In[∘]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
 << KnotTheory`;
]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

pdf

In[∘]:=
```
sign[x_?NumberQ] := Sign[Re[x]]
```

pdf

In[∘]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

pdf

In[∘]:=
```
CF[ℰ_] := Module[{ηs = Union@Cases[ℰ, η_ | η̄_, ∞]},
  Total[CoefficientRules[ℰ, ηs] /. (ps_ → c_) ⧴ Factor[c] Times @@ ηs^ps] ]
```

pdf

In[∘]:=
```
CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_η r, {r, rs}, {η, ηs}]].ηs] ]
```

pdf

In[∘]:=
```
(ℰ_)* := ℰ /. {η̄ → η, η → η̄, ω → ω^-1};
r_Rule⁺ := {r, r*}
```

In[∘]:=
```
{((2 u - ω + 3 ω^-1) η̄1 η2)*, (η1 → ω η2)⁺}
```

Out[∘]=

$$\left\{\left(2\,u-\frac{1}{\omega}+3\,\omega\right)\eta_1\,\bar\eta_2,\ \left\{\eta_1\to\omega\,\eta_2,\ \bar\eta_1\to\frac{\bar\eta_2}{\omega}\right\}\right\}$$

pdf

In[∘]:=
```
RulesOf[η_i_ + rest_.] := (η_i → -rest)⁺;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

In[∘]:=
```
CF[{η1 - η2, η1 - η3}]
```

Out[∘]=

$$\{\eta_1-\eta_3,\ \eta_2-\eta_3\}$$

In[∘]:=
```
RulesOf[η1 + η2 + η3]
```

Out[∘]=

$$\{\eta_1\to-\eta_2-\eta_3,\ \bar\eta_1\to-\bar\eta_2-\bar\eta_3\}$$

pdf

```
In[∘]:= CF[TSI_b_[σ_, pq_]] := TSI_CF[b][σ, CF[pq]]
```

pdf

```
In[∘]:= Kas[P[i_, j_]] := CF@TSI_B[{-i,j}][0, PQ[{}, 0]];
        Bed[P[i_, j_]] := CF@TSI_B[{-i,j}][0, PQ[{}, 0]]
```

The disjoint union in the world of multi-tangles.

pdf

```
In[∘]:= TSI_b1_[σ1_, PQ[rs1_, q1_]] ⋃ TSI_b2_[σ2_, PQ[rs2_, q2_]] ^:=
          CF@TSI_Join[b1,b2][σ1 + σ2, PQ[rs1 ⋃ rs2, q1 + q2]];
```

tex

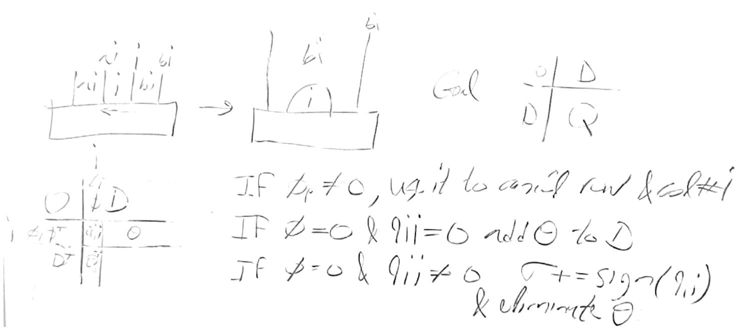
FM for Face Merge:

pdf

```
In[∘]:= FM_i_,j_@TSI_B[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[rs_, q_]] :=
         CF@TSI_B[{ri,li,i,rj,lj,j},bs][σ, PQ[rs ⋃ {η_i - η_j}, q]]
```

```
In[∘]:= Kas[P[1, 2]] ⋃ Kas[P[3, 4]] // FM_-1,4
Out[∘]=
        TSI_B[{-3,4,2,-1}][0, PQ[{η_-1 - η_4}, 0]]
```

pdf



pdf

```
In[∘]:= Cordon_i_@TSI_B[{li___,i_,ri___},bs___][σ_, PQ[rs_, q_]] :=
         Module[{ϕ = ∂_η_i rs, nσ = σ, nrs = rs, nq = q, qii, p},
          Which[
           Or @@ ((# =!= 0) & /@ ϕ), ({p} = FirstPosition[(# === 0) & /@ ϕ, False];
            {nrs, nq} = {rs, q} /. (η_i → -rs⟦p⟧ / ϕ⟦p⟧)^+ /. (η_i → 0)^+),
           (qii = ∂_{η̄_i,η_i} q) =!= 0, (nσ += sign[qii];
            nq = q /. (η_i → -(∂_η̄_i q) / qii)^+ /. (η_i → 0)^+),
           qii === 0, AppendTo[nrs, ∂_η̄_i q]; nq = q /. (η_i → 0)^+];
          CF@TSI_B[Rest@{ri,li},bs][nσ, PQ[nrs, nq] /. (η_First@{ri,li} → η_Last@{ri,li})^+] ]
```

tex

c for contract:

pdf

```
In[∘]:= c_{i_,j_}@TSI_{B[{li___,i_,ri___},{lj___,j_,rj___},bs___]}[ℰ__] := Module[{bi = Last@{ri, li}},
    TSI_{B[{li,i,ri},{lj,j,rj},bs]}[ℰ] // FM_{j,bi} // Cordon_j ];
```

pdf

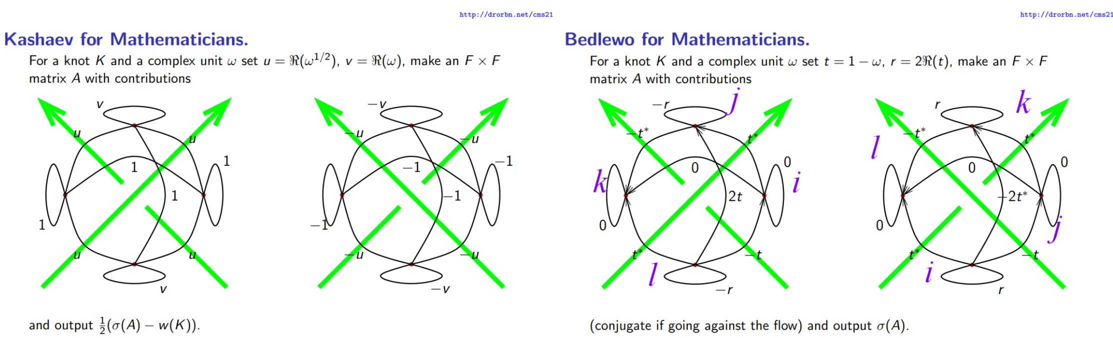
```
In[∘]:= c_{i_,j_}@TSI_{B[{l___,i_,j_,r___},bs___]}[ℰ__] := Cordon_i@TSI_{B[{l,i,j,r},bs]}[ℰ];
c_{i_,j_}@TSI_{B[{j_,m___,i_},bs___]}[ℰ__] := Cordon_i@TSI_{B[{j,m,i},bs]}[ℰ];
c_{i_,j_}@TSI_{B[{l___,j_,i_,r___},bs___]}[ℰ__] := Cordon_j@TSI_{B[{l,j,i,r},bs]}[ℰ];
c_{i_,j_}@TSI_{B[{i_,m___,j_},bs___]}[ℰ__] := Cordon_j@TSI_{B[{i,m,j},bs]}[ℰ];
```

tex

mc for magnetic contract:

pdf

