Expressing signatures using Heavyside theta’s at their jump points.

In[∘]:=
```
HeavisideTheta[u - 7]
```
Out[∘]=
```
HeavisideTheta[-7 + u]
```
In[∘]:=
```
UnitStep[u - 7]
```
Out[∘]=
```
UnitStep[-7 + u]
```
In[∘]:=
```
θ[x_] /; NumberQ[x] := HeavisideTheta[x];
sign[ℰ_] := Module[{p, rs, d, k},
  p = Expand[Numerator[ℰ] Denominator[ℰ]];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Return[Sign[p /. u → 0]]];
  rs = Union@(u /. rs);
  Sign[Coefficient[p, u, Exponent[p, u]]] (-1)^Exponent[p,u] + Sum[
    k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
    If[EvenQ[k], 0, 2 Sign[d]] θ[u - r],
    {r, rs}]
 ]
```
In[∘]:=
```
sign[1 + u^2]
```
Out[∘]=
```
1
```

In[ ]:=

$$f = \mathrm{Expand}\Big[\frac{1}{2}\Big(4 + \mathrm{sign}\big[-2\,(-1+2u^2)\big] + \mathrm{sign}\Big[\frac{2}{3}\,(-3+4u^2)\Big] + \mathrm{sign}\Big[\frac{-7+8u^2}{2\,(-3+4u^2)}\Big] +$$

$$\mathrm{sign}\Big[-\frac{2\,(8-23u^2+16u^4)}{-7+8u^2}\Big] + \mathrm{sign}\Big[-\frac{(-3+4u^2)^2\,(11-28u^2+16u^4)}{-44+155u^2-176u^4+64u^6}\Big] +$$

$$\mathrm{sign}\Big[\frac{-44+155u^2-176u^4+64u^6}{8-23u^2+16u^4}\Big] + \mathrm{sign}\Big[-\frac{-11+76u^2-128u^4+64u^6}{11-28u^2+16u^4}\Big] +$$

$$\mathrm{sign}\Big[-\frac{(-29+160u^2-256u^4+128u^6)\,(11-170u^2+544u^4-640u^6+256u^8)}{4\,(-1+2u^2)\,(-3+4u^2)^2\,(-11+76u^2-128u^4+64u^6)}\Big] +$$

$$\mathrm{sign}\Big[-\frac{(-3+4u^2)\,(-23+152u^2-256u^4+128u^6)}{11-228u^2+864u^4-1152u^6+512u^8}\Big] +$$

$$\mathrm{sign}\Big[-\frac{(-3+4u^2)\,(-23+152u^2-256u^4+128u^6)\,(11-228u^2+864u^4-1152u^6+512u^8)}{2\,(-29+160u^2-256u^4+128u^6)\,(11-170u^2+544u^4-640u^6+256u^8)}\Big]\Big)$$

$$\Big]$$

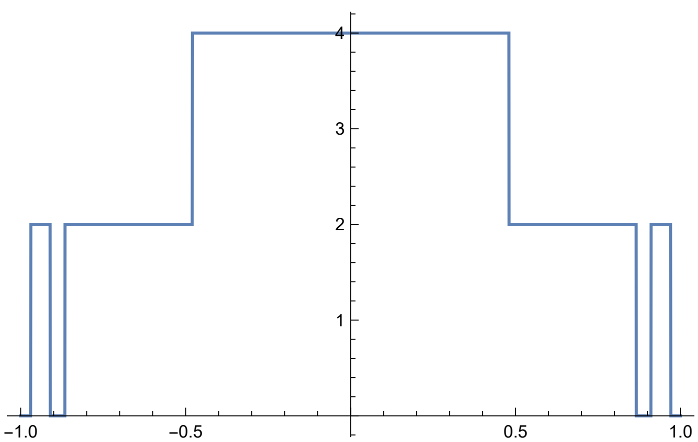
```
Plot[f, {u, -1, 1}, Exclusions → None]
```

