Pensieve header: NOE-1 demo for GWU-1612, using elf conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\GWU-1612"];
```

NOE-1t

```
CF[ℰ_] := Module[{vars = Union@@Cases[ℰ, e_ | l_ | f_, ∞]},
   If[vars === {}, Factor[ℰ],
    Total[CoefficientRules[ℰ, vars] /. (p_ → c_) :↦ Factor[c] Times @@ (vars^p)]
   ]];
```

```
Format[ℰ_𝔼] := "𝔼"@@(CF /@ ℰ);
Format[𝕆[spec_List, ℰ_𝔼]] := "𝕆"[StringJoin@@((x ↦ ToString[x, StandardForm]) /@ spec), ℰ];
```

Logos

```
Λ[k_] := ((t - 1) (2 (α β + δ μ)^2 - α^2 β^2) - 4 e_k l_k f_k δ^2 μ^2 -
      δ (1 + μ) (f_k^2 α^2 + e_k^2 β^2) - e_k^2 f_k^2 δ^3 (1 + 3 μ) -
      2 (α β + 2 δ μ + e_k f_k δ^2 (1 + 2 μ) + 2 l_k δ μ^2) (f_k α + e_k β) - 4 (l_k μ^2 + e_k f_k δ (1 + μ)) (α β + δ μ)) (1 + t) / 4;
```

```
Λ[k] // TeXForm
```

```
\frac{1}{4} (t+1) \left(-\delta  (\mu +1) \left(\beta ^2
   e_k^2+\alpha ^2 f_k^2\right)+\delta ^3 (-(3 \mu +1)) e_k^2
   f_k^2-2 \left(\beta  e_k+\alpha  f_k\right) \left(\alpha  \beta
   +2 \delta  \mu +\delta ^2 (2 \mu +1) e_k f_k+2 \delta  \mu ^2
   l_k\right)-4 (\alpha  \beta +\delta  \mu ) \left(\delta  (\mu +1)
   e_k f_k+\mu ^2 l_k\right)-4 \delta ^2 \mu ^2 e_k f_k l_k+(t-1)
   \left(2 (\alpha  \beta +\delta  \mu )^2-\alpha ^2 \beta
   ^2\right)\right)
```

1Gens

```
R^+_{i_,j_} := 𝔼[1, l_j, e_i f_j, e_i l_i f_j + l_i l_j + e_i^2 f_j^2 / 4];
R^-_{i_,j_} := 𝔼[1, -l_j, -t^-1 e_i f_j, t^-1 e_i l_j f_j - l_i l_j - t^-2 e_i^2 f_j^2 / 4];
(ur_{i_} := 𝔼[t^-1/2, 0, 0, l_i t^-2]; nr_{i_} := 𝔼[t^1/2, 0, 0, -l_i t^2];)
```

1DP

```
DP_{x_→D_α,y_→D_β_}[P_][f_] := (* means P[∂_α,∂_β][f] *)
 Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) :↦ c D[f, {α, m}, {β, n}]]
```

1Util

```
CF[ℰ_𝔼] := Expand /@ Together /@ ℰ;
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] := CF@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

1NOuw

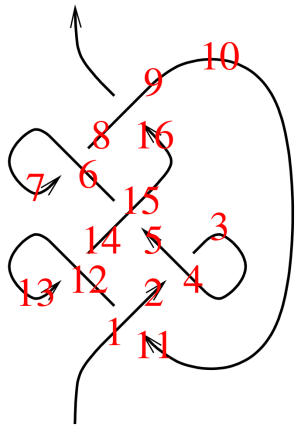
```
N_{f_i_ e_j_→k_}[𝔼[ω_, L_, Q_, P_]] := With[{q = ((1 - t) α β + β e_k + α f_k + δ e_k f_k) / μ}, CF[
     𝔼[μ ω, L, μ ω q + μ (Q /. f_i | e_j → 0), μ^4 ℯ^-q DP_{f_i→D_α,e_j→D_β}[P][ℯ^q] + ω^4 Λ[k]] /. μ → 1 + (t - 1) δ /.
      {α → ω^-1 (∂_{f_i} Q /. e_j → 0), β → ω^-1 (∂_{e_j} Q /. f_i → 0), δ → ω^-1 ∂_{f_i,e_j} Q}]];
```

1NOc

```
N_{1_j_ (x:e|f)_i_→k_}[𝔼[ω_, L_, Q_, P_]] := With[{λ = ∂_{1_j} L, α = ∂_{x_i} Q, q = ℯ^γ β x_k + γ l_k}, CF[
     𝔼[ω, L /. 1_j → 1_k, t^λ α x_k + (Q /. x_i → 0), ℯ^-q DP_{1_j→D_γ,x_i→D_β}[P][ℯ^q] /. {β → α / ω, γ → λ Log[t]}]
    ]];
```