```
In[∘]:= mc@TSI_{B[{li___,i_,ri___},{lj___,j_,rj___},bs___]}[ℰ__] /; j == -i :=
  mc@c_{i,j}@TSI_{B[{li,i,ri},{lj,j,rj},bs]}[ℰ];
mc@TSI_{B[{l___,i_,j_,r___},bs___]}[ℰ__] /; j == -i := mc@Cordon_i@TSI_{B[{l,i,j,r},bs]}[ℰ];
mc@TSI_{B[{j_,m___,i_},bs___]}[ℰ__] /; j == -i := mc@Cordon_i@TSI_{B[{j,m,i},bs]}[ℰ];
mc@TSI_{b_B}[ℰ__] /; (Union @@ b ⋂ (-Union @@ b)) === {} := TSI_b[ℰ]
```



pdf

```
In[∘]:= Kas[x : X[i_, j_, k_, l_]] := Module[{v = 2 u^2 - 1, m, ηs},
  If[PositiveQ@x,
    ηs = {η_{-i}, η_j, η_k, η_{-l}}   ηs = {η_{-i}, η_{-j}, η_k, η_l}
        m = ( 1 u 1 u
              u v u 1
              1 u 1 u
              u 1 u v ) ,     m = -( v u 1 u
                                     u 1 u 1
                                     1 u v u
                                     u 1 u 1 )   ];
  CF@TSI_{B[ηs/.η_{m_}:>m]}[0, PQ[{}, ηs^*.m.ηs]]]
```

pdf

In[∘]:=
```
Bed[x : X[i_, j_, k_, l_]] := Module[{t = 1 - ω, r = 2 - ω - ω^-1, m, ηs},
  If[PositiveQ@x,
   ηs = {η-i, ηj, ηk, η-l}                 ηs = {η-i, η-j, ηk, ηl}
   m = ( 0   t*   0   -t*  )          m = ( r     -t  -2 t*  t*  )
       ( t   -r  -t*  2 t* )   ,          ( -t*    0    t*    0  )
       ( 0   -t   0    t   )              ( -2 t   t    r   -t*  )
       ( -t  2 t  t*  -r   )              ( t      0   -t    0   )  ];
  CF@TSI_B[ηs/.η_m_ :→ m] [0, PQ[{}, ηs*.m.ηs]]]
```

pdf

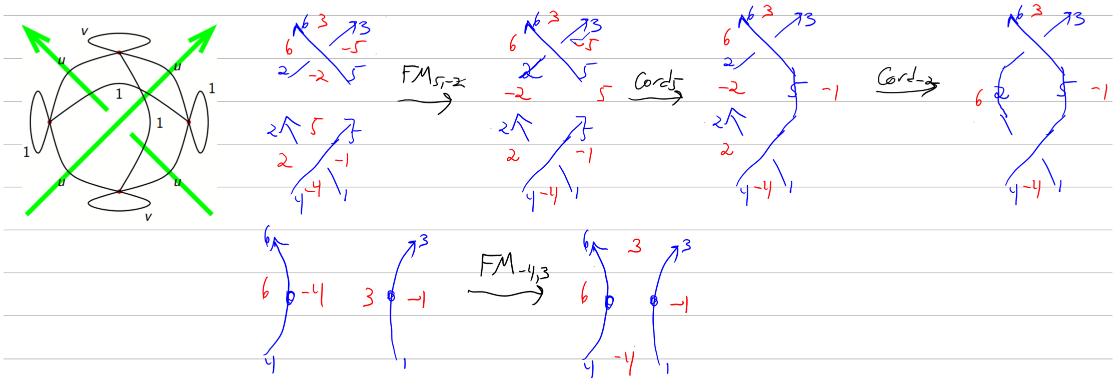
In[∘]:=
```
KasSig[K_] := Module[{pd = PD[K]},
   mc[Union @@ (Kas /@ pd)][[1]] - Sum[If[PositiveQ@x, 1, -1], {x, List @@ pd}] ] / 2
```

pdf

In[∘]:=
```
BedSig[K_] := mc[Union @@ (Bed /@ PD[K])][[1]]
```

pdf

Reidemeister 2



In[∘]:=
```
Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]]
```

Out[∘]=

$$\mathrm{TSI}_{B[\{-5,3,6,-2\},\{-4,-1,5,2\}]}\Big[0,\ \mathrm{PQ}\Big[\{\},\ -\eta_{-5}\bar\eta_{-5}-u\,\eta_{-2}\bar\eta_{-5}-u\,\eta_3\bar\eta_{-5}-\eta_6\bar\eta_{-5}+\left(-1+2u^2\right)\eta_{-4}\bar\eta_{-4}+u\,\eta_{-1}\bar\eta_{-4}+u\,\eta_2\bar\eta_{-4}+\eta_5\bar\eta_{-4}-u\,\eta_{-5}\bar\eta_{-2}+\left(1-2u^2\right)\eta_{-2}\bar\eta_{-2}-\eta_3\bar\eta_{-2}-u\,\eta_6\bar\eta_{-2}+u\,\eta_{-4}\bar\eta_{-1}+\eta_{-1}\bar\eta_{-1}+\eta_2\bar\eta_{-1}+u\,\eta_5\bar\eta_{-1}+u\,\eta_{-4}\bar\eta_2+\eta_{-1}\bar\eta_2+\eta_2\bar\eta_2+u\,\eta_5\bar\eta_2-u\,\eta_{-5}\bar\eta_3-\eta_{-2}\bar\eta_3+\left(1-2u^2\right)\eta_3\bar\eta_3-u\,\eta_6\bar\eta_3+\eta_{-4}\bar\eta_5+u\,\eta_{-1}\bar\eta_5+u\,\eta_2\bar\eta_5+\left(-1+2u^2\right)\eta_5\bar\eta_5-\eta_{-5}\bar\eta_6-u\,\eta_{-2}\bar\eta_6-u\,\eta_3\bar\eta_6-\eta_6\bar\eta_6\Big]\Big]$$

In[∘]:=
```
Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM_{-2,5}
```
Out[∘]=

$$\mathrm{TSI}_{B[\{-5,3,6,-2,2,-4,-1,5\}]}\Big[0,\ \mathrm{PQ}\Big[\{\eta_{-2}-\eta_5\},\ -\eta_{-5}\overline{\eta}_{-5}-u\,\eta_3\overline{\eta}_{-5}-u\,\eta_5\overline{\eta}_{-5}-\eta_6\overline{\eta}_{-5}+\left(-1+2u^2\right)\eta_{-4}\overline{\eta}_{-4}+u\,\eta_{-1}\overline{\eta}_{-4}+u\,\eta_2\overline{\eta}_{-4}+\eta_5\overline{\eta}_{-4}+u\,\eta_{-4}\overline{\eta}_{-1}+\eta_{-1}\overline{\eta}_{-1}+\eta_2\overline{\eta}_{-1}+u\,\eta_5\overline{\eta}_{-1}+u\,\eta_{-4}\overline{\eta}_2+\eta_{-1}\overline{\eta}_2+\eta_2\overline{\eta}_2+u\,\eta_5\overline{\eta}_2-u\,\eta_{-5}\overline{\eta}_3+\left(1-2u^2\right)\eta_3\overline{\eta}_3-\eta_5\overline{\eta}_3-u\,\eta_6\overline{\eta}_3-u\,\eta_{-5}\overline{\eta}_5+\eta_{-4}\overline{\eta}_5+u\,\eta_{-1}\overline{\eta}_5+u\,\eta_2\overline{\eta}_5-\eta_3\overline{\eta}_5-u\,\eta_6\overline{\eta}_5-\eta_{-5}\overline{\eta}_6-u\,\eta_3\overline{\eta}_6-u\,\eta_5\overline{\eta}_6-\eta_6\overline{\eta}_6\Big]\Big]$$

In[∘]:=
```
Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM_{-2,5} // Cordon_5
```
Out[∘]=

$$\mathrm{TSI}_{B[\{-4,-1,3,6,-2,2\}]}\Big[0,\ \mathrm{PQ}\Big[\{\},\ \left(-1+2u^2\right)\eta_{-4}\overline{\eta}_{-4}+\eta_{-2}\overline{\eta}_{-4}+u\,\eta_{-1}\overline{\eta}_{-4}+u\,\eta_2\overline{\eta}_{-4}+\eta_{-4}\overline{\eta}_{-2}+u\,\eta_2\overline{\eta}_{-2}-\eta_3\overline{\eta}_{-2}-u\,\eta_6\overline{\eta}_{-2}+u\,\eta_{-4}\overline{\eta}_{-1}+\eta_2\overline{\eta}_{-1}-u\,\eta_3\overline{\eta}_{-1}-\eta_6\overline{\eta}_{-1}+u\,\eta_{-4}\overline{\eta}_2+u\,\eta_{-2}\overline{\eta}_2+\eta_{-1}\overline{\eta}_2+\eta_2\overline{\eta}_2-\eta_{-2}\overline{\eta}_3-u\,\eta_{-1}\overline{\eta}_3+\left(1-2u^2\right)\eta_3\overline{\eta}_3-u\,\eta_6\overline{\eta}_3-u\,\eta_{-2}\overline{\eta}_6-\eta_{-1}\overline{\eta}_6-u\,\eta_3\overline{\eta}_6-\eta_6\overline{\eta}_6\Big]\Big]$$

