

Pensieve header: NOE-1 demo for GWU-1612, using elf conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\GWU-1612"];
```

## NOE-It

```
CF[ $\mathcal{E}$ ] := Module[{vars = Union@Cases[ $\mathcal{E}$ ,  $e_$  |  $l_$  |  $f_$ ,  $\infty$ ]},  
  If[vars === {}, Factor[ $\mathcal{E}$ ],  
   Total[CoefficientRules[ $\mathcal{E}$ , vars] /. ( $p_ \rightarrow c_$ )  $\Rightarrow$  Factor[ $c$ ] Times @@ (varsp)]  
  ]];
```

```
Format[ $\mathcal{E}$ ] := "E" @@ (CF /@  $\mathcal{E}$ );  
Format[ $\mathbb{O}$ [spec_List,  $\mathcal{E}$ ] ] := " $\mathbb{O}$ "[StringJoin @@ (( $x \mapsto$  ToString[ $x$ , StandardForm]) /@ spec),  $\mathcal{E}$ ];
```

Logos

$$\begin{aligned}\Delta[k] := & ((t-1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 e_k l_k f_k \delta^2 \mu^2 - \\ & \delta (1+\mu) (f_k^2 \alpha^2 + e_k^2 \beta^2) - e_k^2 f_k^2 \delta^3 (1+3 \mu) - \\ & 2 (\alpha \beta + 2 \delta \mu + e_k f_k \delta^2 (1+2 \mu) + 2 l_k \delta \mu^2) (f_k \alpha + e_k \beta) - 4 (l_k \mu^2 + e_k f_k \delta (1+\mu)) (\alpha \beta + \delta \mu) (1+t) / 4;\end{aligned}$$

1Gens

$$\begin{aligned}R_{i,j}^+ &:= E[1, \text{Log}[t] l_j, e_i f_j, e_i l_i f_j + l_i l_j + e_i^2 f_j^2 / 4]; \\ R_{i,j}^- &:= E[1, -\text{Log}[t] l_j, -t^{-1} e_i f_j, t^{-1} e_i l_j f_j - l_i l_j - t^{-2} e_i^2 f_j^2 / 4]; \\ (ur_i) &:= E[t^{-1/2}, 0, 0, l_i t^{-2}]; \quad nr_i := E[t^{1/2}, 0, 0, -l_i t^2];\end{aligned}$$

1DP

```
DP[x_>D $_{\alpha}$ ,y_>D $_{\beta}$ ][P_][f_] := (* means P[ $\partial_{\alpha}, \partial_{\beta}$ ][f] *)  
Total[CoefficientRules[P_, {x, y}] /. ({m_, n_}  $\rightarrow$  c_)  $\Rightarrow$  c D[f, { $\alpha$ , m}, { $\beta$ , n}]]
```

1Util

```
CF[ $\mathcal{E}$ ] := Expand /@ Together /@  $\mathcal{E}$ ;  
 $\mathbb{E}$  /:  $\mathbb{E}[\omega_1, L_1, Q_1, P_1] \mathbb{E}[\omega_2, L_2, Q_2, P_2] := CF @ E[\omega_1 \omega_2, L_1 + L_2, \omega_2 Q_1 + \omega_1 Q_2, \omega_2^4 P_1 + \omega_1^4 P_2];$ 
```

1NOuw

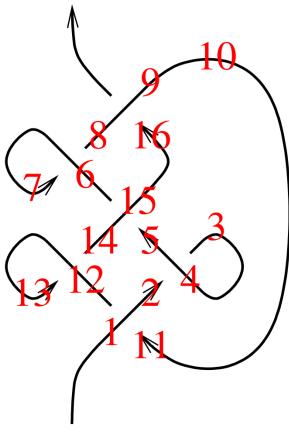
$$\begin{aligned}N_{f_i e_j \rightarrow k} [E[\omega, L, Q, P]] &:= \text{With}[\{q = ((1-t) \alpha \beta + \beta e_k f_k + \delta e_k f_k + \alpha f_k) / \mu\}, CF[ \\ & E[\mu \omega, L, \mu \omega q + \mu (Q /. f_i | e_j \rightarrow 0), \mu^4 e^{-q} DP_{f_i \rightarrow D_{\alpha}, e_j \rightarrow D_{\beta}}[P] [e^q] + \omega^4 \Delta[k]] / . \mu \rightarrow 1 + (t-1) \delta / . \\ & \{\alpha \rightarrow \omega^{-1} (\partial_{f_i} Q /. e_j \rightarrow 0), \beta \rightarrow \omega^{-1} (\partial_{e_j} Q /. f_i \rightarrow 0), \delta \rightarrow \omega^{-1} \partial_{f_i, e_j} Q\}]\};\end{aligned}$$

1NOc

$$\begin{aligned}N_{l_j (x:e|f)_i \rightarrow k} [E[\omega, L, Q, P]] &:= \text{With}[\{q = e^x \beta x_k + \gamma l_k\}, CF[ \\ & E[\omega, \gamma l_k + (L /. l_j \rightarrow 0), \omega e^x \beta x_k + (Q /. x_i \rightarrow 0), e^{-q} DP_{l_j \rightarrow D_y, x_i \rightarrow D_{\beta}}[P] [e^q]] / . \{\gamma \rightarrow \partial_{l_j} L, \beta \rightarrow \omega^{-1} \partial_{x_i} Q\}\}]\};\end{aligned}$$

1m

```
m_{i,j,k}[Z] := Module[{x, z},  
  CF[(Z // N $_{f_i e_j \rightarrow x}$  // N $_{l_i e_x \rightarrow x}$  // N $_{f_x l_j \rightarrow x}$ ) /. z_{i|j|x}  $\rightarrow$  z_k]]
```