1m

```
m_{i_,j_→k_}[Z_𝔼] := Module[{x, z},
  CF[(Z // N_{f_i e_j→x} // N_{1_i e_x→x} // N_{f_x 1_j→x}) /. z_{i_|j_|x} → z_k]]
```



131

```
z1 = 𝕆[{e1, l1, f1, e2, l2, f2, e3, l3, f3, e4, l4, f4, e5, l5, f5, e6, l6, f6,
    e7, l7, f7, e8, l8, f8, e9, l9, f9, e10, l10, f10, e11, l11, f11, e12, l12, f12, e13, l13, f13,
    e14, l14, f14, e15, l15, f15, e16, l16, f16}, R+_{1,11} R-_{4,2} nr3 R+_{15,5} R-_{6,8} ur7 R+_{9,16} nr10 R-_{12,14} ur13]
```

131

$$\mathbb{O}\Big[e_1 l_1 f_1 e_2 l_2 f_2 e_3 l_3 f_3 e_4 l_4 f_4 e_5 l_5 f_5 e_6 l_6 f_6 e_7 l_7 f_7 e_8 l_8 f_8 e_9 l_9 f_9 e_{10} l_{10} f_{10} e_{11} l_{11} f_{11} e_{12} l_{12} f_{12} e_{13} l_{13} f_{13} e_{14} l_{14} f_{14} e_{15} l_{15} f_{15} e_{16} l_{16} f_{16},$$
$$\mathbb{E}\Big[1, -l_2 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_2}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16},$$
$$-\frac{e_4^2 f_2^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + e_1 f_{11} l_1 + \frac{e_4 f_2 l_2}{t} - l_3 - l_2 l_4 + l_7 +$$
$$\frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}\Big]\Big]$$

```
z2 = Last[z1]
```

$$\mathbb{E}\Big[1, -l_2 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_2}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16},$$
$$-\frac{e_4^2 f_2^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + e_1 f_{11} l_1 + \frac{e_4 f_2 l_2}{t} - l_3 - l_2 l_4 + l_7 +$$
$$\frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}\Big]$$

131

```
(Do[z2 = Echo[z2 // m_{1,k→1}], {k, 2, 16}]; z2)
```

131

» $$\mathbb{E}\Big[1, -l_1 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_1}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16},$$
$$-\frac{e_4^2 f_1^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + \frac{e_4 f_1 l_1}{t} + e_1 f_{11} l_1 - l_3 - l_1 l_4 +$$
$$l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}\Big]$$

