]
```

Out[∘]=

$$\text{TSI}_{B[]}\left[\text{sign}\left[-\frac{2(-1+\omega)^2}{\omega}\right]+\text{sign}\left[-\frac{2\left(1-\omega+\omega^2\right)}{\omega}\right],\ \text{PQ}[\{\},0]\right]$$

In[ ]:=
```
ω = -1;
BedSig /@ AllKnots[{3, 8}]
Clear[ω]
```

Out[ ]=
```
{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}
```

In[∘]:= **BedSig /@ AllKnots[{3, 7}]**

Out[∘]=

$$\Big\{\text{sign}\Big[-\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(1-\omega+\omega^2)}{\omega}\Big],\ \text{sign}\Big[\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(1-3\omega+\omega^2)}{\omega}\Big],$$

$$2\,\text{sign}\Big[-\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(1+\omega^2)}{\omega}\Big]+\text{sign}\Big[-\frac{2(1-\omega+\omega^2-\omega^3+\omega^4)}{\omega(1+\omega^2)}\Big],$$

$$\text{sign}\Big[-\frac{4(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2-3\omega+2\omega^2}{\omega}\Big],\ \text{sign}\Big[\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(-2+\omega)(-1+2\omega)}{\omega}\Big],$$

$$\text{sign}\Big[\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\Big]+\text{sign}\Big[-\frac{2(1-3\omega+\omega^2)}{\omega}\Big]+$$

$$\text{sign}\Big[-\frac{2(1-3\omega+3\omega^2-3\omega^3+\omega^4)}{(-1+\omega)^2\omega}\Big],\ \text{sign}\Big[\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\Big]+$$

$$\text{sign}\Big[-\frac{2(1-3\omega+\omega^2)}{\omega}\Big]+\text{sign}\Big[\frac{2(1-3\omega+5\omega^2-3\omega^3+\omega^4)}{(-1+\omega)^2\omega}\Big],$$

$$3\,\text{sign}\Big[-\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(1+\omega^2)}{\omega}\Big]+\text{sign}\Big[-\frac{2(1-\omega+\omega^2)(1+\omega+\omega^2)}{\omega(1+\omega^2)}\Big]+$$

$$\text{sign}\Big[-\frac{2(1-\omega+\omega^2-\omega^3+\omega^4-\omega^5+\omega^6)}{\omega(1-\omega+\omega^2)(1+\omega+\omega^2)}\Big],\ \text{sign}\Big[-\frac{6(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(3-5\omega+3\omega^2)}{3\omega}\Big],$$

$$\text{sign}\Big[\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[\frac{4(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[\frac{2-\omega+2\omega^2}{\omega}\Big]+\text{sign}\Big[\frac{2(2-3\omega+3\omega^2-3\omega^3+2\omega^4)}{\omega(2-\omega+2\omega^2)}\Big],$$

$$\text{sign}\Big[\frac{4(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[\frac{4-7\omega+4\omega^2}{\omega}\Big],\ \text{sign}\Big[-\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(1-\omega+\omega^2)}{\omega}\Big]+$$

$$\text{sign}\Big[-\frac{2(-1+\omega)^2(2-\omega+2\omega^2)}{\omega(1-\omega+\omega^2)}\Big]+\text{sign}\Big[-\frac{2(2-4\omega+5\omega^2-4\omega^3+2\omega^4)}{\omega(2-\omega+2\omega^2)}\Big],$$