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$$\mathbf{z1} = \mathbf{O}[\{\mathbf{e}_1, \mathbf{l}_1, \mathbf{f}_1, \mathbf{e}_2, \mathbf{l}_2, \mathbf{f}_2, \mathbf{e}_3, \mathbf{l}_3, \mathbf{f}_3, \mathbf{e}_4, \mathbf{l}_4, \mathbf{f}_4, \mathbf{e}_5, \mathbf{l}_5, \mathbf{f}_5, \mathbf{e}_6, \mathbf{l}_6, \mathbf{f}_6, \mathbf{e}_7, \mathbf{l}_7, \mathbf{f}_7, \mathbf{e}_8, \mathbf{l}_8, \mathbf{f}_8, \mathbf{e}_9, \mathbf{l}_9, \mathbf{f}_9, \mathbf{e}_{10}, \mathbf{l}_{10}, \mathbf{f}_{10}, \mathbf{e}_{11}, \mathbf{l}_{11}, \mathbf{f}_{11}, \mathbf{e}_{12}, \mathbf{l}_{12}, \mathbf{f}_{12}, \mathbf{e}_{13}, \mathbf{l}_{13}, \mathbf{f}_{13}, \mathbf{e}_{14}, \mathbf{l}_{14}, \mathbf{f}_{14}, \mathbf{e}_{15}, \mathbf{l}_{15}, \mathbf{f}_{15}, \mathbf{e}_{16}, \mathbf{l}_{16}, \mathbf{f}_{16}\}, \mathbf{R}_{1,11}^+ \mathbf{R}_{4,2}^- \mathbf{nr}_3 \mathbf{R}_{15,5}^+ \mathbf{R}_{6,8}^- \mathbf{ur}_7 \mathbf{R}_{9,16}^+ \mathbf{nr}_{10} \mathbf{R}_{12,14}^- \mathbf{ur}_{13}]$$

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$$\begin{aligned} & \mathbb{E}[1, -\text{Log}[t] l_2 + \text{Log}[t] l_5 - \text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, \\ & -\frac{e_4 f_2}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, -\frac{e_4^2 f_2^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + e_1 f_{11} l_1 + \\ & \frac{e_4 f_2 l_2}{t} - l_3 - l_2 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}] \end{aligned}$$

$$\mathbf{z2} = \text{Last}[\mathbf{z1}]$$

$$\begin{aligned} & \mathbb{E}[1, -\text{Log}[t] l_2 + \text{Log}[t] l_5 - \text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, \\ & -\frac{e_4 f_2}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, -\frac{e_4^2 f_2^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + e_1 f_{11} l_1 + \\ & \frac{e_4 f_2 l_2}{t} - l_3 - l_2 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}] \end{aligned}$$

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$$(\text{Do}[\mathbf{z2} = \text{Echo}[\mathbf{z2} // \text{m}_{1,k \rightarrow 1}], \{k, 2, 16\}); \mathbf{z2})$$

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$$\begin{aligned} & \gg \mathbb{E}[1, -\text{Log}[t] l_1 + \text{Log}[t] l_5 - \text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, \\ & -\frac{e_4 f_1}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, -\frac{e_4^2 f_1^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + \frac{e_4 f_1 l_1}{t} + \\ & e_1 f_{11} l_1 - l_3 - l_1 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}] \end{aligned}$$

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$$\begin{aligned} & \gg \mathbb{E}[1, -\text{Log}[t] l_1 + \text{Log}[t] l_5 - \text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, \\ & -\frac{e_4 f_1}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, \frac{e_4 f_1}{t} - \frac{e_4^2 f_1^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 - l_1 + \frac{e_4 f_1 l_1}{t} + \\ & e_1 f_{11} l_1 - l_1 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}] \end{aligned}$$

