```
In[◦]:=  Once[
          SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
          << KnotTheory`;
         ]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

pdf

```
In[◦]:=  SetAttributes[B, Orderless];
         CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

```
In[◦]:=  CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_, ∞]},
            Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) ⧴ Factor[c] Times @@ ηs^ps]]
```

pdf

```
In[◦]:=  CF[{}] = {};
         CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
           CF /@ DeleteCases[
             RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs,
             0] ]
```

pdf

```
In[◦]:=  RuleOf[η_i_ + rest_.] := (η_i → -rest);
         CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
           PQ[nrs, CF[q /. (RuleOf /@ nrs)]] ]
```

```
In[◦]:=  CF[{η_1 - η_2, η_1 - η_3}]
Out[◦]=
         {η_1 - η_3, η_2 - η_3}
```

```
In[◦]:=  RuleOf[η_1 + η_2 + η_3]
Out[◦]=
         η_1 → -η_2 - η_3
```

pdf

```
In[◦]:=  CF[TSI_b_[σ_, pq_]] := TSI_CF[b][σ, CF[pq]]
```

pdf

```
In[◦]:=  Kas[P[i_, j_]] := CF@TSI_B[{-i,j}][0, PQ[{}, 0]]
```

The disjoint union in the world of multi-tangles.

pdf

```
In[◦]:=  TSI_b1_[σ1_, PQ[rs1_, q1_]] ⋃ TSI_b2_[σ2_, PQ[rs2_, q2_]] ^:=
           CF@TSI_Join[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

FM for Face Merge:

pdf

In[●]:=
```
FM_{i_,j_} @TSI_{B[{li___,i_,ri___},{lj___,j_,rj___},bs___]}[σ_, PQ[rs_, q_]] :=
   CF@TSI_{B[{ri,li,i,rj,lj,j},bs]}[σ, PQ[rs ∪ {η_i - η_j}, q]]
```

In[●]:= `Kas[P[1, 2]] ∪ Kas[P[3, 4]] // FM_{-1,4}`

Out[●]=

$TSI_{B[\{-3,4,2,-1\}]}[0, PQ[\{η_{-1} - η_4\}, 0]]$

pdf
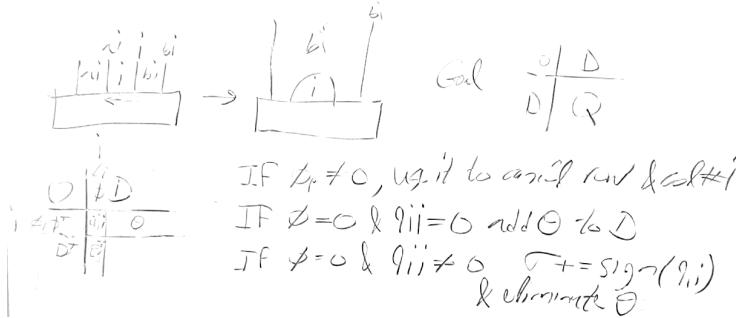
pdf

In[●]:=
```
Cordon_{i_} @TSI_{B[{li___,i_,ri___},bs___]}[σ_, PQ[rs_, q_]] :=
  Module[{bi, ai, φ, nσ, nrs, nq, qii, p},
    ai = First@{ri, li}; bi = Last@{ri, li};
    {nσ, nrs, nq} = {σ, rs, q}; φ = ∂_{η_i} rs;
    If[And @@ ((# == 0) & /@ φ), qii = ∂_{η_i,η_i} q;
     If[qii === 0,
       AppendTo[nrs, ∂_{η_i} q]; nq = q /. η_i → 0,
       (*else*) nσ += Sign[qii]; nq = q /. η_i → - (∂_{η_i} q) / qii /. η_i → 0],
      (*else*) {p} = FirstPosition[(# == 0) & /@ φ, False];
     {nrs, nq} = {rs, q} /. η_i → -rs[[p]] / φ[[p]] /. η_i → 0];
    CF@TSI_{B[Rest@{ri,li},bs]}[nσ, PQ[nrs, nq] /. η_{ai} → η_{bi}] ]
```

tex

c for contract:

pdf

In[●]:=
```
c_{i_,j_} @TSI_{B[{li___,i_,ri___},{lj___,j_,rj___},bs___]}[ℰ__] := Module[{bi = Last@{ri, li}},
    TSI_{B[{li,i,ri},{lj,j,rj},bs]}[ℰ] // FM_{j,bi} // Cordon_j ];
```

pdf

In[●]:=
```
c_{i_,j_} @TSI_{B[{l___,i_,j_,r___},bs___]}[ℰ__] := Cordon_i@TSI_{B[{l,i,j,r},bs]}[ℰ];
c_{i_,j_} @TSI_{B[{j_,m___,i_},bs___]}[ℰ__] := Cordon_i@TSI_{B[{j,m,i},bs]}[ℰ];
c_{i_,j_} @TSI_{B[{l___,j_,i_,r___},bs___]}[ℰ__] := Cordon_j@TSI_{B[{l,j,i,r},bs]}[ℰ];
c_{i_,j_} @TSI_{B[{i_,m___,j_},bs___]}[ℰ__] := Cordon_j@TSI_{B[{i,m,j},bs]}[ℰ];
```

tex

mc for magnetic contract:

pdf

In[ ]:=
$$\texttt{mc@TSI}_{B[\{li\_\_\_,i\_,ri\_\_\_\},\{lj\_\_\_,j\_,rj\_\_\_\},bs\_\_\_]}[\mathcal{E}\_\_] \ /; \ j == -i :=$$
$$\quad \texttt{mc@c}_{i,j}\texttt{@TSI}_{B[\{li,i,ri\},\{lj,j,rj\},bs]}[\mathcal{E}];$$
$$\texttt{mc@TSI}_{B[\{l\_\_\_,i\_,j\_,r\_\_\_\},bs\_\_\_]}[\mathcal{E}\_\_] \ /; \ j == -i := \texttt{mc@Cordon}_i\texttt{@TSI}_{B[\{l,i,j,r\},bs]}[\mathcal{E}];$$
$$\texttt{mc@TSI}_{B[\{j\_,m\_\_\_,i\_\},bs\_\_\_]}[\mathcal{E}\_\_] \ /; \ j == -i := \texttt{mc@Cordon}_i\texttt{@TSI}_{B[\{j,m,i\},bs]}[\mathcal{E}];$$
$$\texttt{mc@TSI}_{b\_B}[\mathcal{E}\_\_] \ /; \ (\texttt{Union @@ } b \cap (-\texttt{Union @@ } b)) \ === \ \{\} := \texttt{TSI}_b[\mathcal{E}]$$

pdf

In[ ]:=
$$\texttt{Kas}[X[i\_, \ j\_, \ k\_, \ l\_]] := \texttt{With}\Big[\{v = 2\,u^2 - 1\}, \ \texttt{CF@If}\Big[\texttt{PositiveQ@X}[i, \ j, \ k, \ l],$$
