Pensieve header: Implementing ρ_1.

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Preliminaries

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This is Rho1.nb of http://drorbn.net/gro22/ap.

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Groningen-220620"];
```

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In[∘]:=
```
Once[<< KnotTheory`; << Rot.m];
```

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```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

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```
Loading Rot.m from http://drorbn.net/gro22/ap to compute rotation numbers.
```

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The Program

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In[∘]:=
```
R1[s_, i_, j_] := s (g[j,i] (g[j+1,j] + g[j,j+1] - g[i,j]) - g[i,i] (g[j,j+1] - 1) - 1/2);
ρ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s, T^s - 1}, {0, -1}})];
   Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
   G = Inverse[A];
   ρ1 = Sum[R1 @@ Cs[[k]], {k, 1, n}] - Sum[φ[[k]] (g[k,k] - 1/2), {k, 1, 2 n}];
   Factor@{Δ, Δ^2 ρ1 /. g[α_, β_] :> G[[α, β]]}];
```

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The First Few Knots

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In[ ]:= `Table[K → ρ[K], {K, AllKnots[{3, 6}]}]`

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KnotTheory: Loading precomputed data in PD4Knots`.

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Out[ ]= $\{\text{Knot}[3,1] \to \{\frac{1-T+T^2}{T}, \frac{(-1+T)^2 (1+T^2)}{T^2}\}, \text{Knot}[4,1] \to \{-\frac{1-3T+T^2}{T}, 0\},$

$\text{Knot}[5,1] \to \{\frac{1-T+T^2-T^3+T^4}{T^2}, \frac{(-1+T)^2 (1+T^2)(2+T^2+2T^4)}{T^4}\},$

$\text{Knot}[5,2] \to \{\frac{2-3T+2T^2}{T}, \frac{(-1+T)^2 (5-4T+5T^2)}{T^2}\},$

$\text{Knot}[6,1] \to \{-\frac{(-2+T)(-1+2T)}{T}, \frac{(-1+T)^2 (1-4T+T^2)}{T^2}\},$

$\text{Knot}[6,2] \to \{-\frac{1-3T+3T^2-3T^3+T^4}{T^2}, \frac{(-1+T)^2 (1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4}\},$

$\text{Knot}[6,3] \to \{\frac{1-3T+5T^2-3T^3+T^4}{T^2}, 0\}\}$

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\def\nbpdfText#1{\vskip -3mm\[\includegraphics[width=0.4\linewidth]{#1}\quad p=1-T^s \]}

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tex

\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}

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\needspace{2in}

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Fast!

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\[\resizebox{\linewidth}{!}{\import{../Waco-2203/}{GST48-Marked.pdf_t}} \]

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In[ ]:= $\text{Timing}@\rho[\text{EPD}[X_{14,1}, \overline{X}_{2,29}, X_{3,40}, X_{43,4}, \overline{X}_{26,5}, X_{6,95}, X_{96,7}, X_{13,8}, \overline{X}_{9,28}, X_{10,41}, X_{42,11}, \overline{X}_{27,12}, X_{30,15}, \overline{X}_{16,61}, \overline{X}_{17,72}, \overline{X}_{18,83}, X_{19,34}, \overline{X}_{89,20}, \overline{X}_{21,92}, \overline{X}_{79,22}, \overline{X}_{68,23}, \overline{X}_{57,24}, \overline{X}_{25,56}, X_{62,31}, X_{73,32}, X_{84,33}, \overline{X}_{50,35}, X_{36,81}, X_{37,70}, X_{38,59}, \overline{X}_{39,54}, X_{44,55}, X_{58,45}, X_{69,46}, X_{80,47}, X_{48,91}, X_{90,49}, X_{51,82}, X_{52,71}, X_{53,60}, \overline{X}_{63,74}, \overline{X}_{64,85}, \overline{X}_{76,65}, \overline{X}_{87,66}, \overline{X}_{67,94}, \overline{X}_{75,86}, \overline{X}_{88,77}, \overline{X}_{78,93}]]$

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Out[ ]= $$\left\{86.2031, \left\{-\frac{(-1+2T-T^2-T^3+2T^4-T^5+T^8)(-1+T^3-2T^4+T^5+T^6-2T^7+T^8)}{T^8}, \frac{1}{T^{16}}(-1+T)^2(5-18T+33T^2-32T^3+2T^4+42T^5-62T^6-8T^7+166T^8-242T^9+108T^{10}+132T^{11}-226T^{12}+148T^{13}-11T^{14}-36T^{15}-11T^{16}+148T^{17}-226T^{18}+132T^{19}+108T^{20}-242T^{21}+166T^{22}-8T^{23}-62T^{24}+42T^{25}+2T^{26}-32T^{27}+33T^{28}-18T^{29}+5T^{30})\right\}\right\}$$

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Strong!