In[∘]:=
```
Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // FM_{-2,5} // Cordon_5 // Cordon_{-2}
```
Out[∘]=

$$\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]]$$

pdf

In[∘]:=
```
{Kas[P[1, 3]] ⋃ Kas[P[4, 6]] // FM_{-4,3}, Kas[X[1, 5, 2, 4]] ⋃ Kas[X[2, 5, 3, 6]] // mc}
```
Out[∘]=
pdf

$$\{\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]],\ \mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]]\}$$

pdf

In[∘]:=
```
{Bed[P[1, 3]] ⋃ Bed[P[4, 6]] // FM_{-4,3}, Bed[X[1, 5, 2, 4]] ⋃ Bed[X[2, 5, 3, 6]] // mc}
```
Out[∘]=
pdf

$$\{\mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]],\ \mathrm{TSI}_{B[\{-4,-1,3,6\}]}[0,\ \mathrm{PQ}[\{\eta_{-4}-\eta_3\},\ 0]]\}$$

pdf

Reidemeister 3

In[∘]:=
```
{u = 7 / 29};
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // c2,-2 // c5,-5 // c8,-8
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // c2,-2 // c5,-5 // c8,-8
Clear[u]
```

Out[∘]=

$$\mathrm{TSI}_{B[\{-7,3,6,9,-1,-4\}]}\Big[-1,\ \mathrm{PQ}\Big[\{\},\ \frac{1486}{645}\eta_{-7}\bar\eta_{-7}+\frac{16289\,\eta_{-4}\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_{-1}\bar\eta_{-7}+\frac{16289\,\eta_{3}\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_{6}\bar\eta_{-7}+\frac{406}{645}\eta_{9}\bar\eta_{-7}+\frac{16289\,\eta_{-7}\bar\eta_{-4}}{18705}+\frac{228046\,\eta_{-4}\bar\eta_{-4}}{542445}+\frac{16289\,\eta_{-1}\bar\eta_{-4}}{18705}+\frac{841}{645}\eta_{3}\bar\eta_{-4}+\frac{406}{645}\eta_{6}\bar\eta_{-4}+\frac{841}{645}\eta_{9}\bar\eta_{-4}+\frac{841}{645}\eta_{-7}\bar\eta_{-1}+\frac{16289\,\eta_{-4}\bar\eta_{-1}}{18705}+\frac{228046\,\eta_{-1}\bar\eta_{-1}}{542445}+\frac{406}{645}\eta_{3}\bar\eta_{-1}+\frac{841}{645}\eta_{6}\bar\eta_{-1}+\frac{16289\,\eta_{9}\bar\eta_{-1}}{18705}+\frac{16289\,\eta_{-7}\bar\eta_{3}}{18705}+\frac{841}{645}\eta_{-4}\bar\eta_{3}+\frac{406}{645}\eta_{-1}\bar\eta_{3}+\frac{228046\,\eta_{3}\bar\eta_{3}}{542445}+\frac{16289\,\eta_{6}\bar\eta_{3}}{18705}+\frac{841}{645}\eta_{9}\bar\eta_{3}+\frac{841}{645}\eta_{-7}\bar\eta_{6}+\frac{406}{645}\eta_{-4}\bar\eta_{6}+\frac{841}{645}\eta_{-1}\bar\eta_{6}+\frac{16289\,\eta_{3}\bar\eta_{6}}{18705}+\frac{228046\,\eta_{6}\bar\eta_{6}}{542445}+\frac{16289\,\eta_{9}\bar\eta_{6}}{18705}+\frac{406}{645}\eta_{-7}\bar\eta_{9}+\frac{841}{645}\eta_{-4}\bar\eta_{9}+\frac{16289\,\eta_{-1}\bar\eta_{9}}{18705}+\frac{841}{645}\eta_{3}\bar\eta_{9}+\frac{16289\,\eta_{6}\bar\eta_{9}}{18705}+\frac{1486}{645}\eta_{9}\bar\eta_{9}\Big]\Big]$$

Out[∘]=

$$\mathrm{TSI}_{B[\{-7,3,6,9,-1,-4\}]}\Big[-1,\ \mathrm{PQ}\Big[\{\},\ \frac{1486}{645}\eta_{-7}\bar\eta_{-7}+\frac{16289\,\eta_{-4}\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_{-1}\bar\eta_{-7}+\frac{16289\,\eta_{3}\bar\eta_{-7}}{18705}+\frac{841}{645}\eta_{6}\bar\eta_{-7}+\frac{406}{645}\eta_{9}\bar\eta_{-7}+\frac{16289\,\eta_{-7}\bar\eta_{-4}}{18705}+\frac{228046\,\eta_{-4}\bar\eta_{-4}}{542445}+\frac{16289\,\eta_{-1}\bar\eta_{-4}}{18705}+\frac{841}{645}\eta_{3}\bar\eta_{-4}+\frac{406}{645}\eta_{6}\bar\eta_{-4}+\frac{841}{645}\eta_{9}\bar\eta_{-4}+\frac{841}{645}\eta_{-7}\bar\eta_{-1}+\frac{16289\,\eta_{-4}\bar\eta_{-1}}{18705}+\frac{228046\,\eta_{-1}\bar\eta_{-1}}{542445}+\frac{406}{645}\eta_{3}\bar\eta_{-1}+\frac{841}{645}\eta_{6}\bar\eta_{-1}+\frac{16289\,\eta_{9}\bar\eta_{-1}}{18705}+\frac{16289\,\eta_{-7}\bar\eta_{3}}{18705}+\frac{841}{645}\eta_{-4}\bar\eta_{3}+\frac{406}{645}\eta_{-1}\bar\eta_{3}+\frac{228046\,\eta_{3}\bar\eta_{3}}{542445}+\frac{16289\,\eta_{6}\bar\eta_{3}}{18705}+\frac{841}{645}\eta_{9}\bar\eta_{3}+\frac{841}{645}\eta_{-7}\bar\eta_{6}+\frac{406}{645}\eta_{-4}\bar\eta_{6}+\frac{841}{645}\eta_{-1}\bar\eta_{6}+\frac{16289\,\eta_{3}\bar\eta_{6}}{18705}+\frac{228046\,\eta_{6}\bar\eta_{6}}{542445}+\frac{16289\,\eta_{9}\bar\eta_{6}}{18705}+\frac{406}{645}\eta_{-7}\bar\eta_{9}+\frac{841}{645}\eta_{-4}\bar\eta_{9}+\frac{16289\,\eta_{-1}\bar\eta_{9}}{18705}+\frac{841}{645}\eta_{3}\bar\eta_{9}+\frac{16289\,\eta_{6}\bar\eta_{9}}{18705}+\frac{1486}{645}\eta_{9}\bar\eta_{9}\Big]\Big]$$

pdf

In[*]:=
```
lhs = Kas[X[4, 2, 5, 1]] ⋃ Kas[X[7, 3, 8, 2]] ⋃ Kas[X[8, 6, 9, 5]] // mc;
rhs = Kas[X[7, 5, 8, 4]] ⋃ Kas[X[8, 2, 9, 1]] ⋃ Kas[X[5, 3, 6, 2]] // mc;
{lhs⟦1⟧, rhs⟦1⟧}