$$\quad \texttt{TSI}_{B[\{-i,j,k,-l\}]}\Big[0, \ \texttt{PQ}\Big[\{\},$$
$$\quad\quad \eta_{-i}^2 + 2\,u\,\eta_{-i}\,\eta_j + v\,\eta_j^2 + 2\,\eta_{-i}\,\eta_k + 2\,u\,\eta_j\,\eta_k + \eta_k^2 + 2\,u\,\eta_{-i}\,\eta_{-l} + 2\,\eta_j\,\eta_{-l} + 2\,u\,\eta_k\,\eta_{-l} + v\,\eta_{-l}^2\Big]\Big],$$
$$\quad \texttt{TSI}_{B[\{-i,-j,k,l\}]}\Big[0, \ \texttt{PQ}\Big[\{\},$$
$$\quad\quad -v\,\eta_{-i}^2 - 2\,u\,\eta_{-i}\,\eta_{-j} - \eta_{-j}^2 - 2\,\eta_{-i}\,\eta_k - 2\,u\,\eta_{-j}\,\eta_k - v\,\eta_k^2 - 2\,u\,\eta_{-i}\,\eta_l - 2\,\eta_{-j}\,\eta_l - 2\,u\,\eta_k\,\eta_l - \eta_l^2\Big]\Big] \ \Big]\Big]$$

pdf

Reidemeister 2
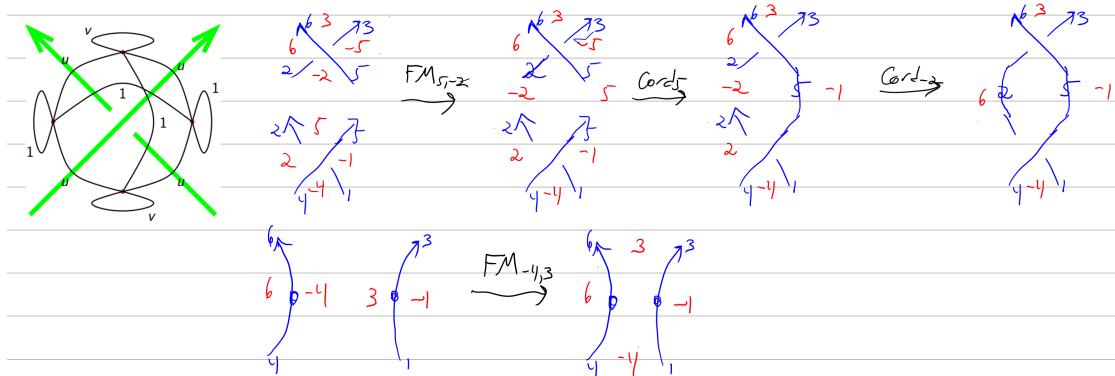
In[ ]:= $\texttt{Kas}[X[1, \ 5, \ 2, \ 4]] \ \bigcup \texttt{Kas}[X[2, \ 5, \ 3, \ 6]]$

Out[ ]=
$$\texttt{TSI}_{B[\{-5,3,6,-2\},\{-4,-1,5,2\}]}\Big[0,$$
$$\texttt{PQ}\Big[\{\}, \ -\eta_{-5}^2 + \left(-1 + 2\,u^2\right)\eta_{-4}^2 - 2\,u\,\eta_{-5}\,\eta_{-2} + \left(1 - 2\,u^2\right)\eta_{-2}^2 + 2\,u\,\eta_{-4}\,\eta_{-1} + \eta_{-1}^2 +$$
$$\quad 2\,u\,\eta_{-4}\,\eta_2 + 2\,\eta_{-1}\,\eta_2 + \eta_2^2 - 2\,u\,\eta_{-5}\,\eta_3 - 2\,\eta_{-2}\,\eta_3 + \left(1 - 2\,u^2\right)\eta_3^2 + 2\,\eta_{-4}\,\eta_5 +$$
$$\quad 2\,u\,\eta_{-1}\,\eta_5 + 2\,u\,\eta_2\,\eta_5 + \left(-1 + 2\,u^2\right)\eta_5^2 - 2\,\eta_{-5}\,\eta_6 - 2\,u\,\eta_{-2}\,\eta_6 - 2\,u\,\eta_3\,\eta_6 - \eta_6^2\Big]\Big]$$

In[ ]:= $\texttt{Kas}[X[1, \ 5, \ 2, \ 4]] \ \bigcup \texttt{Kas}[X[2, \ 5, \ 3, \ 6]] \ // \ \texttt{FM}_{-2,5}$

Out[ ]=
$$\texttt{TSI}_{B[\{-5,3,6,-2,2,-4,-1,5\}]}\Big[0, \ \texttt{PQ}\Big[\{\eta_{-2} - \eta_5\},$$
$$-\eta_{-5}^2 + \left(-1 + 2\,u^2\right)\eta_{-4}^2 + 2\,u\,\eta_{-4}\,\eta_{-1} + \eta_{-1}^2 + 2\,u\,\eta_{-4}\,\eta_2 + 2\,\eta_{-1}\,\eta_2 + \eta_2^2 - 2\,u\,\eta_{-5}\,\eta_3 + \left(1 - 2\,u^2\right)\eta_3^2 -$$
$$\quad 2\,u\,\eta_{-5}\,\eta_5 + 2\,\eta_{-4}\,\eta_5 + 2\,u\,\eta_{-1}\,\eta_5 + 2\,u\,\eta_2\,\eta_5 - 2\,\eta_3\,\eta_5 - 2\,\eta_{-5}\,\eta_6 - 2\,u\,\eta_3\,\eta_6 - 2\,u\,\eta_5\,\eta_6 - \eta_6^2\Big]\Big]$$

$In[\circ]:=$ **Kas[X[1, 5, 2, 4]] $\bigcup$ Kas[X[2, 5, 3, 6]] // FM$_{-2,5}$ // Cordon$_5$**

$Out[\circ]=$

$\mathrm{TSI}_{B[\{-4,-1,3,6,-2,2\}]}\Big[0,$

$\quad \mathrm{PQ}\Big[\{\}, \ \left(-1 + 2\,u^2\right)\eta_{-4}^2 + 2\,\eta_{-4}\,\eta_{-2} + 2\,u\,\eta_{-4}\,\eta_{-1} + 2\,u\,\eta_{-4}\,\eta_2 + 2\,u\,\eta_{-2}\,\eta_2 + 2\,\eta_{-1}\,\eta_2 + \eta_2^2 -$

$\qquad 2\,\eta_{-2}\,\eta_3 - 2\,u\,\eta_{-1}\,\eta_3 + \left(1 - 2\,u^2\right)\eta_3^2 - 2\,u\,\eta_{-2}\,\eta_6 - 2\,\eta_{-1}\,\eta_6 - 2\,u\,\eta_3\,\eta_6 - \eta_6^2\Big]\Big]$

$In[\circ]:=$ **Kas[X[1, 5, 2, 4]] $\bigcup$ Kas[X[2, 5, 3, 6]] // FM$_{-2,5}$ // Cordon$_5$ // Cordon$_{-2}$**

$Out[\circ]=$

$\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0, \mathrm{PQ}[\{\eta_{-4} - \eta_3\}, 0]]$

pdf

$In[\circ]:=$ **{Kas[P[1, 3]] $\bigcup$ Kas[P[4, 6]] // FM$_{-4,3}$, Kas[X[1, 5, 2, 4]] $\bigcup$ Kas[X[2, 5, 3, 6]] // mc}**

$Out[\circ]=$
pdf

$\{\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0, \mathrm{PQ}[\{\eta_{-4} - \eta_3\}, 0]], \ \mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0, \mathrm{PQ}[\{\eta_{-4} - \eta_3\}, 0]]\}$

pdf

Reidemeister 3

$In[\circ]:=$ **{u = 7 / 29};**
lhs = Kas[X[4, 2, 5, 1]] $\bigcup$ Kas[X[7, 3, 8, 2]] $\bigcup$ Kas[X[8, 6, 9, 5]] // c$_{2,-2}$ // c$_{5,-5}$ // c$_{8,-8}$
rhs = Kas[X[7, 5, 8, 4]] $\bigcup$ Kas[X[8, 2, 9, 1]] $\bigcup$ Kas[X[5, 3, 6, 2]] // c$_{2,-2}$ // c$_{5,-5}$ // c$_{8,-8}$
Clear[u]

$Out[\circ]=$

$\mathrm{TSI}_{B[\{-7,3,6,9,-1,-4\}]}\Big[-1, \mathrm{PQ}\Big[\{\},$

$\quad \dfrac{1486\,\eta_{-7}^2}{645} + \dfrac{32\,578\,\eta_{-7}\,\eta_{-4}}{18\,705} + \dfrac{228\,046\,\eta_{-4}^2}{542\,445} + \dfrac{1682}{645}\,\eta_{-7}\,\eta_{-1} + \dfrac{32\,578\,\eta_{-4}\,\eta_{-1}}{18\,705} + \dfrac{228\,046\,\eta_{-1}^2}{542\,445} + \dfrac{32\,578\,\eta_{-7}\,\eta_3}{18\,705} +$

