In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Groningen-220620"];
<< Rho1.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/gro22/ap to compute rotation numbers.
```

pdf

In[∘]:=
$$\delta_{i\_,j\_} := \mathrm{If}[i == j, 1, 0];$$
$$\mathrm{gRules}_{s\_,i\_,j\_} := \{g_{i\beta\_} \mapsto \delta_{i\beta} + T^s g_{i+1,\beta} + (1 - T^s)\, g_{j+1,\beta},\ g_{j\beta\_} \mapsto \delta_{j\beta} + g_{j+1,\beta},\ g_{\alpha\_,i} \mapsto T^{-s}\,(g_{\alpha,i+1} - \delta_{\alpha,i+1}),\ g_{\alpha\_j} \mapsto g_{\alpha,j+1} - (1 - T^s)\, g_{\alpha i} - \delta_{\alpha,j+1}\}$$

Proof of Reidemeister 3:

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In[∘]:=
$$\mathrm{lhs} = R_1[1, 20, 30] + R_1[1, 10, 31] + R_1[1, 11, 21] \mathbin{//.} \mathrm{gRules}_{1,20,30} \cup \mathrm{gRules}_{1,10,31} \cup \mathrm{gRules}_{1,11,21};$$
$$\mathrm{rhs} = R_1[1, 10, 20] + R_1[1, 11, 30] + R_1[1, 21, 31] \mathbin{//.} \mathrm{gRules}_{1,10,20} \cup \mathrm{gRules}_{1,11,30} \cup \mathrm{gRules}_{1,21,31};$$
$$\mathrm{Simplify}[\mathrm{lhs} == \mathrm{rhs}]$$

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Out[∘]= True

tex

Next comes Reid1, where we use results from an earlier example:

In[∘]:=
$$\begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix} \mathbin{//} \mathrm{Inverse} \mathbin{//} \mathrm{MatrixForm}$$

Out[∘]//MatrixForm=
$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & T & -T \\ 0 & 0 & 1 \end{pmatrix}$$

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In[∘]:=
$$R_1[1, 2, 1] - 1\,(g_{22} - 1/2)\ \ /.\ g_{\alpha\_,\beta\_} \mapsto \begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}[\![\alpha, \beta]\!]$$

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Out[∘]=
$$\frac{1}{T^2} - \frac{1}{T} - \frac{-1 + \frac{1}{T}}{T}$$

tex

Invariance under the other moves is proven similarly.

Alternative R_1's:

In[∘]:=
$$\mathrm{Simplify}[R_1[s, i, j] == s\,((1 - T^s)\, g_{ji}\,(g_{ji} - g_{ii}) + 2\, g_{jj}\, g_{ji} - g_{ji}\, g_{ij} - g_{jj}\, g_{ii} - g_{ji} + g_{ii} - 1/2) \mathbin{//.} \mathrm{gRules}_{s,i,j}]$$

Out[∘]=
$$i \neq j \,||\, s\,(-1 + T^s)\,(g_{1+i,1+i} - g_{1+j,1+i}) == 0$$

```
In[•]:= Simplify[R1[s, i, j] ==
          s ((g_{j,j+1} - g_{j,j}) (g_{ji} - g_{ii}) + 2 g_{jj} g_{ji} - g_{ji} g_{ij} - g_{jj} g_{ii} - g_{ji} + g_{ii} - 1 / 2) //. gRules_{s,i,j}]
Out[•]= True

In[•]:= Simplify[R1[s, i, j] == s (g_{j,i} (-1 - g_{i,j} + g_{j,j} + g_{j,1+j}) - g_{i,i} (-1 + g_{j,1+j}) - 1 / 2) //. gRules_{s,i,j}]
Out[•]= True

In[•]:= Simplify[R1[s, i, j] == s (g_{ji} (g_{jj} + g_{j,j+1} - g_{ij} - 1) - g_{ii} (g_{j,j+1} - 1) - 1 / 2) //. gRules_{s,i,j}]
Out[•]= True

In[•]:= Simplify[R1[s, i, j] == s (g_{ji} (g_{j+1,j} + g_{j,j+1} - g_{ij}) - g_{ii} (g_{j,j+1} - 1) - 1 / 2) //. gRules_{s,i,j}]
Out[•]= True

In[•]:= Simplify[(g_{ji} (g_{j+1,j} + g_{j,j+1} - g_{ij}) - g_{ii} (g_{j,j+1} - 1) - 1 / 2)]
```

Out[•]= $-\frac{1}{2} - g_{i,i}\left(-1 + g_{j,1+j}\right) + g_{j,i}\left(-g_{i,j} + g_{j,1+j} + g_{1+j,j}\right)$

```
In[•]:= Simplify[{g_{jj}, g_{j,j+1}, g_{ij}, g_{j+1,j} + g_{j,j+1}} //. gRules_{s,i,j}]
```

Out[•]= $\{-T^{-s}\left(-1 + T^{s}\right)\left(\text{If}[i == j, 1, 0] - g_{1+j,1+i}\right) + g_{1+j,1+j},$
$g_{1+j,1+j}, \text{If}[i == j, 1, 0] - T^{s}\,\text{If}[i == j, 1, 0] + \left(-1 + T^{s}\right)\left(-1 + g_{1+i,1+i}\right) +$
$T^{s} g_{1+i,1+j} + \left(1 - T^{s}\right)\left(-1 - T^{-s}\left(-1 + T^{s}\right)\left(\text{If}[i == j, 1, 0] - g_{1+j,1+i}\right) + g_{1+j,1+j}\right),$
$T^{-s}\left(-\left(\left(-1 + T^{s}\right)\text{If}[i == j, 1, 0]\right) + \left(-1 + T^{s}\right) g_{1+j,1+i} + T^{s}\left(-1 + 2\, g_{1+j,1+j}\right)\right)\}$