Simplify[lhs⟦2, 2⟧ == rhs⟦2, 2⟧]
```

Out[*]=
pdf

{sign[(-1 + 2 u) (1 + 2 u)], sign[(-1 + 2 u) (1 + 2 u)]}

Out[*]=
pdf

True

In[*]:=
```
lhs⟦2, 2⟧
```

Out[*]=

$$\frac{2\left(-1+2u^2\right)\eta_{-7}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-4}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}-\frac{\eta_{-1}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{3}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{6}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{9}\overline{\eta}_{-7}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-7}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}+\frac{2u^2\left(-3+4u^2\right)\eta_{-4}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}+$$
$$\frac{u\left(-3+4u^2\right)\eta_{-1}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-\frac{\eta_{3}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{6}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-\frac{\eta_{9}\overline{\eta}_{-4}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{-7}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-4}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{2u^2\left(-3+4u^2\right)\eta_{-1}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{3}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{6}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{9}\overline{\eta}_{-1}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{-7}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-\frac{\eta_{-4}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-$$
$$\frac{2u\,\eta_{-1}\overline{\eta}_{3}}{(-1+2u)(1+2u)}+\frac{2u^2\left(-3+4u^2\right)\eta_{3}\overline{\eta}_{3}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{6}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-\frac{\eta_{9}\overline{\eta}_{3}}{(-1+2u)(1+2u)}-$$
$$\frac{\eta_{-7}\overline{\eta}_{6}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{-4}\overline{\eta}_{6}}{(-1+2u)(1+2u)}-\frac{\eta_{-1}\overline{\eta}_{6}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{3}\overline{\eta}_{6}}{(-1+2u)(1+2u)}+$$
$$\frac{2u^2\left(-3+4u^2\right)\eta_{6}\overline{\eta}_{6}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{9}\overline{\eta}_{6}}{(-1+2u)(1+2u)}-\frac{2u\,\eta_{-7}\overline{\eta}_{9}}{(-1+2u)(1+2u)}-\frac{\eta_{-4}\overline{\eta}_{9}}{(-1+2u)(1+2u)}+$$
$$\frac{u\left(-3+4u^2\right)\eta_{-1}\overline{\eta}_{9}}{(-1+2u)(1+2u)}-\frac{\eta_{3}\overline{\eta}_{9}}{(-1+2u)(1+2u)}+\frac{u\left(-3+4u^2\right)\eta_{6}\overline{\eta}_{9}}{(-1+2u)(1+2u)}+\frac{2\left(-1+2u^2\right)\eta_{9}\overline{\eta}_{9}}{(-1+2u)(1+2u)}$$

pdf

In[∘]:=
```
lhs = Bed[X[4, 2, 5, 1]] ⋃ Bed[X[7, 3, 8, 2]] ⋃ Bed[X[8, 6, 9, 5]] // mc;
rhs = Bed[X[7, 5, 8, 4]] ⋃ Bed[X[8, 2, 9, 1]] ⋃ Bed[X[5, 3, 6, 2]] // mc;
{lhs⟦1⟧, rhs⟦1⟧}
lhs⟦2, 2⟧ == rhs⟦2, 2⟧
```

Out[∘]=
pdf

$$\left\{\text{sign}\left[\frac{2\ (-1+\omega)^2}{\omega}\right],\ \text{sign}\left[\frac{2\ (-1+\omega)^2}{\omega}\right]\right\}$$

Out[∘]=
pdf

True

In[∘]:=
```
lhs⟦2, 2⟧
```

Out[∘]=

$$-\frac{(-1+\omega)\ \eta_{-4}\ \overline{\eta}_{-7}}{\omega}+\frac{(-1+\omega)\ \eta_3\ \overline{\eta}_{-7}}{\omega}+(-1+\omega)\ \eta_{-7}\ \overline{\eta}_{-4}+\frac{\left(1+\omega^2\right)\ \eta_{-4}\ \overline{\eta}_{-4}}{\omega}-\frac{(1+\omega)\ \eta_{-1}\ \overline{\eta}_{-4}}{\omega}-$$
$$2\ \omega\ \eta_3\ \overline{\eta}_{-4}+2\ \eta_6\ \overline{\eta}_{-4}+(-1-\omega)\ \eta_{-4}\ \overline{\eta}_{-1}+\frac{\left(1+\omega^2\right)\ \eta_{-1}\ \overline{\eta}_{-1}}{\omega}+2\ \omega\ \eta_3\ \overline{\eta}_{-1}-2\ \omega\ \eta_6\ \overline{\eta}_{-1}+$$
$$\frac{(-1+\omega)\ \eta_9\ \overline{\eta}_{-1}}{\omega}+(1-\omega)\ \eta_{-7}\ \overline{\eta}_3-\frac{2\ \eta_{-4}\ \overline{\eta}_3}{\omega}+\frac{2\ \eta_{-1}\ \overline{\eta}_3}{\omega}+\frac{\left(1+\omega^2\right)\ \eta_3\ \overline{\eta}_3}{\omega}-\frac{(1+\omega)\ \eta_6\ \overline{\eta}_3}{\omega}+2\ \eta_{-4}\ \overline{\eta}_6-$$
$$\frac{2\ \eta_{-1}\ \overline{\eta}_6}{\omega}+(-1-\omega)\ \eta_3\ \overline{\eta}_6+\frac{\left(1+\omega^2\right)\ \eta_6\ \overline{\eta}_6}{\omega}-\frac{(-1+\omega)\ \eta_9\ \overline{\eta}_6}{\omega}+(1-\omega)\ \eta_{-1}\ \overline{\eta}_9+(-1+\omega)\ \eta_6\ \overline{\eta}_9$$

Kashaev for Knots

In[∘]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
(*u=0;*)
mc[Union @@ (Kas /@ PD@Knot[3, 1])]
Clear[u]
```