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$$\begin{aligned} & \gg \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_1 + \text{Log}[t] l_5 - \text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{e_1 f_1}{t^2} + \frac{e_{15} f_5}{t} - \frac{e_6 f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & \left. -\frac{e_1^2 f_1^2}{4 t^6} + \frac{e_{15}^2 f_5^2}{4 t^4} - \frac{e_6^2 f_8^2}{4 t^6} - \frac{e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{l_1}{4 t^4} + \frac{e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} + \right. \\ & \left. \frac{e_6 f_8 l_8}{t^5} - \frac{l_6 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^5} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} + \frac{e_{15} f_5 l_{15}}{t^4} + \frac{l_5 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4}\right] \end{aligned}$$

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$$\begin{aligned} & \gg \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{e_1 f_1}{t} + \frac{e_{15} f_5}{t} - \frac{e_6 f_8}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & \left. -\frac{e_1^2 f_1^2}{4 t^4} + \frac{e_{15}^2 f_5^2}{4 t^4} - \frac{e_6^2 f_8^2}{4 t^6} - \frac{e_1^2 f_1 f_{11}}{t^4} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{l_1}{4 t^4} + \frac{e_1 f_1 l_1}{t^4} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} + \frac{e_6 f_8 l_8}{t^5} - \right. \\ & \left. \frac{l_6 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^4} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_5 l_{15}}{t^4} + \frac{l_5 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4}\right] \end{aligned}$$

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$$\begin{aligned} \gg & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{e_1 f_1}{t} + \frac{e_{15} f_1}{t} - \frac{e_1 f_8}{t} + \frac{(-1+t) e_{15} f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & -\frac{e_1^2 f_1^2}{4 t^4} + \frac{e_{15}^2 f_1^2}{4 t^4} - \frac{(1+t) e_1 e_{15} f_1 f_8}{t^5} + \frac{e_{15}^2 f_1 f_8}{t^4} - \frac{e_1^2 f_8^2}{4 t^4} - \frac{e_1 e_{15} f_8^2}{t^5} + \frac{(-1+t) (1+t) e_{15}^2 f_8^2}{4 t^6} - \\ & \frac{e_1^2 f_1 f_{11}}{t^4} - \frac{e_1^2 f_8 f_{11}}{t^4} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} + \frac{e_1 f_1 l_1}{t^4} + \frac{(1+t) e_{15} f_8 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} + \\ & \frac{e_1 f_1 l_8}{t^4} - \frac{e_{15} f_1 l_8}{t^4} + \frac{e_1 f_8 l_8}{t^4} - \frac{(-1+t) e_{15} f_8 l_8}{t^5} - \frac{l_1 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^4} - \frac{e_1 f_8 l_{11}}{t^4} + \frac{l_1 l_{11}}{t^4} + \\ & \left. \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^4} - \frac{e_1 f_8 l_{15}}{t^4} + \frac{(-1+t) e_{15} f_8 l_{15}}{t^5} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} \gg & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_8 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{e_1 f_1}{t} + \frac{e_{15} f_1}{t} - \frac{e_1 f_8}{t} + \frac{(-1+t) e_{15} f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & -\frac{e_1 f_1}{t^4} + \frac{e_{15} f_1}{t^4} - \frac{e_1^2 f_1^2}{4 t^4} + \frac{e_{15}^2 f_1^2}{4 t^4} - \frac{(1+t) e_1 e_{15} f_1 f_8}{t^5} + \frac{e_{15}^2 f_1 f_8}{t^4} - \frac{e_1^2 f_8^2}{4 t^4} - \frac{e_1 e_{15} f_8^2}{t^5} + \frac{(-1+t) (1+t) e_{15}^2 f_8^2}{4 t^6} - \\ & \frac{e_1^2 f_1 f_{11}}{t^4} - \frac{e_1^2 f_8 f_{11}}{t^4} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{2 