Out[ ]=

$$-2\,\theta\Big[-\frac{\sqrt{3}}{2}+u\Big] + 2\,\theta\Big[\frac{\sqrt{3}}{2}+u\Big] + 2\,\theta\big[u - \boxed{-0.970\ldots}\big] - 2\,\theta\big[u - \boxed{-0.910\ldots}\big] +$$

$$2\,\theta\big[u - \boxed{-0.480\ldots}\big] - 2\,\theta\big[u - \boxed{0.480\ldots}\big] + 2\,\theta\big[u - \boxed{0.910\ldots}\big] - 2\,\theta\big[u - \boxed{0.970\ldots}\big]$$

Out[ ]=



In[ ]:=

$$\mathcal{E} = -\frac{(-29+160u^2-256u^4+128u^6)\,(11-170u^2+544u^4-640u^6+256u^8)}{4\,(-1+2u^2)\,(-3+4u^2)^2\,(-11+76u^2-128u^4+64u^6)}$$

Out[ ]=

$$-\frac{(-29+160u^2-256u^4+128u^6)\,(11-170u^2+544u^4-640u^6+256u^8)}{4\,(-1+2u^2)\,(-3+4u^2)^2\,(-11+76u^2-128u^4+64u^6)}$$

In[∘]:=
```
p = Expand[Numerator[ℰ] Denominator[ℰ]]
```
Out[∘]=

$$126\,324 - 4\,111\,536\,u^2 + 55\,241\,840\,u^4 - 418\,854\,464\,u^6 + 2\,037\,750\,016\,u^8 - 6\,802\,006\,016\,u^{10} + 16\,169\,902\,080\,u^{12} - 27\,900\,198\,912\,u^{14} + 35\,116\,810\,240\,u^{16} - 31\,970\,033\,664\,u^{18} + 20\,526\,923\,776\,u^{20} - 8\,824\,815\,616\,u^{22} + 2\,281\,701\,376\,u^{24} - 268\,435\,456\,u^{26}$$

In[∘]:=
```
roots = Union@(u /. Solve[p == 0, u, Reals])
```
Out[∘]=

$$\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \boxed{-0.462\ldots}, \boxed{0.462\ldots}, \boxed{-0.561\ldots}, \boxed{0.561\ldots}, \boxed{-0.787\ldots}, \boxed{-0.293\ldots}, \boxed{0.293\ldots}, \boxed{0.787\ldots}\right\}$$

In[∘]:=
```
Table[RootReduce[p /. u → r] > 0, {r, roots}]
```
Out[∘]=
```
{False, False, False, False, False, False, False, False, False, False, False, False}
```

