Operators are Objects

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The (Semi-)Category $\mathcal{DO}$. Hence we care about the monoidal (semi-)category $\mathcal{DO}$ whose objects are finite sets B and whose morphisms are $\mathrm{mor}_{\mathcal{DO}}(B, B') := \mathrm{Hom}_{\mathbb{Q}}(\mathcal{S}(B) \to \mathcal{S}(B')) = \mathcal{S}(B^*, B')$ (by convention, $x^* = \xi$, $y^* = \eta$, etc.).

The Composition Law. If

$$\mathcal{S}(B_0) \xrightarrow[\tilde f \in \mathbb{Q}[\![\zeta_{0i}, z_{1j}]\!]]{f} \mathcal{S}(B_1) \xrightarrow[\tilde g \in \mathbb{Q}[\![\zeta_{1j}, z_{2k}]\!]]{g} \mathcal{S}(B_2)$$

then $\widetilde{(g \circ f)} = \left(\tilde g|_{\zeta_{1j} \to \partial_{z_{1j}}} \tilde f \right)_{z_{1j}=0} = \left(\tilde f|_{z_{1j} \to \partial_{\zeta_{1j}}} \tilde g \right)_{\zeta_{1j}=0}$.

Note. For $f \in \mathcal{S}(z)$ and $g \in \mathcal{S}(\zeta)$, $\langle f, g \rangle = f(\partial_\zeta) g|_{\zeta=0} = g(\partial_z) f|_{z=0}$.

Proposition. If $F : \mathcal{S}(B) \to \mathcal{S}(B')$ is linear and "continuous", then $\tilde F = F\left(\exp\left(\sum_{z_i \in B} \zeta_i z_i\right)\right)$.

reinsert

$= f/\!/g =$

operators are objects

$$F \in \mathrm{Hom}_{\mathbb{Q}}(S(B) \longrightarrow S(B'))$$
$$= S(B)^* \otimes S(B')$$
$$\overset{*}{=} S(B^*) \otimes S(B')$$
$$= S(B^* \sqcup B')$$
$$\tilde F \in \mathbb{Q}[\zeta_i, z_i]$$

$*$ $B^* := \{z_i^* = \zeta_i : z_i \in B\}$

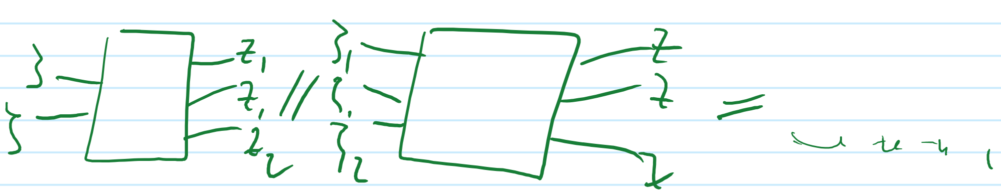
$\langle z_i^m, \zeta_i^n \rangle = \delta_{mn} n!$

$\langle \prod z_i^{m_i}, \zeta_i^{n_i} \rangle = \prod \delta_{m_i n_i} n_i!$

& in general,

(get w/ $\zeta \to \zeta_i$, $z \to z_i$)

w/ picture added:



Example The first example of the next box:

$$e^{\zeta z}, \quad e^{\sum z_i \zeta_i}$$

Proposition:

in 1-variable case

$$\tilde{F} = F\left(e^{\sum z_i \zeta_i}\right) = \sum_{n=0}^{\infty} F(z^n) \frac{1}{n!} \zeta^n$$