$\quad \dfrac{1682}{645}\,\eta_{-4}\,\eta_3 + \dfrac{812}{645}\,\eta_{-1}\,\eta_3 + \dfrac{228\,046\,\eta_3^2}{542\,445} + \dfrac{1682}{645}\,\eta_{-7}\,\eta_6 + \dfrac{812}{645}\,\eta_{-4}\,\eta_6 + \dfrac{1682}{645}\,\eta_{-1}\,\eta_6 + \dfrac{32\,578\,\eta_3\,\eta_6}{18\,705} +$

$\quad \dfrac{228\,046\,\eta_6^2}{542\,445} + \dfrac{812}{645}\,\eta_{-7}\,\eta_9 + \dfrac{1682}{645}\,\eta_{-4}\,\eta_9 + \dfrac{32\,578\,\eta_{-1}\,\eta_9}{18\,705} + \dfrac{1682\,\eta_3\,\eta_9}{645} + \dfrac{32\,578\,\eta_6\,\eta_9}{18\,705} + \dfrac{1486\,\eta_9^2}{645}\Big]\Big]$

$Out[\circ]=$

$\mathrm{TSI}_{B[\{-7,3,6,9,-1,-4\}]}\Big[-1, \mathrm{PQ}\Big[\{\},$

$\quad \dfrac{1486\,\eta_{-7}^2}{645} + \dfrac{32\,578\,\eta_{-7}\,\eta_{-4}}{18\,705} + \dfrac{228\,046\,\eta_{-4}^2}{542\,445} + \dfrac{1682}{645}\,\eta_{-7}\,\eta_{-1} + \dfrac{32\,578\,\eta_{-4}\,\eta_{-1}}{18\,705} + \dfrac{228\,046\,\eta_{-1}^2}{542\,445} + \dfrac{32\,578\,\eta_{-7}\,\eta_3}{18\,705} +$

$\quad \dfrac{1682}{645}\,\eta_{-4}\,\eta_3 + \dfrac{812}{645}\,\eta_{-1}\,\eta_3 + \dfrac{228\,046\,\eta_3^2}{542\,445} + \dfrac{1682}{645}\,\eta_{-7}\,\eta_6 + \dfrac{812}{645}\,\eta_{-4}\,\eta_6 + \dfrac{1682}{645}\,\eta_{-1}\,\eta_6 + \dfrac{32\,578\,\eta_3\,\eta_6}{18\,705} +$

$\quad \dfrac{228\,046\,\eta_6^2}{542\,445} + \dfrac{812}{645}\,\eta_{-7}\,\eta_9 + \dfrac{1682}{645}\,\eta_{-4}\,\eta_9 + \dfrac{32\,578\,\eta_{-1}\,\eta_9}{18\,705} + \dfrac{1682\,\eta_3\,\eta_9}{645} + \dfrac{32\,578\,\eta_6\,\eta_9}{18\,705} + \dfrac{1486\,\eta_9^2}{645}\Big]\Big]$

pdf

```
In[◦]:= lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // mc;
       rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // mc;
       {lhs〚1〛, rhs〚1〛}
       Simplify[lhs〚2, 2〛 == rhs〚2, 2〛]
```

Out[◦]=

pdf

$$\{\text{Sign}[(-1 + 2\,u)\,(1 + 2\,u)], \text{Sign}[(-1 + 2\,u)\,(1 + 2\,u)]\}$$

Out[◦]=

pdf

True

```
In[◦]:= lhs〚2, 2〛
```

Out[◦]=

$$\frac{2\,(-1 + 2\,u^2)\,\eta_{-7}^2}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,u\,(-3 + 4\,u^2)\,\eta_{-7}\,\eta_{-4}}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,u^2\,(-3 + 4\,u^2)\,\eta_{-4}^2}{(-1 + 2\,u)\,(1 + 2\,u)} -$$

$$\frac{2\,\eta_{-7}\,\eta_{-1}}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,u\,(-3 + 4\,u^2)\,\eta_{-4}\,\eta_{-1}}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,u^2\,(-3 + 4\,u^2)\,\eta_{-1}^2}{(-1 + 2\,u)\,(1 + 2\,u)} +$$

$$\frac{2\,u\,(-3 + 4\,u^2)\,\eta_{-7}\,\eta_3}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{2\,\eta_{-4}\,\eta_3}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{4\,u\,\eta_{-1}\,\eta_3}{(-1 + 2\,u)\,(1 + 2\,u)} +$$

$$\frac{2\,u^2\,(-3 + 4\,u^2)\,\eta_3^2}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{2\,\eta_{-7}\,\eta_6}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{4\,u\,\eta_{-4}\,\eta_6}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{2\,\eta_{-1}\,\eta_6}{(-1 + 2\,u)\,(1 + 2\,u)} +$$

$$\frac{2\,u\,(-3 + 4\,u^2)\,\eta_3\,\eta_6}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,u^2\,(-3 + 4\,u^2)\,\eta_6^2}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{4\,u\,\eta_{-7}\,\eta_9}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{2\,\eta_{-4}\,\eta_9}{(-1 + 2\,u)\,(1 + 2\,u)} +$$

$$\frac{2\,u\,(-3 + 4\,u^2)\,\eta_{-1}\,\eta_9}{(-1 + 2\,u)\,(1 + 2\,u)} - \frac{2\,\eta_3\,\eta_9}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,u\,(-3 + 4\,u^2)\,\eta_6\,\eta_9}{(-1 + 2\,u)\,(1 + 2\,u)} + \frac{2\,(-1 + 2\,u^2)\,\eta_9^2}{(-1 + 2\,u)\,(1 + 2\,u)}$$

Kashaev for Knots

```
In[◦]:= -KnotSignature /@ AllKnots[{3, 8}]
```

⚫ KnotTheory: Loading precomputed data in PD4Knots`.