Out[∘]=

$$\text{TSI}_{\text{B}[]}\left[-1+\text{sign}\left[\frac{1}{2}\ \left(3-4\ u^2\right)\right]+\text{sign}\left[-\frac{2}{3}\ \left(-3+4\ u^2\right)\right],\ \text{PQ}[\{\},\ 0]\right]$$

In[∘]:=
```
(*u=0;*)
KasSig /@ AllKnots[{3, 7}]
Clear[u]
```

Out[∘]=

$$\left\{\frac{1}{2}\ \left(2+\text{sign}\left[\frac{1}{2}\ \left(3-4\ u^2\right)\right]+\text{sign}\left[-\frac{2}{3}\ \left(-3+4\ u^2\right)\right]\right),\right.$$

$$\frac{1}{2}\left(-1+\text{sign}\left[3-2u^2\right]+\text{sign}\left[\frac{1}{2}\left(-5+4u^2\right)\right]+\text{sign}\left[\frac{-5+4u^2}{-3+2u^2}\right]\right),$$

$$\frac{1}{2}\left(5+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{2\left(-1+2u^2\right)\left(-5+6u^2\right)}{-5+8u^2}\right]+\right.$$
$$\left.\text{sign}\left[-\frac{-5+8u^2}{-1+2u^2}\right]+\text{sign}\left[-\frac{5-20u^2+16u^4}{2\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(5-20u^2+16u^4\right)}{2\left(-1+2u^2\right)\left(-5+6u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(3+\text{sign}\left[\frac{1}{4}\left(7-8u^2\right)\right]+\text{sign}\left[-\frac{4}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[-\frac{-7+8u^2}{-3+4u^2}\right]\right),$$

$$\frac{1}{2}\left(1+\text{sign}\left[\frac{1}{2}\left(9-8u^2\right)\right]+\text{sign}\left[-\frac{-5+4u^2}{-3+2u^2}\right]+\right.$$
$$\left.\text{sign}\left[\frac{2\left(-3+2u^2\right)}{-3+4u^2}\right]+\text{sign}\left[-3+4u^2\right]+\text{sign}\left[-\frac{-9+8u^2}{2\left(-5+4u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(2+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{\left(-1+2u^2\right)\left(-5+4u^2\right)\left(-3+4u^2\right)}{2\left(5-10u^2+4u^4\right)}\right]+\right.$$
$$\text{sign}\left[\frac{2\left(5-10u^2+4u^4\right)}{5-12u^2+8u^4}\right]+\text{sign}\left[\frac{5-12u^2+8u^4}{-1+2u^2}\right]+$$
$$\left.\text{sign}\left[-\frac{11-28u^2+16u^4}{\left(-1+2u^2\right)\left(-5+4u^2\right)}\right]+\text{sign}\left[-\frac{11-28u^2+16u^4}{2\left(-3+4u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(\text{sign}\left[4\left(-1+u\right)\left(1+u\right)\right]+\text{sign}\left[\frac{-3+2u^2}{\left(-1+u\right)\left(1+u\right)}\right]+\text{sign}\left[\frac{\left(-7+4u^2\right)\left(-3+4u^2\right)}{4\left(-3+2u^2\right)}\right]+\right.$$
$$\text{sign}\left[-\frac{13-28u^2+16u^4}{8\left(-1+u\right)\left(1+u\right)}\right]+\text{sign}\left[-\frac{2\left(-1+u\right)\left(1+u\right)\left(13-28u^2+16u^4\right)}{-14+45u^2-48u^4+16u^6}\right]+$$
$$\left.\text{sign}\left[-\frac{4\left(-14+45u^2-48u^4+16u^6\right)}{\left(-7+4u^2\right)\left(-3+4u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(7+2\,\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{2\left(-1+2u^2\right)\left(-7+8u^2\right)}{-7+10u^2}\right]+\text{sign}\left[-\frac{-7+10u^2}{-1+2u^2}\right]+\right.$$
$$\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(7-30u^2+24u^4\right)}{2\left(-1+2u^2\right)\left(-7+8u^2\right)}\right]+\text{sign}\left[-\frac{-7+56u^2-112u^4+64u^6}{2\left(5-20u^2+16u^4\right)}\right]+$$
$$\left.\text{sign}\left[-\frac{\left(5-20u^2+16u^4\right)\left(-7+56u^2-112u^4+64u^6\right)}{2\left(-1+2u^2\right)\left(-3+4u^2\right)\left(7-30u^2+24u^4\right)}\right]\right),\ \frac{1}{2}\left(4+\text{sign}\left[\frac{1}{6}\left(11-12u^2\right)\right]+\right.$$
$$\left.\text{sign}\left[-\frac{2}{3}\left(-9+11u^2\right)\right]+\text{sign}\left[-\frac{3\left(-11+12u^2\right)}{2\left(-21+23u^2\right)}\right]+\text{sign}\left[-\frac{-21+23u^2}{-9+11u^2}\right]\right),$$