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```
{NumberOfKnots[{3, 12}],
 Length@Union@Table[ρ[K], {K, AllKnots[{3, 12}]}],
 Length@Union@Table[{HOMFLYPT[K], Kh[K]}, {K, AllKnots[{3, 12}]}]}
```

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Out[ ]= {2977, 2882, 2785}

In[ ]:= `2977 - {2882, 2785}`

Out[ ]= {95, 192}

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So the pair (Δ,ρ_1) attains 2,882 distinct values on the 2,977 prime knots with up to 12 crossings (a deficit of 95), whereas the pair (HOMFLYPT, Khovanov Homology) attains only 2,785 distinct values on the same knots (a deficit of 192).

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\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}

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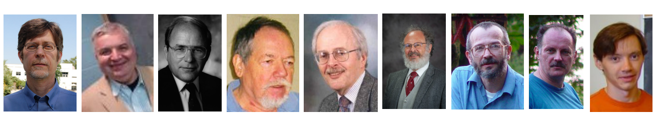

Hoste Ocneanu Millett Freyd Lickorish Yetter Przytycki Traczyk Khovanov

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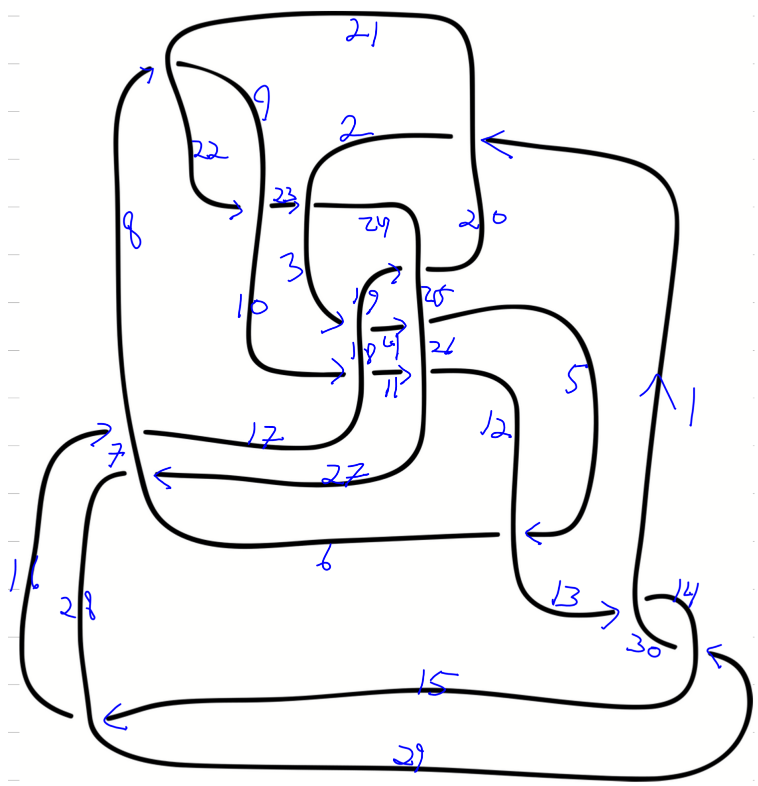
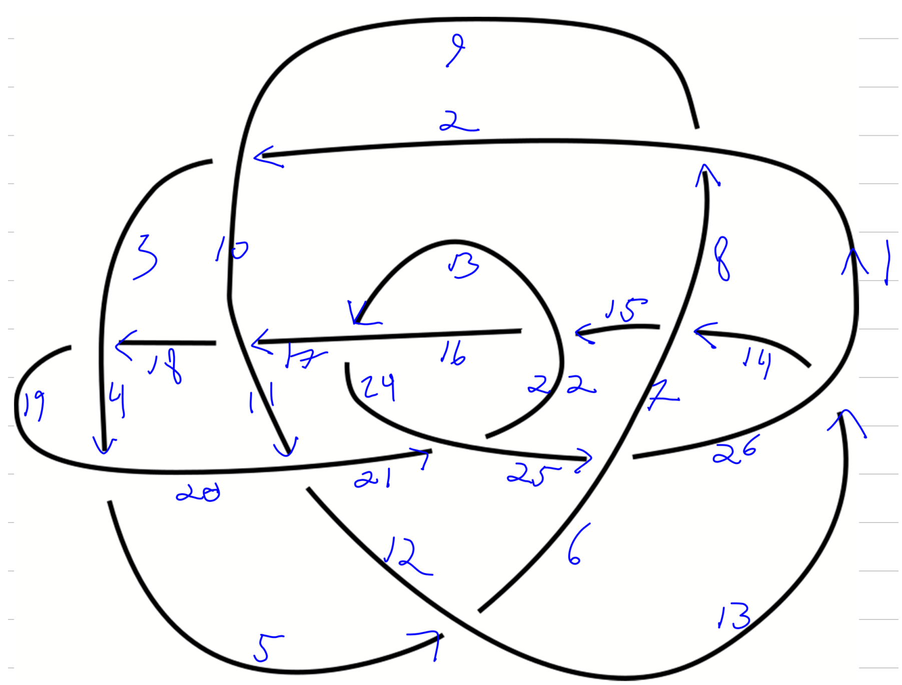
\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}



In[ ]:= $\texttt{Timing@}\rho\left[\text{EPD}\left[X_{20,1}, \overline{X}_{18,3}, X_{25,4}, \overline{X}_{12,5}, X_{21,8}, \overline{X}_{17,10}, X_{26,11}, \overline{X}_{30,13}, \overline{X}_{28,15}, \overline{X}_{7,16}, X_{24,19}, X_{9,22}, X_{2,23}, X_{6,27}, \overline{X}_{14,29}\right]\right]$

Out[ ]= {1.28125, {1, 0}}



In[•]:= Timing@ρ[EPD[$\overline{X}_{9,2}$, $\overline{X}_{19,4}$, $X_{12,5}$, $\overline{X}_{1,8}$, $\overline{X}_{20,11}$, $X_{26,13}$, $X_{7,14}$, $X_{22,15}$, $\overline{X}_{10,17}$, $\overline{X}_{3,18}$, $X_{24,21}$, $X_{16,23}$, $\overline{X}_{6,25}$]]

Out[•]= {0.796875, {1, 0}}