Out[◦]=

$$\{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,$$
$$0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2\}$$

pdf

```
In[◦]:= KasSig[K_] := Module[{pd = PD[K]},
           mc[Union @@ (Kas /@ pd)]〚1〛 - Sum[If[PositiveQ@x, 1, -1], {x, List @@ pd}]] / 2
```

```
In[◦]:= (*u=0;*)
       mc[Union @@ (Kas /@ PD@Knot[3, 1])]
       Clear[u]
```

Out[◦]=

$$\text{TSI}_{B[\,]}\left[-1 + \text{Sign}[3 - 4\,u^2] - \text{Sign}[-3 + 4\,u^2],\, \text{PQ}[\{\}, 0]\right]$$

In[◦]:= (*u=0;*)
KasSig /@ **AllKnots**[{3, 8}]
Clear[u]

Out[◦]=

$$\left\{\frac{1}{2} \left(2 + \text{Sign}\left[3 - 4\,u^2\right] - \text{Sign}\left[-3 + 4\,u^2\right]\right),\right.$$

$$\frac{1}{2} \left(-1 + \text{Sign}\left[3 - 2\,u^2\right] + \text{Sign}\left[-5 + 4\,u^2\right] + \frac{\text{Sign}\left[-5 + 4\,u^2\right]}{\text{Sign}\left[-3 + 2\,u^2\right]}\right),$$

$$\frac{1}{2} \left(5 - \text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\left(-5 + 6\,u^2\right)\right]}{\text{Sign}\left[-5 + 8\,u^2\right]} - \frac{\text{Sign}\left[-5 + 8\,u^2\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \right.$$
$$\left.\frac{\text{Sign}\left[5 - 20\,u^2 + 16\,u^4\right]}{\text{Sign}\left[-3 + 4\,u^2\right]} - \frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(5 - 20\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-5 + 6\,u^2\right]}\right),$$

$$\frac{1}{2} \left(3 + \text{Sign}\left[7 - 8\,u^2\right] - \text{Sign}\left[-3 + 4\,u^2\right] - \frac{\text{Sign}\left[-7 + 8\,u^2\right]}{\text{Sign}\left[-3 + 4\,u^2\right]}\right),$$

$$\frac{1}{2} \left(1 + \text{Sign}\left[9 - 8\,u^2\right] - \frac{\text{Sign}\left[-5 + 4\,u^2\right]}{\text{Sign}\left[-3 + 2\,u^2\right]} + \frac{\text{Sign}\left[-3 + 2\,u^2\right]}{\text{Sign}\left[-3 + 4\,u^2\right]} + \text{Sign}\left[-3 + 4\,u^2\right] - \frac{\text{Sign}\left[-9 + 8\,u^2\right]}{\text{Sign}\left[-5 + 4\,u^2\right]}\right),$$

$$\frac{1}{2} \left(2 - \text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\left(-5 + 4\,u^2\right)\left(-3 + 4\,u^2\right)\right]}{\text{Sign}\left[5 - 10\,u^2 + 4\,u^4\right]} + \frac{\text{Sign}\left[5 - 10\,u^2 + 4\,u^4\right]}{\text{Sign}\left[5 - 12\,u^2 + 8\,u^4\right]} + \right.$$
$$\left.\frac{\text{Sign}\left[5 - 12\,u^2 + 8\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[11 - 28\,u^2 + 16\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-5 + 4\,u^2\right]} - \frac{\text{Sign}\left[11 - 28\,u^2 + 16\,u^4\right]}{\text{Sign}\left[-3 + 4\,u^2\right]}\right),$$

$$\frac{1}{2} \left(\text{Sign}\left[\left(-1 + u\right)\left(1 + u\right)\right] + \frac{\text{Sign}\left[-3 + 2\,u^2\right]}{\text{Sign}\left[-1 + u\right]\,\text{Sign}\left[1 + u\right]} + \frac{\text{Sign}\left[\left(-7 + 4\,u^2\right)\left(-3 + 4\,u^2\right)\right]}{\text{Sign}\left[-3 + 2\,u^2\right]} - \right.$$
$$\frac{\text{Sign}\left[13 - 28\,u^2 + 16\,u^4\right]}{\text{Sign}\left[-1 + u\right]\,\text{Sign}\left[1 + u\right]} - \frac{\text{Sign}\left[\left(-1 + u\right)\left(1 + u\right)\left(13 - 28\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[-14 + 45\,u^2 - 48\,u^4 + 16\,u^6\right]} - $$
$$\left.\frac{\text{Sign}\left[-14 + 45\,u^2 - 48\,u^4 + 16\,u^6\right]}{\text{Sign}\left[-7 + 4\,u^2\right]\,\text{Sign}\left[-3 + 4\,u^2\right]}\right),$$

$$\frac{1}{2} \left(7 - 2\,\text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\left(-7 + 8\,u^2\right)\right]}{\text{Sign}\left[-7 + 10\,u^2\right]} - \frac{\text{Sign}\left[-7 + 10\,u^2\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \right.$$
$$\frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(7 - 30\,u^2 + 24\,u^4\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-7 + 8\,u^2\right]} - \frac{\text{Sign}\left[-7 + 56\,u^2 - 112\,u^4 + 64\,u^6\right]}{\text{Sign}\left[5 - 20\,u^2 + 16\,u^4\right]} - $$
$$\left.\frac{\text{Sign}\left[\left(5 - 20\,u^2 + 16\,u^4\right)\left(-7 + 56\,u^2 - 112\,u^4 + 64\,u^6\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-3 + 4\,u^2\right]\,\text{Sign}\left[7 - 30\,u^2 + 24\,u^4\right]}\right),$$

$$\frac{1}{2} \left(4 + \text{Sign}\left[11 - 12\,u^2\right] - \text{Sign}\left[-9 + 11\,u^2\right] - \frac{\text{Sign}\left[-11 + 12\,u^2\right]}{\text{Sign}\left[-21 + 23\,u^2\right]} - \frac{\text{Sign}\left[-21 + 23\,u^2\right]}{\text{Sign}\left[-9 + 11\,u^2\right]}\right),$$

$$\frac{1}{2}\left(-5 + \text{Sign}\left[-1 + 2\,u^2\right] + \text{Sign}\left[-3 + 4\,u^2\right] + \frac{\text{Sign}\left[13 - 36\,u^2 + 24\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-3 + 4\,u^2\right]} + \right.$$
$$\left. \frac{\text{Sign}\left[13 - 44\,u^2 + 32\,u^4\right]}{\text{Sign}\left[-7 + 8\,u^2\right]} + \frac{\text{Sign}\left[\left(-7 + 8\,u^2\right)\,\left(13 - 44\,u^2 + 32\,u^4\right)\right]}{\text{Sign}\left[13 - 36\,u^2 + 24\,u^4\right]}\right),$$

$$\frac{1}{2}\left(-4 + \text{Sign}\left[-3 + 4\,u^2\right] + \frac{\text{Sign}\left[-7 + 8\,u^2\right]}{\text{Sign}\left[-3 + 4\,u^2\right]} + \text{Sign}\left[-15 + 16\,u^2\right] + \frac{\text{Sign}\left[-15 + 16\,u^2\right]}{\text{Sign}\left[-7 + 8\,u^2\right]}\right),$$