$$\frac{1}{2}\left(-5+\text{sign}\left[2\left(-1+2u^2\right)\right]+\text{sign}\left[\frac{4}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{13-36u^2+24u^4}{\left(-1+2u^2\right)\left(-3+4u^2\right)}\right]+\text{sign}\left[\frac{13-44u^2+32u^4}{2\left(-7+8u^2\right)}\right]+\text{sign}\left[\frac{\left(-7+8u^2\right)\left(13-44u^2+32u^4\right)}{4\left(13-36u^2+24u^4\right)}\right]\right),$$

$$\frac{1}{2}\left(-4+\text{sign}\left[\frac{4}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{-7+8u^2}{-3+4u^2}\right]+\text{sign}\left[\frac{1}{4}\left(-15+16u^2\right)\right]+\text{sign}\left[\frac{-15+16u^2}{2\left(-7+8u^2\right)}\right]\right),$$

$$\frac{1}{2}\left(7+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[-\frac{2\left(-2+3u^2\right)}{-1+2u^2}\right]+\text{sign}\left[-\frac{2\left(-1+2u^2\right)\left(-4+5u^2\right)}{-2+3u^2}\right]+\text{sign}\left[-\frac{12-35u^2+24u^4}{\left(-1+2u^2\right)\left(-4+5u^2\right)}\right]+\text{sign}\left[-\frac{\left(-1+2u^2\right)\left(15-42u^2+28u^4\right)}{12-35u^2+24u^4}\right]+\text{sign}\left[-\frac{17-48u^2+32u^4}{2\left(-5+8u^2\right)}\right]+\text{sign}\left[-\frac{\left(-5+8u^2\right)\left(17-48u^2+32u^4\right)}{4\left(-1+2u^2\right)\left(15-42u^2+28u^4\right)}\right]\right),$$

$$\frac{1}{2}\left(2+\text{sign}\left[3-2u^2\right]+\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[\frac{19-36u^2+16u^4}{15-28u^2+16u^4}\right]+\text{sign}\left[\frac{-27+82u^2-88u^4+32u^6}{2\left(-3+2u^2\right)\left(-1+2u^2\right)}\right]+\text{sign}\left[-\frac{\left(19-36u^2+16u^4\right)\left(15-28u^2+16u^4\right)}{\left(-5+4u^2\right)\left(-27+112u^2-144u^4+64u^6\right)}\right]+\text{sign}\left[-\frac{\left(-5+4u^2\right)\left(-27+112u^2-144u^4+64u^6\right)}{2\left(-27+82u^2-88u^4+32u^6\right)}\right]\right),$$

$$\frac{1}{2}\left(\text{sign}\left[-3+2u^2\right]+\text{sign}\left[\frac{11-20u^2+8u^4}{-3+2u^2}\right]+\text{sign}\left[-\frac{21-36u^2+16u^4}{11-28u^2+16u^4}\right]+\text{sign}\left[-\frac{\left(21-36u^2+16u^4\right)\left(11-28u^2+16u^4\right)}{2\left(-5+4u^2\right)\left(11-18u^2+8u^4\right)}\right]+\text{sign}\left[\frac{2\left(11-18u^2+8u^4\right)}{11-24u^2+16u^4}\right]+\text{sign}\left[-\frac{\left(-5+4u^2\right)\left(11-24u^2+16u^4\right)}{2\left(11-20u^2+8u^4\right)}\right]\right)\Big\}$$

In[#]:=
```
u = 1 / 2;
KasSig /@ AllKnots[{3, 8}]
Clear[u]
```

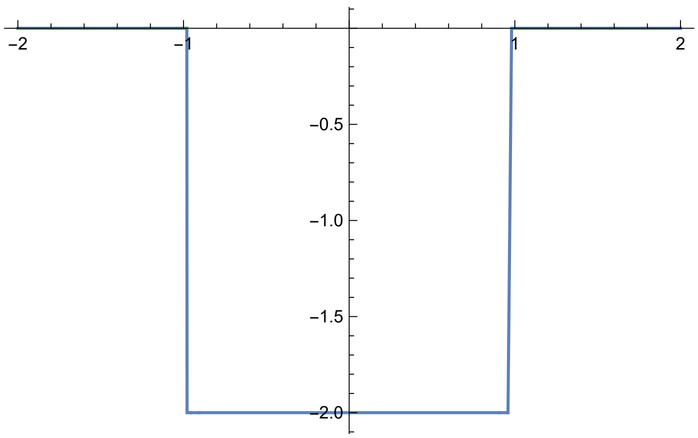
Out[#]=
```
{2, 0, 4, 2, 0, 2, 0, 4, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -4, 0, 2}
```

In[•]:=
```
f = KasSig[Knot[9, 5]]
Plot[f, {u, -2, 2}]
```

Out[•]=

$$\frac{1}{2}\left(-5+\text{sign}\left[\frac{2}{3}\left(-9+11\,u^2\right)\right]+\text{sign}\left[\frac{3\left(-11+12\,u^2\right)}{2\left(-21+23\,u^2\right)}\right]+\text{sign}\left[\frac{-21+23\,u^2}{-9+11\,u^2}\right]+\text{sign}\left[\frac{1}{6}\left(-23+24\,u^2\right)\right]+\text{sign}\left[\frac{-23+24\,u^2}{2\left(-11+12\,u^2\right)}\right]\right)$$