```
In[•]:= K = PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]];

In[•]:= {Cs, r} = List @@ RVK[K]
```

Set: Lists {Cs, r} and {PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]} are not the same shape.

```
Out[•]= {PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]}

In[•]:= n = Length[Cs]

Out[•]= 0

In[•]:= A = IdentityMatrix[2 n + 1]

Out[•]= {{1}}
```

In[•]:= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = $\begin{pmatrix} -T^s & T^s - 1 \\ 0 & -1 \end{pmatrix}$, {c, Cs}]

Do: Iterator {c, Cs} does not have appropriate bounds.

Out[•]= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = {{$-T^s$, $T^s - 1$}, {0, -1}}, {c, Cs}]

```
In[•]:= A // MatrixForm

Out[•]//MatrixForm=
( 1 )

In[•]:= A // MatrixForm // TeXForm

Out[•]//TeXForm=
\left(
\begin{array}{c}
 1 \\
\end{array}
\right)
```

In[•]:= Δ = $T^{(-\text{Total}[r]-\text{Total}[\text{First}/@\text{Cs}])/2}$ Det[A]

Out[•]= $T^{\frac{1}{2}(-\text{Total}[\text{Cs}]-\text{Total}[r])}$

```
In[•]:= G = Inverse[A];

In[•]:= G // MatrixForm

Out[•]//MatrixForm=
( 1 )

In[•]:= G // Simplify // MatrixForm

Out[•]//MatrixForm=
( 1 )
```

In[ ]:=
```
G // Simplify // MatrixForm // TeXForm
```

Out[ ]//TeXForm=
```
\left(
\begin{array}{c}
 1 \\
\end{array}
\right)
```