$$\frac{1}{2}\left(7 - \text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[-2 + 3\,u^2\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\,\left(-4 + 5\,u^2\right)\right]}{\text{Sign}\left[-2 + 3\,u^2\right]} - \right.$$
$$\frac{\text{Sign}\left[12 - 35\,u^2 + 24\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-4 + 5\,u^2\right]} - \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\,\left(15 - 42\,u^2 + 28\,u^4\right)\right]}{\text{Sign}\left[12 - 35\,u^2 + 24\,u^4\right]} - $$
$$\left. \frac{\text{Sign}\left[17 - 48\,u^2 + 32\,u^4\right]}{\text{Sign}\left[-5 + 8\,u^2\right]} - \frac{\text{Sign}\left[\left(-5 + 8\,u^2\right)\,\left(17 - 48\,u^2 + 32\,u^4\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[15 - 42\,u^2 + 28\,u^4\right]}\right),$$

$$\frac{1}{2}\left(2 + \text{Sign}\left[3 - 2\,u^2\right] - \text{Sign}\left[-1 + 2\,u^2\right] + \frac{\text{Sign}\left[19 - 36\,u^2 + 16\,u^4\right]}{\text{Sign}\left[15 - 28\,u^2 + 16\,u^4\right]} + \right.$$
$$\frac{\text{Sign}\left[-27 + 82\,u^2 - 88\,u^4 + 32\,u^6\right]}{\text{Sign}\left[-3 + 2\,u^2\right]\,\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[\left(19 - 36\,u^2 + 16\,u^4\right)\,\left(15 - 28\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[-5 + 4\,u^2\right]\,\text{Sign}\left[-27 + 112\,u^2 - 144\,u^4 + 64\,u^6\right]} - $$
$$\left. \frac{\text{Sign}\left[\left(-5 + 4\,u^2\right)\,\left(-27 + 112\,u^2 - 144\,u^4 + 64\,u^6\right)\right]}{\text{Sign}\left[-27 + 82\,u^2 - 88\,u^4 + 32\,u^6\right]}\right),$$

$$\frac{1}{2}\left(\text{Sign}\left[-3 + 2\,u^2\right] + \frac{\text{Sign}\left[11 - 20\,u^2 + 8\,u^4\right]}{\text{Sign}\left[-3 + 2\,u^2\right]} - \frac{\text{Sign}\left[21 - 36\,u^2 + 16\,u^4\right]}{\text{Sign}\left[11 - 28\,u^2 + 16\,u^4\right]} - \right.$$
$$\frac{\text{Sign}\left[\left(21 - 36\,u^2 + 16\,u^4\right)\,\left(11 - 28\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[-5 + 4\,u^2\right]\,\text{Sign}\left[11 - 18\,u^2 + 8\,u^4\right]} + $$
$$\left. \frac{\text{Sign}\left[11 - 18\,u^2 + 8\,u^4\right]}{\text{Sign}\left[11 - 24\,u^2 + 16\,u^4\right]} - \frac{\text{Sign}\left[\left(-5 + 4\,u^2\right)\,\left(11 - 24\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[11 - 20\,u^2 + 8\,u^4\right]}\right),$$

$$\frac{1}{2}\left(3 + \text{Sign}\left[13 - 12\,u^2\right] + \text{Sign}\left[-3 + 2\,u^2\right] - \frac{\text{Sign}\left[-5 + 4\,u^2\right]}{\text{Sign}\left[-3 + 2\,u^2\right]} - \frac{\text{Sign}\left[-9 + 8\,u^2\right]}{\text{Sign}\left[-5 + 4\,u^2\right]} - \frac{\text{Sign}\left[-13 + 12\,u^2\right]}{\text{Sign}\left[-9 + 8\,u^2\right]}\right),$$

$$\frac{1}{2}\left(5 - 2\,\text{Sign}\left[-1 + 2\,u^2\right] + \frac{\text{Sign}\left[7 - 12\,u^2 + 4\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\,\left(-7 + 34\,u^2 - 44\,u^4 + 16\,u^6\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[7 - 12\,u^2 + 4\,u^4\right]} - \right.$$
$$\frac{\text{Sign}\left[\left(5 - 20\,u^2 + 16\,u^4\right)\,\left(-7 + 46\,u^2 - 72\,u^4 + 32\,u^6\right)\right]}{\text{Sign}\left[-3 + 4\,u^2\right]\,\text{Sign}\left[-7 + 34\,u^2 - 44\,u^4 + 16\,u^6\right]} - $$
$$\left. \frac{\text{Sign}\left[-17 + 96\,u^2 - 144\,u^4 + 64\,u^6\right]}{\text{Sign}\left[5 - 20\,u^2 + 16\,u^4\right]} - \frac{\text{Sign}\left[-17 + 96\,u^2 - 144\,u^4 + 64\,u^6\right]}{\text{Sign}\left[-7 + 46\,u^2 - 72\,u^4 + 32\,u^6\right]}\right),$$

$$\frac{1}{2}\left(-2 + \text{Sign}\left[5 - 4\,u^2\right] + \frac{\text{Sign}\left[-9 + 8\,u^2\right]}{\text{Sign}\left[-5 + 4\,u^2\right]} + \text{Sign}\left[-17 + 16\,u^2\right] + \frac{\text{Sign}\left[-17 + 16\,u^2\right]}{\text{Sign}\left[-9 + 8\,u^2\right]}\right),$$

$$\frac{1}{2}\left(2 + \text{Sign}\left[3 - 4\,u^2\right] - \text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\left(7 - 16\,u^2 + 8\,u^4\right)\right]}{\text{Sign}\left[6 - 15\,u^2 + 8\,u^4\right]} - \right.$$
$$\frac{\text{Sign}\left[6 - 15\,u^2 + 8\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\text{Sign}\left[-3 + 4\,u^2\right]} + \frac{\text{Sign}\left[19 - 52\,u^2 + 32\,u^4\right]}{\text{Sign}\left[-3 + 4\,u^2\right]\text{Sign}\left[-7 + 8\,u^2\right]} +$$
$$\left.\frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(-7 + 8\,u^2\right)\left(19 - 52\,u^2 + 32\,u^4\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\text{Sign}\left[7 - 16\,u^2 + 8\,u^4\right]}\right),$$

$$\frac{1}{2}\left(-5 + \text{Sign}\left[-1 + 2\,u^2\right] + \frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\left(-5 + 4\,u^2\right)\left(-3 + 4\,u^2\right)\right]}{\text{Sign}\left[5 - 10\,u^2 + 4\,u^4\right]} - \frac{\text{Sign}\left[5 - 10\,u^2 + 4\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} + \right.$$
$$\frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(7 - 24\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[11 - 28\,u^2 + 16\,u^4\right]} + \frac{\text{Sign}\left[21 - 56\,u^2 + 32\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\text{Sign}\left[-5 + 4\,u^2\right]} +$$