l_1}{t^4} + \frac{e_1 f_1 l_1}{t^4} + \frac{(1+t) e_{15} f_8 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \\ & \frac{e_1 f_1 l_8}{t^4} - \frac{e_{15} f_1 l_8}{t^4} + \frac{e_1 f_8 l_8}{t^4} - \frac{(-1+t) e_{15} f_8 l_8}{t^5} - \frac{l_1 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^4} - \frac{e_1 f_8 l_{11}}{t^4} + \frac{l_1 l_{11}}{t^4} + \\ & \left. \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^4} - \frac{e_1 f_8 l_{15}}{t^4} + \frac{(-1+t) e_{15} f_8 l_{15}}{t^5} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} \gg & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_1 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{(1+t) e_1 f_1}{t^2} + \frac{e_{15} f_1}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & -\frac{(5+4 t+t^2) e_1^2 f_1^2}{4 t^6} + \frac{e_{15}^2 f_1^2}{4 t^4} - \frac{(1+t) e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{2 l_1}{t^4} + \frac{(3+t) e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{2 l_1^2}{t^4} + \\ & \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{(1+t) e_1 f_1 l_{11}}{t^5} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^5} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \left. \right] \end{aligned}$$

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$$\begin{aligned} \gg & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_1 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{(1+t) e_1 f_1}{t^2} + \frac{e_{15} f_1}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_1 f_{16}}{t} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & -\frac{(5+4 t+t^2) e_1^2 f_1^2}{4 t^6} + \frac{e_{15}^2 f_1^2}{4 t^4} - \frac{(1+t) e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} - \frac{(1+t) e_1^2 f_1 f_{16}}{t^4} + \frac{(2+3 t) e_1 e_{15} f_1 f_{16}}{t^4} - \\ & \frac{(-1+2 t) e_1^2 f_1 f_{16}}{t^4} + \frac{e_1^2 f_{11} f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4 t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t) (-1+3 t) e_{15}^2 f_{16}^2}{4 t^4} + \frac{2 l_1}{t^4} + \frac{(3+t) e_1 f_1 l_1}{t^5} + \\ & \frac{e_1 f_{11} l_1}{t^4} + \frac{e_1 f_{16} l_1}{t^3} - \frac{2 e_{15} f_{16} l_1}{t^3} - \frac{2 l_1^2}{t^4} - \frac{(1+t) e_1 f_1 l_{11}}{t^5} + \frac{e_1 f_{16} l_{11}}{t^3} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \\ & \frac{(1+t) e_1 f_1 l_{15}}{t^5} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \left. \right] \end{aligned}$$

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$$\begin{aligned} \gg & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_1 + \text{Log}[t] l_{11} - \text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -\frac{(1+t) e_1 f_1}{t^2} + \frac{e_{15} f_1}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_1 f_{16}}{t} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & \frac{(1+t) e_1 f_1}{t^5} - \frac{e_{15} f_1}{t^4} - \frac{(5+4 t+t^2) e_1^2 f_1^2}{4 t^6} + \frac{e_{15}^2 f_1^2}{4 t^4} - \frac{(1+t) e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} - \frac{(1+t) e_1^2 f_1 f_{16}}{t^4} + \\ & \frac{(2+3 t) e_1 e_{15} f_1 f_{16}}{t^4} - \frac{(-1+2 t) e_1^2 f_1 f_{16}}{t^4} + \frac{e_1^2 f_{11} f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4 