Out[•]=

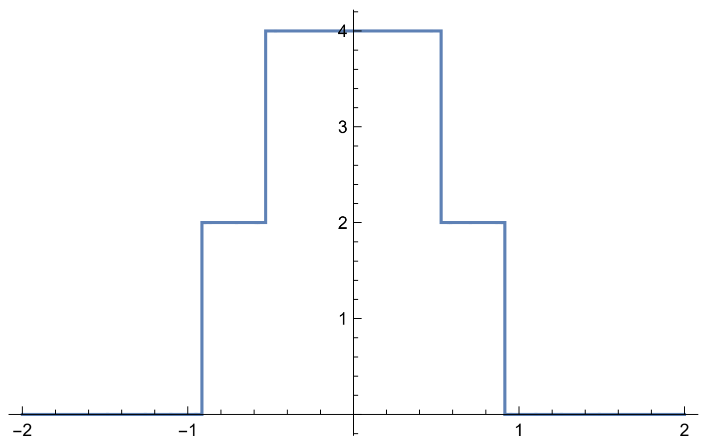


In[∘]:=
```
f = KasSig[Knot[8, 2]]
Plot[f, {u, -2, 2}, PlotPoints → 1000]
```

Out[∘]=

$$\frac{1}{2}\left(5+2\,\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[\frac{7-12u^2+4u^4}{-1+2u^2}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-7+34u^2-44u^4+16u^6\right)}{2\left(-1+2u^2\right)\left(7-12u^2+4u^4\right)}\right]+\text{sign}\left[-\frac{\left(5-20u^2+16u^4\right)\left(-7+46u^2-72u^4+32u^6\right)}{2\left(-3+4u^2\right)\left(-7+34u^2-44u^4+16u^6\right)}\right]+\text{sign}\left[-\frac{-17+96u^2-144u^4+64u^6}{2\left(5-20u^2+16u^4\right)}\right]+\text{sign}\left[-\frac{-17+96u^2-144u^4+64u^6}{-7+46u^2-72u^4+32u^6}\right]\right)$$

Out[∘]=



In[∘]:=
```
f = KasSig[Knot[12, Alternating, 422]]
Plot[f, {u, -1, 1}, PlotPoints → 1000]
```

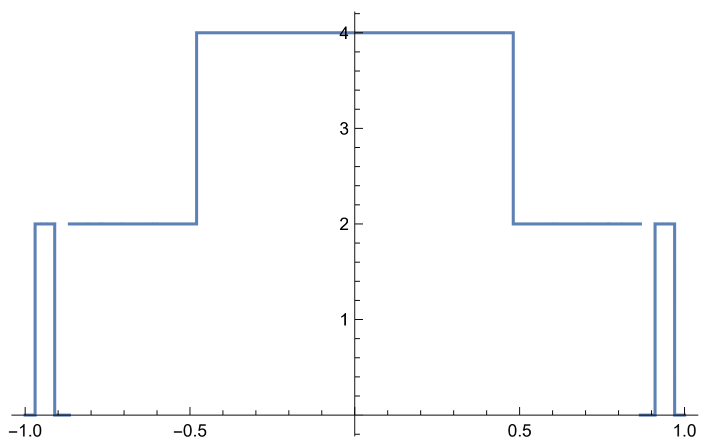
KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[∘]=

$$\frac{1}{2}\Bigg(4+2\,\text{sign}\left[-2\left(-1+2u^2\right)\right]+\text{sign}\left[\frac{2}{3}\left(-3+4u^2\right)\right]+\text{sign}\left[\frac{1}{2}\left(-11+8u^2\right)\right]+$$
$$\text{sign}\left[\frac{-7+8u^2}{2\left(-3+4u^2\right)}\right]+\text{sign}\left[-\frac{4(-1+u)^2(1+u)^2\left(69-192u^2+128u^4\right)}{\left(-1+2u^2\right)\left(-44+155u^2-176u^4+64u^6\right)}\right]+$$
$$\text{sign}\left[-\frac{4\left(-44+155u^2-176u^4+64u^6\right)}{\left(-11+8u^2\right)\left(-7+8u^2\right)}\right]+\text{sign}\left[-\frac{\left(-3+4u^2\right)\left(-23+152u^2-256u^4+128u^6\right)}{32(-1+u)^2(1+u)^2\left(-3+8u^2\right)}\right]+$$
$$\text{sign}\left[-\frac{2\left(-3+4u^2\right)\left(-3+8u^2\right)\left(-23+152u^2-256u^4+128u^6\right)}{483-3280u^2+7936u^4-8192u^6+3072u^8}\right]+$$
$$\text{sign}\left[-\frac{483-3280u^2+7936u^4-8192u^6+3072u^8}{4\left(-1+2u^2\right)\left(69-192u^2+128u^4\right)}\right]\Bigg)$$