$$\left.\frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(7 - 24\,u^2 + 16\,u^4\right)\right]}{\text{Sign}\left[21 - 68\,u^2 + 48\,u^4\right]} + \frac{\text{Sign}\left[\left(11 - 28\,u^2 + 16\,u^4\right)\left(21 - 68\,u^2 + 48\,u^4\right)\right]}{\text{Sign}\left[-3 + 4\,u^2\right]\text{Sign}\left[21 - 56\,u^2 + 32\,u^4\right]}\right),$$

$$\frac{1}{2}\left(5 + \text{Sign}\left[-3 + 2\,u^2\right] - \text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[-2 + 3\,u^2\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[-5 + 4\,u^2\right]}{\text{Sign}\left[-3 + 2\,u^2\right]} - \right.$$
$$\frac{\text{Sign}\left[18 - 47\,u^2 + 28\,u^4\right]}{\text{Sign}\left[-2 + 3\,u^2\right]} - \frac{\text{Sign}\left[23 - 56\,u^2 + 32\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\text{Sign}\left[-5 + 4\,u^2\right]} -$$
$$\left.\frac{\text{Sign}\left[\left(-1 + 2\,u^2\right)\left(23 - 56\,u^2 + 32\,u^4\right)\right]}{\text{Sign}\left[18 - 47\,u^2 + 28\,u^4\right]}\right),$$

$$\frac{1}{2}\left(-2 - \frac{\text{Sign}\left[-3 + 4\,u^2\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[11 - 22\,u^2 + 8\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} + \frac{\text{Sign}\left[-23 + 104\,u^2 - 144\,u^4 + 64\,u^6\right]}{\text{Sign}\left[13 - 28\,u^2 + 16\,u^4\right]} + \right.$$
$$\frac{\text{Sign}\left[\left(13 - 28\,u^2 + 16\,u^4\right)\left(-23 + 104\,u^2 - 144\,u^4 + 64\,u^6\right)\right]}{\text{Sign}\left[-1 + u\right]\text{Sign}\left[1 + u\right]\text{Sign}\left[-23 + 88\,u^2 - 112\,u^4 + 48\,u^6\right]} +$$
$$\frac{\text{Sign}\left[\left(-1 + u\right)\left(1 + u\right)\left(-23 + 88\,u^2 - 112\,u^4 + 48\,u^6\right)\right]}{\text{Sign}\left[22 - 103\,u^2 + 176\,u^4 - 128\,u^6 + 32\,u^8\right]} +$$
$$\left.\frac{\text{Sign}\left[22 - 103\,u^2 + 176\,u^4 - 128\,u^6 + 32\,u^8\right]}{\text{Sign}\left[-3 + 4\,u^2\right]\text{Sign}\left[11 - 22\,u^2 + 8\,u^4\right]}\right),$$

$$\frac{1}{2}\left(-1 + \text{Sign}\left[7 - 4\,u^2\right] - \text{Sign}\left[-1 + 2\,u^2\right] - \frac{\text{Sign}\left[-3 + 4\,u^2\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} + \frac{\text{Sign}\left[25 - 56\,u^2 + 32\,u^4\right]}{\text{Sign}\left[-7 + 8\,u^2\right]} + \right.$$
$$\frac{\text{Sign}\left[\left(-7 + 8\,u^2\right)\left(25 - 56\,u^2 + 32\,u^4\right)\right]}{\text{Sign}\left[-1 + u\right]\text{Sign}\left[1 + u\right]\text{Sign}\left[91 - 200\,u^2 + 112\,u^4\right]} +$$

$$\frac{\text{Sign}\left[(-1+u)\ (1+u)\ \left(91-200\,u^2+112\,u^4\right)\right]}{\text{Sign}\left[-49+151\,u^2-152\,u^4+48\,u^6\right]}+\frac{\text{Sign}\left[-49+151\,u^2-152\,u^4+48\,u^6\right]}{\text{Sign}\left[-7+4\,u^2\right]\text{Sign}\left[-3+4\,u^2\right]}\Biggr),$$

$$\frac{1}{2}\left(-\text{Sign}\left[-1+2\,u^2\right]-\frac{\text{Sign}\left[-2+3\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]}-\frac{\text{Sign}\left[(-2+u^2)\ (-5+8\,u^2)\right]}{\text{Sign}\left[-2+3\,u^2\right]}+\right.$$

$$\frac{\text{Sign}\left[-10+37\,u^2-44\,u^4+16\,u^6\right]}{\text{Sign}\left[-2+u^2\right]\text{Sign}\left[-5+8\,u^2\right]}+$$

$$\frac{\text{Sign}\left[(5-8\,u-4\,u^2+8\,u^3)\ (-5-8\,u+4\,u^2+8\,u^3)\ (5-20\,u^2+16\,u^4)\right]}{\text{Sign}\left[-3+4\,u^2\right]\text{Sign}\left[-10+45\,u^2-68\,u^4+32\,u^6\right]}+$$

$$\frac{\text{Sign}\left[(-3+4\,u^2)\ (-10+45\,u^2-68\,u^4+32\,u^6)\right]}{\text{Sign}\left[-10+37\,u^2-44\,u^4+16\,u^6\right]}+$$

$$\left.\frac{\text{Sign}\left[(5-8\,u-4\,u^2+8\,u^3)\ (-5-8\,u+4\,u^2+8\,u^3)\right]}{\text{Sign}\left[-23+104\,u^2-144\,u^4+64\,u^6\right]}-\frac{\text{Sign}\left[-23+104\,u^2-144\,u^4+64\,u^6\right]}{\text{Sign}\left[5-20\,u^2+16\,u^4\right]}\right),$$

$$\frac{1}{2}\left(-2+\text{Sign}\left[7-4\,u^2\right]+\text{Sign}\left[-3+4\,u^2\right]-\frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]}+\frac{\text{Sign}\left[-3+4\,u^2\right]^3}{\text{Sign}[-1+u]^2\,\text{Sign}[1+u]^2}+\right.$$

$$\left.\frac{\text{Sign}[-1+u]^2\,\text{Sign}[1+u]^2\,\text{Sign}\left[-3+4\,u^2\right]^2}{\text{Sign}\left[-1+2\,u^2\right]\text{Sign}\left[-14+45\,u^2-48\,u^4+16\,u^6\right]}+\frac{\text{Sign}\left[-14+45\,u^2-48\,u^4+16\,u^6\right]}{\text{Sign}\left[-7+4\,u^2\right]\text{Sign}\left[-3+4\,u^2\right]}\right),$$

$$\frac{1}{2}\left(4-\text{Sign}\left[-2+3\,u^2\right]+\frac{\text{Sign}\left[(-2+u^2)\ (-5+6\,u^2)\right]}{\text{Sign}\left[-2+3\,u^2\right]}-\frac{\text{Sign}\left[(-3+4\,u^2)\ (-9+8\,u^2)\right]}{\text{Sign}\left[-7+8\,u^2\right]}-\right.$$

$$\frac{\text{Sign}\left[13-22\,u^2+8\,u^4\right]}{\text{Sign}\left[-2+u^2\right]\text{Sign}\left[-5+6\,u^2\right]}-\frac{\text{Sign}\left[(-3+4\,u^2)\ (-9+8\,u^2)\right]}{\text{Sign}\left[13-30\,u^2+16\,u^4\right]}-$$

$$\left.\frac{\text{Sign}\left[(-7+8\,u^2)\ (13-30\,u^2+16\,u^4)\right]}{\text{Sign}\left[13-22\,u^2+8\,u^4\right]}\right),$$

$$\frac{1}{2}\left(1+\text{Sign}\left[3-4\,u^2\right]+\text{Sign}\left[-3+2\,u^2\right]-\frac{\text{Sign}\left[29-44\,u^2+16\,u^4\right]}{\text{Sign}\left[19-36\,u^2+16\,u^4\right]}-\frac{\text{Sign}\left[21-36\,u^2+16\,u^4\right]}{\text{Sign}\left[-3+2\,u^2\right]}+\right.$$