t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t) (-1+3 t) e_{15}^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} + \\ & \frac{(3+t) e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} + \frac{e_1 f_{16} l_1}{t^3} - \frac{2 e_{15} f_{16} l_1}{t^3} - \frac{2 l_1^2}{t^4} - \frac{(1+t) e_1 f_1 l_{11}}{t^5} + \frac{e_1 f_{16} l_{11}}{t^3} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \\ & \frac{l_{12} l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^5} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \left. \right] \end{aligned}$$

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$$\begin{aligned} \gg & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -e_1 f_1 + e_{15} f_1 - \frac{e_{12} f_{14}}{t^2} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & -\frac{e_{15} f_1}{t^3} + \frac{(-4+3 t^2) e_1^2 f_1^2}{4 t^4} - \frac{(1+t) e_1 e_{15} f_1^2}{t^3} + \frac{e_{15}^2 f_1^2}{4 t^2} - \frac{e_{12}^2 f_{14}^2}{4 t^6} - \frac{(1+2 t) e_1^2 f_1 f_{16}}{t^3} + \\ & \frac{2 (1+2 t) e_1 e_{15} f_1 f_{16}}{t^3} - \frac{(-1+2 t) e_1^2 f_1 f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4 t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t) (-1+3 t) e_{15}^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} - \\ & \frac{(-2+t) e_1 f_1 l_1}{t^4} + \frac{e_{15} f_1 l_1}{t^3} + \frac{2 e_1 f_{16} l_1}{t^3} - \frac{2 e_{15} f_{16} l_1}{t^3} - \frac{l_1^2}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^4} + \\ & \frac{e_{15} f_1 l_{15}}{t^3} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^3} + \frac{l_1 l_{16}}{t^4} \left. \right] \end{aligned}$$

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$$\begin{aligned} & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -e_1 f_1 + e_{15} f_1 - \frac{(1-t+t^2) e_1 f_{14}}{t^2} + \frac{(-1+t) e_{15} f_{14}}{t} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & -\frac{e_{15} f_1}{t^3} + \frac{(-4+3 t^2) e_1^2 f_1^2}{4 t^4} - \frac{(1+t) e_1 e_{15} f_1^2}{t^3} + \frac{e_{15}^2 f_1^2}{4 t^2} - \frac{(-1+t) e_{15} f_{14}}{t^4} + \frac{(-1+t) (1+2 t) e_1^2 f_1 f_{14}}{t^4} - \\ & \frac{(-1+t+3 t^2) e_1 e_{15} f_1 f_{14}}{t^4} + \frac{e_{15}^2 f_1 f_{14}}{t^2} + \frac{(-1+t^2-4 t^3+3 t^4) e_1^2 f_{14}^2}{4 t^6} - \frac{(1-t+t^3) e_1 e_{15} f_{14}^2}{t^5} + \frac{(-1+t) (1+t) e_{15}^2 f_{14}^2}{4 t^4} - \\ & \frac{(1+2 t) e_1^2 f_1 f_{16}}{t^3} + \frac{2 (1+2 t) e_1 e_{15} f_1 f_{16}}{t^3} - \frac{(-1+2 t) e_{15}^2 f_1 f_{16}}{t^3} - \frac{(1-t+2 t^2) e_1^2 f_{14} f_{16}}{t^4} + \frac{2 (-1+2 t) e_1 e_{15} f_{14} f_{16}}{t^3} - \\ & \frac{(-1+t) (-1+2 t) e_{15}^2 f_{14} f_{16}}{t^4} + \frac{e_1^2 f_{16}^2}{4 t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t) (-1+3 t) e_{15}^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} - \frac{(-2+t) e_1 f_1 l_1}{t^4} + \\ & \frac{e_{15} f_1 l_1}{t^3} - \frac{2 (-1+t) e_1 f_{14} l_1}{t^4} + \frac{2 e_{15} f_{14} l_1}{t^3} + \frac{2 e_1 f_{16} l_1}{t^3} - \frac{2 e_{15} f_{16} l_1}{t^3} - \frac{l_1^2}{t^4} + \frac{l_{13}}{t^4} + \frac{e_1 f_1 l_{14}}{t^3} - \frac{e_{15} f_1 l_{14}}{t^3} + \\ & \frac{(1-t+t^2) e_1 f_{14} l_{14}}{t^5} - \frac{(-1+t) e_{15} f_{14} l_{14}}{t^4} - \frac{l_1 