In[∘]:=
```
RootReduce@Table[p /. u → r, {r, roots}]
```
Out[∘]=
```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

In[∘]:=
```
Sum[
 k = 1; While[(d = RootReduce[D[p, {u, k}] /. u → r]) == 0, ++k];
 If[EvenQ[k], 0, Sign[d]] θ[r],
 {r, roots}]
```
Out[∘]=

$$-\theta\left[-\frac{1}{\sqrt{2}}\right] + \theta\left[\frac{1}{\sqrt{2}}\right] - \theta\left[\boxed{-0.462\ldots}\right] + \theta\left[\boxed{0.462\ldots}\right] + \theta\left[\boxed{-0.561\ldots}\right] - \theta\left[\boxed{0.561\ldots}\right] + \theta\left[\boxed{-0.787\ldots}\right] + \theta\left[\boxed{-0.293\ldots}\right] - \theta\left[\boxed{0.293\ldots}\right] - \theta\left[\boxed{0.787\ldots}\right]$$

In[∘]:=
```
KasSig[Knot[10, 1]]
```
Out[∘]=
```
0
```

In[∘]:=
```
Table[K → Expand@KasSig[K], {K, AllKnots[{3, 8}]}] // Column
```
Out[∘]=

Knot[3, 1] → $-2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right] + 2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]$

Knot[4, 1] → 0

Knot[5, 1] → $2\,\theta\left[u - \boxed{-0.951\ldots}\right] + 2\,\theta\left[u - \boxed{-0.588\ldots}\right] - 2\,\theta\left[u - \boxed{0.588\ldots}\right] - 2\,\theta\left[u - \boxed{0.951\ldots}\right]$

Knot[5, 2] → $-2\,\theta\left[-\frac{\sqrt{\frac{7}{2}}}{2}+u\right] + 2\,\theta\left[\frac{\sqrt{\frac{7}{2}}}{2}+u\right]$

Knot[6, 1] → 0

Knot[6, 2] → $2\,\theta\left[u - \boxed{-0.772\ldots}\right] - 2\,\theta\left[u - \boxed{0.772\ldots}\right]$

Knot[6, 3] → 0

$\text{Knot}[7,1] \to 2\,\theta[u - (-0.975\ldots)] + 2\,\theta[u - (-0.782\ldots)] + 2\,\theta[u - (-0.434\ldots)] - 2\,\theta[u - 0.434\ldots] - 2\,\theta[u - 0.782\ldots] - 2\,\theta[u - 0.975\ldots]$

$\text{Knot}[7,2] \to -2\,\theta\left[-\frac{\sqrt{\frac{11}{3}}}{2} + u\right] + 2\,\theta\left[\frac{\sqrt{\frac{11}{3}}}{2} + u\right]$

$\text{Knot}[7,3] \to -2\,\theta[u - (-0.972\ldots)] - 2\,\theta[u - (-0.656\ldots)] + 2\,\theta[u - 0.656\ldots] + 2\,\theta[u - 0.972\ldots]$

$\text{Knot}[7,4] \to 2\,\theta\left[-\frac{\sqrt{15}}{4} + u\right] - 2\,\theta\left[\frac{\sqrt{15}}{4} + u\right]$

$\text{Knot}[7,5] \to 2\,\theta[u - (-0.963\ldots)] + 2\,\theta[u - (-0.757\ldots)] - 2\,\theta[u - 0.757\ldots] - 2\,\theta[u - 0.963\ldots]$

$\text{Knot}[7,6] \to 2\,\theta[u - (-0.920\ldots)] - 2\,\theta[u - 0.920\ldots]$

$\text{Knot}[7,7] \to 0$

$\text{Knot}[8,1] \to 0$

$\text{Knot}[8,2] \to 2\,\theta[u - (-0.915\ldots)] + 2\,\theta[u - (-0.529\ldots)] - 2\,\theta[u - 0.529\ldots] - 2\,\theta[u - 0.915\ldots]$

$\text{Knot}[8,3] \to 0$

$\text{Knot}[8,4] \to 2\,\theta[u - (-0.745\ldots)] - 2\,\theta[u - 0.745\ldots]$

$\text{Knot}[8,5] \to 2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\theta[u - (-0.630\ldots)] + 2\,\theta[u - 0.630\ldots]$

$\text{Knot}[8,6] \to 2\,\theta[u - (-0.811\ldots)] - 2\,\theta[u - 0.811\ldots]$

$\text{Knot}[8,7] \to -2\,\theta[u - (-0.647\ldots)] + 2\,\theta[u - 0.647\ldots]$

$\text{Knot}[8,8] \to 0$

$\text{Knot}[8,9] \to 0$

$\text{Knot}[8,10] \to 2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$

$\text{Knot}[8,11] \to -2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$

$\text{Knot}[8,12] \to 0$

$\text{Knot}[8,13] \to 0$

$\text{Knot}[8,14] \to 2\,\theta[u - (-0.907\ldots)] - 2\,\theta[u - 0.907\ldots]$

$\text{Knot}[8,15] \to -2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[-\frac{\sqrt{\frac{11}{3}}}{2} + u\right] + 2\,\theta\left[\frac{\sqrt{\frac{11}{3}}}{2} + u\right]$

$\text{Knot}[8,16] \to 2\,\theta[u - (-0.749\ldots)] - 2\,\theta[u - 0.749\ldots]$

$\text{Knot}[8,17] \to 0$

$\text{Knot}[8,18] \to 0$

$\text{Knot}[8,19] \to 2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\theta[u - (-0.966\ldots)] - 2\,\theta[u - (-0.259\ldots)] + 2\,\theta[u - 0.259\ldots] + 2\,\theta[u - 0.966\ldots]$

$\text{Knot}[8,20] \to 0$

$\text{Knot}[8,21] \to -2\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\,\theta\left[\frac{\sqrt{3}}{2} + u\right]$