$$\frac{\text{Sign}\left[-77+220\,u^2-208\,u^4+64\,u^6\right]}{\text{Sign}\left[21-36\,u^2+16\,u^4\right]}+\frac{\text{Sign}\left[(29-44\,u^2+16\,u^4)\ (19-36\,u^2+16\,u^4)\right]}{\text{Sign}\left[399-1436\,u^2+1936\,u^4-1152\,u^6+256\,u^8\right]}-$$

$$\left.\frac{\text{Sign}\left[399-1436\,u^2+1936\,u^4-1152\,u^6+256\,u^8\right]}{\text{Sign}\left[-3+4\,u^2\right]\text{Sign}\left[-77+220\,u^2-208\,u^4+64\,u^6\right]}\right),$$

$$\frac{1}{2}\left(-1+\text{Sign}\left[7-4\,u^2\right]-\text{Sign}\left[-1+2\,u^2\right]-\frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]}+\frac{\text{Sign}\left[29-60\,u^2+32\,u^4\right]}{\text{Sign}\left[13-28\,u^2+16\,u^4\right]}+\right.$$

$$\frac{\text{Sign}\left[(13-28\,u^2+16\,u^4)\ (29-60\,u^2+32\,u^4)\right]}{\text{Sign}[-1+u]\,\text{Sign}[1+u]\,\text{Sign}\left[91-200\,u^2+112\,u^4\right]}+$$

$$\frac{\text{Sign}\left[\,(-1+u)\,(1+u)\,\left(91-200\,u^2+112\,u^4\right)\,\right]}{\text{Sign}\left[-49+151\,u^2-152\,u^4+48\,u^6\right]}+\frac{\text{Sign}\left[-49+151\,u^2-152\,u^4+48\,u^6\right]}{\text{Sign}\left[-7+4\,u^2\right]\,\text{Sign}\left[-3+4\,u^2\right]}\Bigg),$$

$$\frac{1}{2}\left(3+\text{Sign}\left[3-4\,u^2\right]+\frac{\text{Sign}\left[-5+4\,u^2\right]}{\text{Sign}\left[-3+4\,u^2\right]}+\text{Sign}\left[-3+4\,u^2\right]-\frac{\text{Sign}\left[-11+12\,u^2\right]}{\text{Sign}\left[-3+4\,u^2\right]}-\right.$$

$$\frac{\text{Sign}\left[\,(-1+u)\,(1+u)\,\left(-3+4\,u^2\right)\,\left(-33+28\,u^2\right)\,\right]}{\text{Sign}\left[-5+4\,u^2\right]\,\text{Sign}\left[-11+12\,u^2\right]}-\frac{\text{Sign}\left[31-64\,u^2+32\,u^4\right]}{\text{Sign}\left[-9+8\,u^2\right]}-$$

$$\left.\frac{\text{Sign}\left[\,\left(-9+8\,u^2\right)\,\left(31-64\,u^2+32\,u^4\right)\,\right]}{\text{Sign}[-1+u]\,\text{Sign}[1+u]\,\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[-33+28\,u^2\right]}\right),$$

$$\frac{1}{2}\left(8+\text{Sign}\left[11-20\,u^2\right]+\text{Sign}\left[7-12\,u^2\right]-\text{Sign}\left[-1+2\,u^2\right]-\text{Sign}\left[-3+4\,u^2\right]-\frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]}-\right.$$

$$\left.\frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-7+12\,u^2\right]}-\frac{\text{Sign}\left[-11+12\,u^2\right]}{\text{Sign}\left[-7+12\,u^2\right]}-\frac{\text{Sign}\left[\,\left(-11+12\,u^2\right)\,\left(-7+12\,u^2\right)\,\right]}{\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[-11+20\,u^2\right]}\right),$$

$$\frac{1}{2}\left(3-\text{Sign}\left[-1+2\,u^2\right]+\text{Sign}\left[-9+8\,u^2\right]-\right.$$

$$\frac{\text{Sign}\left[\,(-1+u)\,(1+u)\,\left(-3+4\,u^2\right)\,\left(-11+52\,u^2-72\,u^4+32\,u^6\right)\,\right]}{\text{Sign}\left[11-28\,u^2+16\,u^4\right]\,\text{Sign}\left[13-28\,u^2+16\,u^4\right]}-$$

$$\frac{\text{Sign}\left[-35+132\,u^2-160\,u^4+64\,u^6\right]}{\text{Sign}\left[-11+76\,u^2-128\,u^4+64\,u^6\right]}-$$

$$\frac{\text{Sign}\left[\,\left(-35+132\,u^2-160\,u^4+64\,u^6\right)\,\left(-11+76\,u^2-128\,u^4+64\,u^6\right)\,\right]}{\text{Sign}[-1+u]\,\text{Sign}[1+u]\,\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[-11+52\,u^2-72\,u^4+32\,u^6\right]}+$$

$$\left.\frac{\text{Sign}\left[\,\left(11-28\,u^2+16\,u^4\right)\,\left(13-28\,u^2+16\,u^4\right)\,\right]}{\text{Sign}\left[-39+200\,u^2-288\,u^4+128\,u^6\right]}-\frac{\text{Sign}\left[-39+200\,u^2-288\,u^4+128\,u^6\right]}{\text{Sign}\left[-1+2\,u^2\right]\,\text{Sign}\left[-9+8\,u^2\right]}\right),$$

$$\frac{1}{2}\left(-\text{Sign}\left[\,(-1+u)\,(1+u)\,\right]+\text{Sign}\left[3-4\,u^2\right]-\frac{\text{Sign}\left[12-21\,u^2+8\,u^4\right]}{\text{Sign}[-1+u]\,\text{Sign}[1+u]\,\text{Sign}\left[-3+4\,u^2\right]}-\right.$$

$$\frac{\text{Sign}\left[-17+56\,u^2-56\,u^4+16\,u^6\right]}{\text{Sign}\left[12-21\,u^2+8\,u^4\right]}+\frac{\text{Sign}\left[\,\left(-3+4\,u^2\right)\,\left(-17+52\,u^2-52\,u^4+16\,u^6\right)\,\right]}{\text{Sign}\left[-17+56\,u^2-56\,u^4+16\,u^6\right]}+$$

$$\frac{\text{Sign}\left[\,\left(5-12\,u^2+8\,u^4\right)\,\left(-17+64\,u^2-80\,u^4+32\,u^6\right)\,\right]}{\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[-17+52\,u^2-52\,u^4+16\,u^6\right]}+\frac{\text{Sign}\left[-37+132\,u^2-160\,u^4+64\,u^6\right]}{\text{Sign}\left[-17+84\,u^2-128\,u^4+64\,u^6\right]}+$$

$$\left.\frac{\text{Sign}\left[\,\left(-37+132\,u^2-160\,u^4+64\,u^6\right)\,\left(-17+84\,u^2-128\,u^4+64\,u^6\right)\,\right]}{\text{Sign}\left[5-12\,u^2+8\,u^4\right]\,\text{Sign}\left[-17+64\,u^2-80\,u^4+32\,u^6\right]}\right),$$

$$\frac{1}{2}\left(\frac{\text{Sign}\left[\,(-1+u)\,(1+u)\,\right]}{\text{Sign}\left[-3+4\,u^2\right]}+\text{Sign}\left[-3+4\,u^2\right]+\frac{\text{Sign}\left[\,\left(-3+2\,u^2\right)\,\left(-3+4\,u^2\right)\,\right]}{\text{Sign}[-1+u]\,\text{Sign}[1+u]}-\right.$$

$$\frac{\text{Sign}\left[\left(-3+4\,u^2\right)\,\left(-11+8\,u^2\right)\right]}{\text{Sign}\left[-3+2\,u^2\right]} - \frac{\text{Sign}\left[\left(-5+4\,u^2\right)\,\left(-3+4\,u^2\right)\right]}{\text{Sign}\left[7-24\,u^2+16\,u^4\right]} -$$

$$\frac{\text{Sign}\left[\left(-5+4\,u^2\right)\,\left(-3+4\,u^2\right)\,\left(7-24\,u^2+16\,u^4\right)\right]}{\text{Sign}[-1+u]\,\text{Sign}[1+u]\,\text{Sign}\left[7-16\,u^2+8\,u^4\right]} -$$