l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^3} - \frac{e_1 f_{14} l_{15}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{15}}{t^4} + \\ & \left. \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^4} + \frac{e_{15} f_1 l_{16}}{t^3} - \frac{e_1 f_{14} l_{16}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_{14} + \text{Log}[t] l_{16}, -e_1 f_1 + e_{15} f_1 - \frac{(1-t+t^2) e_1 f_{14}}{t^2} + \frac{(-1+t) e_{15} f_{14}}{t} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & -\frac{e_1 f_1}{t^3} + \frac{(-4+3 t^2) e_1^2 f_1^2}{4 t^4} - \frac{(1+t) e_1 e_{15} f_1^2}{t^3} + \frac{e_{15}^2 f_1^2}{4 t^2} - \frac{(-1+t) e_{15} f_{14}}{t^4} + \frac{(-1+t) (1+2 t) e_1^2 f_1 f_{14}}{t^4} - \\ & \frac{(-1+t+3 t^2) e_1 e_{15} f_1 f_{14}}{t^4} + \frac{e_{15}^2 f_1 f_{14}}{t^2} + \frac{(-1+t^2-4 t^3+3 t^4) e_1^2 f_{14}^2}{4 t^6} - \frac{(1-t+t^3) e_1 e_{15} f_{14}^2}{t^5} + \frac{(-1+t) (1+t) e_{15}^2 f_{14}^2}{4 t^4} - \\ & \frac{(1+2 t) e_1^2 f_1 f_{16}}{t^3} + \frac{2 (1+2 t) e_1 e_{15} f_1 f_{16}}{t^3} - \frac{(-1+2 t) e_{15}^2 f_1 f_{16}}{t^3} - \frac{(1-t+2 t^2) e_1^2 f_{14} f_{16}}{t^4} + \frac{2 (-1+2 t) e_1 e_{15} f_{14} f_{16}}{t^3} - \\ & \frac{(-1+t) (-1+2 t) e_{15}^2 f_{14} f_{16}}{t^4} + \frac{e_1^2 f_{16}^2}{4 t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t) (-1+3 t) e_{15}^2 f_{16}^2}{4 t^4} + \frac{2 l_1}{t^4} - \frac{(-2+t) e_1 f_1 l_1}{t^4} + \frac{e_{15} f_1 l_1}{t^3} - \\ & \frac{2 (-1+t) e_1 f_{14} l_1}{t^4} + \frac{2 e_{15} f_{14} l_1}{t^3} + \frac{2 e_1 f_{16} l_1}{t^3} - \frac{2 e_{15} f_{16} l_1}{t^3} - \frac{l_1^2}{t^4} + \frac{e_1 f_1 l_{14}}{t^3} - \frac{e_{15} f_1 l_{14}}{t^3} + \frac{(1-t+t^2) e_1 f_{14} l_{14}}{t^5} - \\ & \frac{(-1+t) e_{15} f_{14} l_{14}}{t^4} - \frac{l_1 l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^3} - \frac{e_1 f_{14} l_{15}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{15}}{t^4} + \frac{e_1 f_{16} l_{15}}{t^3} - \\ & \left. \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^4} + \frac{e_{15} f_1 l_{16}}{t^3} - \frac{e_1 f_{14} l_{16}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \mathbb{E}\left[\frac{1}{t}, -\text{Log}[t] l_1 + \text{Log}[t] l_{16}, -\frac{(1+t^2) e_1 f_1}{t^2} + e_{15} f_1 + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & -\frac{e_{15} f_1}{t^3} + \frac{(-5-8 t+3 t^4) e_1^2 f_1^2}{4 t^6} - \frac{(1+t+t^2) e_1 e_{15} f_1^2}{t^4} + \frac{e_{15}^2 f_1^2}{4 t^2} - \frac{(2+t+2 t^2) e_1^2 f_1 f_{16}}{t^4} + \\ & \frac{2 (1+t+2 t^2) e_1 e_{15} f_1 f_{16}}{t^4} - \frac{(-1+2 t) e_{15}^2 f_1 f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4 t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t) (-1+3 t) e_{15}^2 f_{16}^2}{4 t^4} + \\ & \frac{2 l_1}{t^4} - \frac{(-3+t) (1+t) e_1 f_1 l_1}{t^5} + \frac{e_{15} f_1 l_1}{t^3} + \frac{2 e_1 f_{16} l_1}{t^3} - \frac{2 e_{15} f_{16} l_1}{t^3} - \frac{2 l_1^2}{t^4} - \frac{(1+t+t^2) e_1 f_1 l_{15}}{t^5} + \\ & \left. \frac{e_{15} f_1 l_{15}}{t^3} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t+t^2) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^3} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \mathbb{E}\left[\frac{1-t+t^2}{t}, -\text{Log}[t] l_1 + \text{Log}[t] l_{16}, -\frac{(1-t+t^2) e_1 f_1}{t^2} + \frac{(1-t+t^2) e_1 