$$\text{sign}\Big[-\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(-1+\omega)^2(1-4\omega+\omega^2)}{\omega(1-3\omega+\omega^2)}\Big]+\text{sign}\Big[\frac{2(1-3\omega+\omega^2)}{\omega}\Big]+$$

$$\text{sign}\Big[-\frac{2(1-5\omega+7\omega^2-5\omega^3+\omega^4)}{\omega(1-4\omega+\omega^2)}\Big],\ \text{sign}\Big[\frac{2(-1+\omega)^2}{\omega}\Big]+\text{sign}\Big[-\frac{2(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\Big]+$$

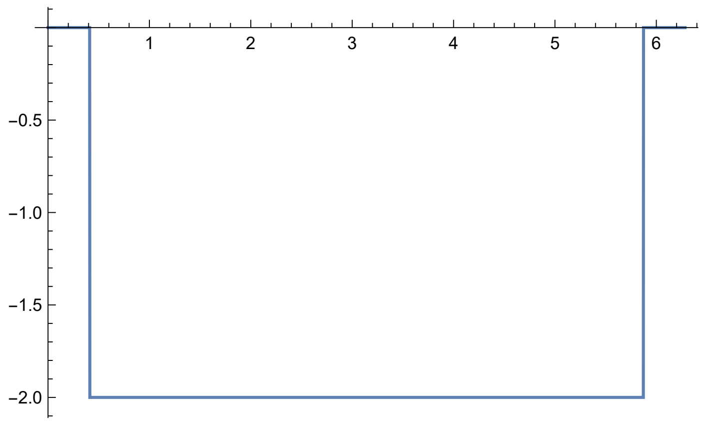
$$\text{sign}\Big[-\frac{2(1-3\omega+\omega^2)}{\omega}\Big]+\text{sign}\Big[\frac{2(1-5\omega+9\omega^2-5\omega^3+\omega^4)}{(-1+\omega)^2\omega}\Big]\Big\}$$

In[ ]:=
```
f = BedSig[Knot[9, 5]] /. ω → e^(i t)
Plot[f, {t, 0, 2 π}]
```

Out[ ]=

$$\text{sign}\left[6\, e^{-i t} \left(-1+e^{i t}\right)^2\right]+\text{sign}\left[\frac{2}{3}\, e^{-i t} \left(6-11\, e^{i t}+6\, e^{2 i t}\right)\right]$$

Out[ ]=

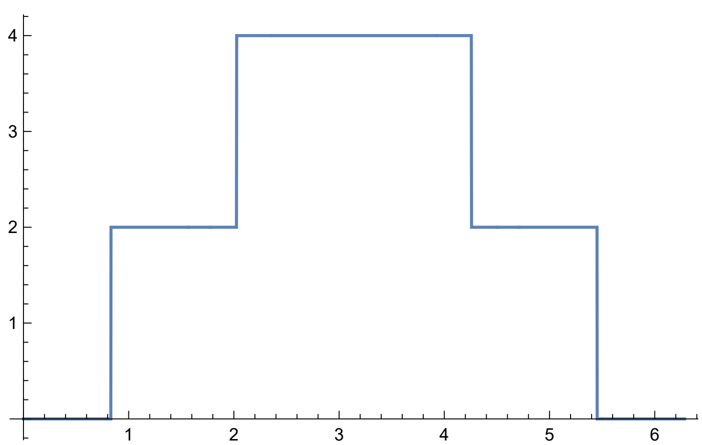


In[ ]:=
```
f = BedSig[Knot[8, 2]] /. ω → e^(i t)
Plot[f, {t, 0, 2 π}]
```

Out[ ]=

$$\text{sign}\left[-2\, e^{-i t} \left(-1+e^{i t}\right)^2\right]+\text{sign}\left[2\, e^{-i t} \left(-1+e^{i t}\right)^2\right]+\text{sign}\left[-\frac{2\, e^{-i t} \left(-1+e^{i t}\right)^4}{1-3\, e^{i t}+e^{2 i t}}\right]+$$
$$\text{sign}\left[-2\, e^{-i t} \left(1-3\, e^{i t}+e^{2 i t}\right)\right]+\text{sign}\left[-\frac{2\, e^{-i t} \left(1-2\, e^{i t}+e^{2 i t}-2\, e^{3 i t}+e^{4 i t}\right)}{\left(-1+e^{i t}\right)^2}\right]+$$
$$\text{sign}\left[-\frac{2\, e^{-i t} \left(1-3\, e^{i t}+3\, e^{2 i t}-3\, e^{3 i t}+3\, e^{4 i t}-3\, e^{5 i t}+e^{6 i t}\right)}{1-2\, e^{i t}+e^{2 i t}-2\, e^{3 i t}+e^{4 i t}}\right]$$

Out[ ]=



In[ ]:=

```
f = BedSig[Knot[12, Alternating, 422]] /. ω → ⅇ^(ⅈ t)
Plot[f, {t, 0, 2 π}, PlotPoints → 1000]
```

Out[ ]=

$$2\,\mathrm{sign}\left[-2\,e^{-i t}\left(-1+e^{i t}\right)^2\right]+\mathrm{sign}\left[2\,e^{-i t}\left(-1+e^{i t}\right)^2\right]+\mathrm{sign}\left[4\,e^{-i t}\left(-1+e^{i t}\right)^2\right]+$$
$$\mathrm{sign}\left[-e^{-i t}\left(-2+e^{i t}\right)\left(-1+2\,e^{i t}\right)\right]+\mathrm{sign}\left[-\frac{4\,e^{-i t}\left(1-e^{i t}+e^{2 i t}\right)^2}{\left(-2+e^{i t}\right)\left(-1+2\,e^{i t}\right)}\right]+$$
$$\mathrm{sign}\left[-\frac{2\,e^{-i t}\left(1-e^{i t}+e^{2 i t}\right)\left(2-4\,e^{i t}+4\,e^{2 i t}-3\,e^{3 i t}+4\,e^{4 i t}-4\,e^{5 i t}+2\,e^{6 i t}\right)}{2-4\,e^{i t}+6\,e^{2 i t}-5\,e^{3 i t}+6\,e^{4 i t}-4\,e^{5 i t}+2\,e^{6 i t}}\right]+$$
$$\mathrm{sign}\left[-\frac{e^{-i t}\left(2-4\,e^{i t}+6\,e^{2 i t}-5\,e^{3 i t}+6\,e^{4 i t}-4\,e^{5 i t}+2\,e^{6 i t}\right)}{\left(1-e^{i t}+e^{2 i t}\right)^2}\right]$$

Out[ ]=