$$\frac{\text{Sign}\left[(-1+u)\,(1+u)\,\left(-3+4\,u^2\right)\,\left(7-16\,u^2+8\,u^4\right)\right]}{\text{Sign}\left[-77+220\,u^2-208\,u^4+64\,u^6\right]} + \frac{\text{Sign}\left[-77+220\,u^2-208\,u^4+64\,u^6\right]}{\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[-11+8\,u^2\right]}\Bigg),$$

$$\frac{1}{2}\Bigg(-8+2\,\text{Sign}\left[-1+2\,u^2\right] + \frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]} + \text{Sign}\left[1+4\,u^2\right] +$$

$$\frac{\text{Sign}\left[\left(-3+4\,u^2\right)\,\left(1-16\,u^2+16\,u^4\right)\right]}{\text{Sign}\left[1-8\,u^2+8\,u^4\right]} + \frac{\text{Sign}\left[1+u^2-16\,u^4+16\,u^6\right]}{\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[1+4\,u^2\right]} +$$

$$\frac{\text{Sign}\left[\left(1-8\,u^2+8\,u^4\right)\,\left(1-16\,u^2+16\,u^4\right)\right]}{\text{Sign}\left[-1+12\,u^2-34\,u^4+24\,u^6\right]} + \frac{\text{Sign}\left[\left(-3+4\,u^2\right)\,\left(-1+12\,u^2-34\,u^4+24\,u^6\right)\right]}{\text{Sign}\left[-1+2\,u^2\right]\,\text{Sign}\left[1+u^2-16\,u^4+16\,u^6\right]}\Bigg),$$

$$\frac{1}{2}\Bigg(\text{Sign}[(-1+2\,u)\,(1+2\,u)] - \text{Sign}\left[-1+2\,u^2\right] + \frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}[-1+2\,u]\,\text{Sign}[1+2\,u]} - \frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]}\Bigg),$$

$$\frac{1}{2}\Bigg(4+\text{Sign}\left[7-12\,u^2\right] - \text{Sign}\left[-1+2\,u^2\right] + \frac{\text{Sign}\left[-5+4\,u^2\right]}{\text{Sign}[-1+2\,u]\,\text{Sign}[1+2\,u]} - \text{Sign}\left[-3+4\,u^2\right] -$$

$$\frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-1+2\,u^2\right]} - \frac{\text{Sign}\left[-3+4\,u^2\right]}{\text{Sign}\left[-7+12\,u^2\right]} - \frac{\text{Sign}\left[(-1+2\,u)\,(1+2\,u)\,\left(-5+4\,u^2\right)\right]}{\text{Sign}\left[-3+4\,u^2\right]\,\text{Sign}\left[-5+12\,u^2\right]} + \text{Sign}\left[-5+12\,u^2\right]\Bigg)\Bigg\}$$

```
In[•]:= u = 1 / 2;
      KasSig /@ AllKnots[{3, 8}]
      Clear[u]
```

```
Out[•]=
      {2, 0, 4, 2, 0, 2, 0, 4, 2, -4, -2, 4, 2, 0, 0, 4,
       0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -4, 0, 2}
```

In[ ]:= **f = KasSig[Knot[9, 5]]**
Plot[f, {u, −2, 2}]

Out[ ]=

$$\frac{1}{2}\left(-5 + \text{Sign}\left[-9 + 11\,u^2\right] + \frac{\text{Sign}\left[-11 + 12\,u^2\right]}{\text{Sign}\left[-21 + 23\,u^2\right]} + \right.$$
$$\left. \frac{\text{Sign}\left[-21 + 23\,u^2\right]}{\text{Sign}\left[-9 + 11\,u^2\right]} + \text{Sign}\left[-23 + 24\,u^2\right] + \frac{\text{Sign}\left[-23 + 24\,u^2\right]}{\text{Sign}\left[-11 + 12\,u^2\right]}\right)$$

Out[ ]=
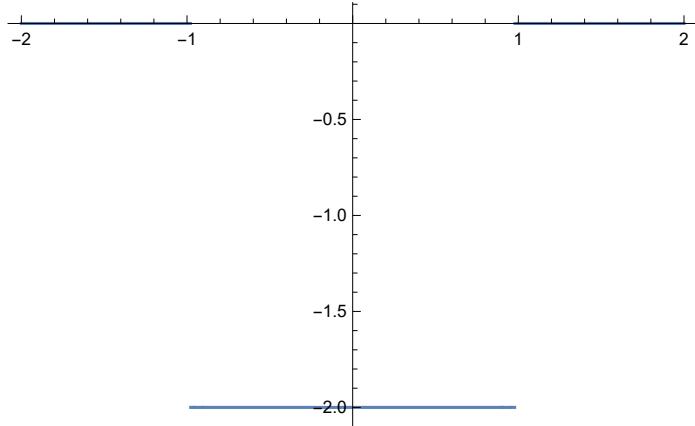
In[ ]:= **f = KasSig[Knot[8, 2]]**
Plot[f, {u, −2, 2}]

Out[ ]=

$$\frac{1}{2}\left(5 - 2\,\text{Sign}\left[-1 + 2\,u^2\right] + \frac{\text{Sign}\left[7 - 12\,u^2 + 4\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]} - \frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(-7 + 34\,u^2 - 44\,u^4 + 16\,u^6\right)\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[7 - 12\,u^2 + 4\,u^4\right]} - \right.$$
$$\frac{\text{Sign}\left[\left(5 - 20\,u^2 + 16\,u^4\right)\left(-7 + 46\,u^2 - 72\,u^4 + 32\,u^6\right)\right]}{\text{Sign}\left[-3 + 4\,u^2\right]\,\text{Sign}\left[-7 + 34\,u^2 - 44\,u^4 + 16\,u^6\right]} - $$
$$\left. \frac{\text{Sign}\left[-17 + 96\,u^2 - 144\,u^4 + 64\,u^6\right]}{\text{Sign}\left[5 - 20\,u^2 + 16\,u^4\right]} - \frac{\text{Sign}\left[-17 + 96\,u^2 - 144\,u^4 + 64\,u^6\right]}{\text{Sign}\left[-7 + 46\,u^2 - 72\,u^4 + 32\,u^6\right]}\right)$$

Out[ ]=
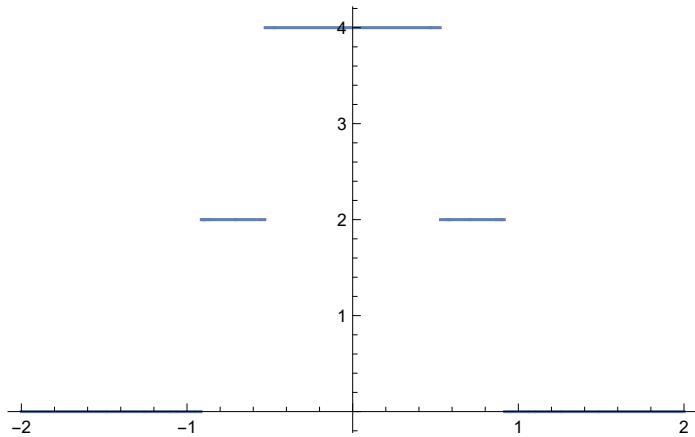
In[ ]:= **f = KasSig[Knot[12, Alternating, 422]]**
Plot[f, {u, -1, 1}]

    KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

    KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[ ]=

$$\frac{1}{2}\left(4 - 2\,\text{Sign}\left[-1 + 2\,u^2\right] + \text{Sign}\left[-3 + 4\,u^2\right] + \text{Sign}\left[-11 + 8\,u^2\right] + \right.$$

$$\frac{\text{Sign}\left[-7 + 8\,u^2\right]}{\text{Sign}\left[-3 + 4\,u^2\right]} - \frac{\text{Sign}\left[-1 + u\right]^2\,\text{Sign}\left[1 + u\right]^2\,\text{Sign}\left[69 - 192\,u^2 + 128\,u^4\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[-44 + 155\,u^2 - 176\,u^4 + 64\,u^6\right]} -$$

$$\frac{\text{Sign}\left[-44 + 155\,u^2 - 176\,u^4 + 64\,u^6\right]}{\text{Sign}\left[-11 + 8\,u^2\right]\,\text{Sign}\left[-7 + 8\,u^2\right]} - \frac{\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(-23 + 152\,u^2 - 256\,u^4 + 128\,u^6\right)\right]}{\text{Sign}\left[-1 + u\right]^2\,\text{Sign}\left[1 + u\right]^2\,\text{Sign}\left[-3 + 8\,u^2\right]} -$$

$$\text{Sign}\left[\left(-3 + 4\,u^2\right)\left(-3 + 8\,u^2\right)\left(-23 + 152\,u^2 - 256\,u^4 + 128\,u^6\right)\right]\,\Big/\,\text{Sign}\Big[$$

$$\left.483 - 3280\,u^2 + 7936\,u^4 - 8192\,u^6 + 3072\,u^8\right] - \frac{\text{Sign}\left[483 - 3280\,u^2 + 7936\,u^4 - 8192\,u^6 + 3072\,u^8\right]}{\text{Sign}\left[-1 + 2\,u^2\right]\,\text{Sign}\left[69 - 192\,u^2 + 128\,u^4\right]}\right)$$

Out[ ]=