f_{16}}{t}, \right. \\ & \frac{(-1+t) (1-t+t^2)^2 (1-t+2 t^2)}{t^3} + \frac{(-1-t+t^2) (1-t+t^2)^3 e_1 f_1}{t^5} - \frac{(1-t+t^2)^3 (1+3 t+t^2) e_1^2 f_1^2}{4 t^6} - \\ & \frac{2 (1-t+t^2)^3 e_1 f_{16}}{t^2} + \frac{(-1+2 t) (1-t+t^2)^3 e_1^2 f_1 f_{16}}{t^5} - \frac{(1-t+t^2)^3 (-1+t+3 t^2) e_1^2 f_{16}^2}{4 t^4} - \\ & \frac{(-1+t) (2+t) (1-t+t^2)^3 l_1}{t^4} + \frac{(1-t+t^2)^3 e_1 f_1 l_1}{t^5} + \frac{(1-t+t^2)^3 (1-2 t+2 t^2) e_1 f_{16} l_1}{t^4} - \frac{(1-t+t^2)^4 l_1^2}{t^4} - \\ & \left. \frac{(-1+t) (1-t+t^2)^3 l_{16}}{t^3} - \frac{(1-t+t^2)^3 e_1 f_1 l_{16}}{t^5} - \frac{(-1+t) (1-t+t^2)^3 e_1 f_{16} l_{16}}{t^3} + \frac{(1-t+t^2)^4 l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\mathbb{E}\left[\frac{1-t+t^2}{t}, 0, 0, \frac{(-1+t) (1-t+t^2)^2 (1-t+2 t^2)}{t^3} - \frac{2 (1+t) (1-t+t^2)^3 e_1 f_1}{t^4} - \frac{2 (-1+t) (1+t) (1-t+t^2)^3 l_1}{t^4} \right]$$

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$$\mathbb{E}\left[\frac{1-t+t^2}{t}, 0, 0, \frac{(-1+t) (1-t+t^2)^2 (1-t+2 t^2)}{t^3} - \frac{2 (1+t) (1-t+t^2)^3 e_1 f_1}{t^4} - \frac{2 (-1+t) (1+t) (1-t+t^2)^3 l_1}{t^4} \right]$$

# Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```

ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
    DeleteFile[temp],
    (* else *) Print["Exporting " <> fname <> "..."];
    If[exists, DeleteFile[fname]];
    RenameFile[temp, fname]
  ];
  fname
]

SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
TagProperties[_] := {};
TagProperties["131"] = {PageWidth → 3.2 / 0.66};
Options[CellExport] = {
  PageWidth → 4 / 0.66, CellFilter → Identity, ExportDirectory → "Snips",
  ExportBaseFilename → Automatic, ExportFormat → ".pdf", ExportOptions → {}, Split → False
};
CellExport[tag_String, opts___Rule] := CellExport[
  NotebookGet[EvaluationNotebook[]],
  tag, opts
];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{OptionValue[ExportDirectory] /. Automatic → Directory[], OptionValue[ExportBaseFilename] /. Automatic → tag}];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
    nb, c_Cell /; FreeQ[List @@ c, Cell] && !FreeQ[c, CellTags → tag],
    Infinity
  ]];
  If[! OptionValue[Split],
    If[Length[cells] ≥ 1,
      If[Length[cells] == 1,
        cells = Join[First[cells]],
        Cell[PageWidth → 1.2 × 72 OptionValue[PageWidth], Background → {White, Opacity[0]}]],
      cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
    ];
    ConditionalExport[
      filename <> format, cells,
      ImageResolution → 300,
      OptionValue[ExportOptions]
    ]
  ],
  k = 0;
  Table[
    ++k;

```

```
ConditionalExport[  
    filename <> "-" <> ToString[k] <> format,  
    Append[cell, PageWidth -> 72 OptionValue[PageWidth]],  
    ImageResolution -> 300,  
    OptionValue[ExportOptions]  
,  
    {cell, cells}  
,  
]  
];  
  
ExportCells := (  
    nb = NotebookGet[EvaluationNotebook[]];  
    tags = Cases[nb, (CellTags -> tag_String) :> tag, Infinity] // Union;  
    Print[tags];  
    CellExport /@ tags;  
    Print["Done."]  
)
```

### ExportCells

```
{0817, 0m, 0mDemo, 0MetaAssoc, 0NO, 0NODemo, 0Q0, 0R, 0R3,  
0R3Left, 0Util, 131, 1DP, 1Gens, 1m, 1N0c, 1NOuw, 1Util, Logos, T0, ZT0}
```

Exporting Snips\1Gens.pdf...

Done.