Out[∘]=



Bedlewo for Knots

In[∘]:=
```
-KnotSignature /@ AllKnots[{3, 8}]
```

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4, 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
mc[Union @@ (Bed /@ PD@Knot[3, 1])]
```

Out[∘]=

$$\text{TSI}_{B[]}\left[\text{sign}\left[-\frac{2(-1+\omega)^2}{\omega}\right]+\text{sign}\left[-\frac{2\left(1-\omega+\omega^2\right)}{\omega}\right],\ \text{PQ}[\{\},0]\right]$$

In[∘]:=
```
ω = -1;
BedSig /@ AllKnots[{3, 8}]
Clear[ω]
```

Out[∘]=

{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[∘]:=
```
BedSig /@ AllKnots[{3, 7}]
```

Out[∘]=

$$\Big\{\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2)}{\omega}\Big],\ \mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{2\,(1-3\,\omega+\omega^2)}{\omega}\Big],$$

$$2\,\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2)}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2-\omega^3+\omega^4)}{\omega\,(1-\omega+\omega^2)}\Big],$$

$$\mathrm{sign}\Big[-\frac{4\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2-3\,\omega+2\,\omega^2}{\omega}\Big],\ \mathrm{sign}\Big[\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(-2+\omega)\,(-1+2\,\omega)}{\omega}\Big],$$

$$\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2)}{\omega}\Big]+$$

$$\mathrm{sign}\Big[-\frac{2\,(1-3\,\omega+3\,\omega^2-3\,\omega^3+\omega^4)}{\omega\,(1-\omega+\omega^2)}\Big],\ \mathrm{sign}\Big[\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^4}{\omega\,(1-\omega+\omega^2)}\Big]+$$

$$\mathrm{sign}\Big[\frac{2\,(1-\omega+\omega^2)}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-3\,\omega+5\,\omega^2-3\,\omega^3+\omega^4)}{(-1+\omega)^2\,\omega}\Big],$$

$$3\,\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2)}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2-\omega^3+\omega^4)}{\omega\,(1-\omega+\omega^2)}\Big]+$$

$$\mathrm{sign}\Big[-\frac{2\,(1-\omega+\omega^2-\omega^3+\omega^4-\omega^5+\omega^6)}{\omega\,(1-\omega+\omega^2-\omega^3+\omega^4)}\Big],\ \mathrm{sign}\Big[-\frac{6\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(3-5\,\omega+3\,\omega^2)}{3\,\omega}\Big],$$

$$\mathrm{sign}\Big[\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{4\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{2-3\,\omega+2\,\omega^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{2\,(2-3\,\omega+3\,\omega^2-3\,\omega^3+2\,\omega^4)}{\omega\,(2-3\,\omega+2\,\omega^2)}\Big],$$

$$\mathrm{sign}\Big[\frac{4\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{4-7\,\omega+4\,\omega^2}{\omega}\Big],\ \mathrm{sign}\Big[-\frac{4\,(-1+\omega)^2}{\omega}\Big]+$$

$$\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2-\omega+2\,\omega^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(2-4\,\omega+5\,\omega^2-4\,\omega^3+2\,\omega^4)}{\omega\,(2-\omega+2\,\omega^2)}\Big],$$

$$2\,\mathrm{sign}\Big[-\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[\frac{2\,(1-3\,\omega+\omega^2)}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-5\,\omega+7\,\omega^2-5\,\omega^3+\omega^4)}{\omega\,(1-3\,\omega+\omega^2)}\Big],$$

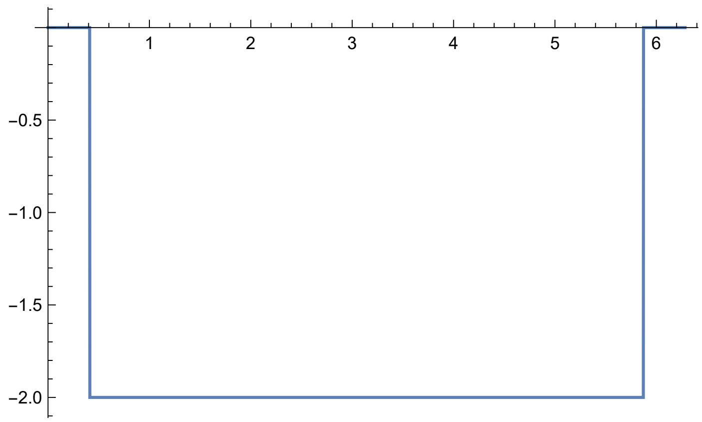
$$2\,\mathrm{sign}\Big[\frac{2\,(-1+\omega)^2}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-3\,\omega+\omega^2)}{\omega}\Big]+\mathrm{sign}\Big[-\frac{2\,(1-5\,\omega+9\,\omega^2-5\,\omega^3+\omega^4)}{\omega\,(1-3\,\omega+\omega^2)}\Big]\Big\}$$

In[∘]:=
```
f = BedSig[Knot[9, 5]] /. ω → ⅇ^(ⅈ t)
Plot[f, {t, 0, 2 π}]
```

Out[∘]=

$$\text{sign}\left[6\, e^{-i t}\left(-1+e^{i t}\right)^2\right]+\text{sign}\left[\frac{2}{3}\, e^{-i t}\left(6-11\, e^{i t}+6\, e^{2 i t}\right)\right]$$

Out[∘]=

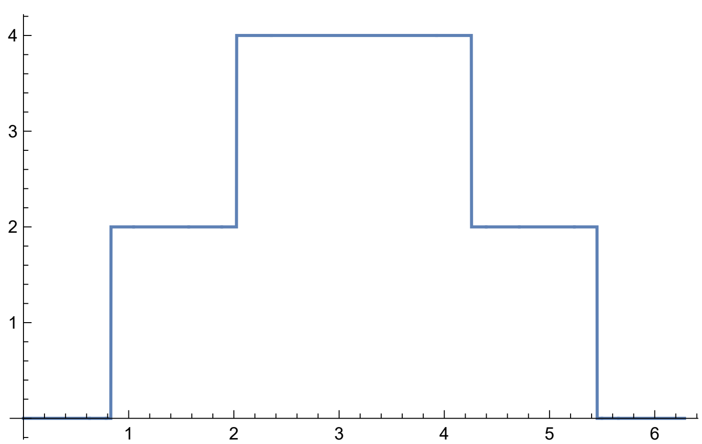


In[∘]:=
```
f = BedSig[Knot[8, 2]] /. ω → ⅇ^(ⅈ t)
Plot[f, {t, 0, 2 π}]
```

Out[∘]=

$$2\,\text{sign}\left[-2\, e^{-i t}\left(-1+e^{i t}\right)^2\right]+\text{sign}\left[2\, e^{-i t}\left(-1+e^{i t}\right)^2\right]+\text{sign}\left[-2\, e^{-i t}\left(1-e^{i t}+e^{2 i t}\right)\right]+\text{sign}\left[-\frac{2\, e^{-i t}\left(1-e^{i t}+e^{2 i t}-e^{3 i t}+e^{4 i t}\right)}{1-e^{i t}+e^{2 i t}}\right]+\text{sign}\left[-\frac{2\, e^{-i t}\left(1-3\, e^{i t}+3\, e^{2 i t}-3\, e^{3 i t}+3\, e^{4 i t}-3\, e^{5 i t}+e^{6 i t}\right)}{1-e^{i t}+e^{2 i t}-e^{3 i t}+e^{4 i t}}\right]$$

Out[∘]=



In[∘]:=
```
f = BedSig[Knot[12, Alternating, 422]] /. ω → e^(i t)
Plot[f, {t, 0, 2 π}, PlotPoints → 1000]
```

Out[∘]=

$$2\,\text{sign}\left[-2\,e^{-i t}\left(-1+e^{i t}\right)^2\right]+\text{sign}\left[2\,e^{-i t}\left(-1+e^{i t}\right)^2\right]+\text{sign}\left[-\frac{4\,e^{-i t}\left(-1+e^{i t}\right)^4}{2-3\,e^{i t}+2\,e^{2 i t}}\right]+\text{sign}\left[-e^{-i t}\left(2-3\,e^{i t}+2\,e^{2 i t}\right)\right]+\text{sign}\left[2\,e^{-i t}\left(2-3\,e^{i t}+2\,e^{2 i t}\right)\right]+\text{sign}\left[-\frac{2\,e^{-i t}\left(-1+e^{i t}\right)^2\left(2+e^{i t}+2\,e^{2 i t}\right)}{2-3\,e^{i t}+2\,e^{2 i t}}\right]+\text{sign}\left[-\frac{2\,e^{-i t}\left(1-e^{i t}+e^{2 i t}\right)\left(2-4\,e^{i t}+4\,e^{2 i t}-3\,e^{3 i t}+4\,e^{4 i t}-4\,e^{5 i t}+2\,e^{6 i t}\right)}{\left(-1+e^{i t}\right)^4\left(2+e^{i t}+2\,e^{2 i t}\right)}\right]$$

Out[∘]=