131

» $$\mathbb{E}\Big[1, -l_1 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_1}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16},$$
$$\frac{e_4 f_1}{t} - \frac{e_4^2 f_1^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 - l_1 + \frac{e_4 f_1 l_1}{t} + e_1 f_{11} l_1 - l_1 l_4 +$$
$$l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_1 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_1 f_1}{t^2} + \frac{e_{15} f_5}{t} - \frac{e_6 f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t},$$
$$-\frac{e_1^2 f_1^2}{4 t^6} + \frac{e_{15}^2 f_5^2}{4 t^4} - \frac{e_6^2 f_8^2}{4 t^6} - \frac{e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} + \frac{e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} +$$
$$\frac{e_6 f_8 l_8}{t^5} - \frac{l_6 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^5} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} + \frac{e_{15} f_5 l_{15}}{t^4} + \frac{l_5 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_8+l_{11}-l_{14}+l_{16}, -\frac{e_1 f_1}{t}+\frac{e_{15} f_1}{t}-\frac{e_6 f_8}{t^2}+\frac{e_1 f_{11}}{t}-\frac{e_{12} f_{14}}{t^2}+\frac{e_9 f_{16}}{t},$$
$$-\frac{e_1^2 f_1^2}{4t^4}+\frac{e_{15}^2 f_1^2}{4t^4}-\frac{e_6^2 f_8^2}{4t^6}-\frac{e_1^2 f_1 f_{11}}{t^4}+\frac{e_1^2 f_{11}^2}{4t^4}-\frac{e_{12}^2 f_{14}^2}{4t^6}+\frac{e_9^2 f_{16}^2}{4t^4}+\frac{l_1}{t^4}+\frac{e_1 f_1 l_1}{t^4}+\frac{e_1 f_{11} l_1}{t^4}-\frac{l_1^2}{t^4}+\frac{l_7}{t^4}+\frac{e_6 f_8 l_8}{t^5}-$$
$$\frac{l_6 l_8}{t^4}+\frac{e_9 f_{16} l_9}{t^4}-\frac{l_{10}}{t^4}-\frac{e_1 f_1 l_{11}}{t^4}+\frac{l_1 l_{11}}{t^4}+\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-\frac{l_{12} l_{14}}{t^4}-\frac{e_1 f_1 l_{15}}{t^4}+\frac{e_{15} f_1 l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}+\frac{l_9 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_8+l_{11}-l_{14}+l_{16}, -\frac{e_1 f_1}{t}+\frac{e_{15} f_1}{t}-\frac{e_1 f_8}{t}+\frac{(-1+t)\, e_{15} f_8}{t^2}+\frac{e_1 f_{11}}{t}-\frac{e_{12} f_{14}}{t^2}+\frac{e_9 f_{16}}{t},$$
$$-\frac{e_1^2 f_1^2}{4t^4}+\frac{e_{15}^2 f_1^2}{4t^4}-\frac{(1+t)\, e_1 e_{15} f_1 f_8}{t^5}+\frac{e_{15}^2 f_1 f_8}{t^4}-\frac{e_1^2 f_8^2}{4t^4}-\frac{e_1 e_{15} f_8^2}{t^5}+\frac{(-1+t)\,(1+t)\, e_{15}^2 f_8^2}{4t^6}-$$
$$\frac{e_1^2 f_1 f_{11}}{t^4}-\frac{e_1^2 f_8 f_{11}}{t^4}+\frac{e_1^2 f_{11}^2}{4t^4}-\frac{e_{12}^2 f_{14}^2}{4t^6}+\frac{e_9^2 f_{16}^2}{4t^4}+\frac{l_1}{t^4}+\frac{e_1 f_1 l_1}{t^4}+\frac{(1+t)\, e_{15} f_8 l_1}{t^5}+\frac{e_1 f_{11} l_1}{t^4}-\frac{l_1^2}{t^4}+\frac{l_7}{t^4}+$$
$$\frac{e_1 f_1 l_8}{t^4}-\frac{e_{15} f_1 l_8}{t^4}+\frac{e_1 f_8 l_8}{t^4}-\frac{(-1+t)\, e_{15} f_8 l_8}{t^5}-\frac{l_1 l_8}{t^4}+\frac{e_9 f_{16} l_9}{t^4}-\frac{l_{10}}{t^4}-\frac{e_1 f_1 l_{11}}{t^4}-\frac{e_1 f_8 l_{11}}{t^4}+\frac{l_1 l_{11}}{t^4}+$$
$$\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-\frac{l_{12} l_{14}}{t^4}-\frac{e_1 f_1 l_{15}}{t^4}+\frac{e_{15} f_1 l_{15}}{t^4}-\frac{e_1 f_8 l_{15}}{t^4}+\frac{(-1+t)\, e_{15} f_8 l_{15}}{t^5}+\frac{l_1 l_{15}}{t^4}+\frac{l_9 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_8+l_{11}-l_{14}+l_{16}, -\frac{e_1 f_1}{t}+\frac{e_{15} f_1}{t}-\frac{e_1 f_8}{t}+\frac{(-1+t)\, e_{15} f_8}{t^2}+\frac{e_1 f_{11}}{t}-\frac{e_{12} f_{14}}{t^2}+\frac{e_9 f_{16}}{t},$$
$$-\frac{e_1 f_1}{t^4}+\frac{e_{15} f_1}{t^4}-\frac{e_1^2 f_1^2}{4t^4}+\frac{e_{15}^2 f_1^2}{4t^4}-\frac{(1+t)\, e_1 e_{15} f_1 f_8}{t^5}+\frac{e_{15}^2 f_1 f_8}{t^4}-\frac{e_1^2 f_8^2}{4t^4}-\frac{e_1 e_{15} f_8^2}{t^5}+\frac{(-1+t)\,(1+t)\, e_{15}^2 f_8^2}{4t^6}-$$
$$\frac{e_1^2 f_1 f_{11}}{t^4}-\frac{e_1^2 f_8 f_{11}}{t^4}+\frac{e_1^2 f_{11}^2}{4t^4}-\frac{e_{12}^2 f_{14}^2}{4t^6}+\frac{e_9^2 f_{16}^2}{4t^4}+\frac{2\, l_1}{t^4}+\frac{e_1 f_1 l_1}{t^4}+\frac{(1+t)\, e_{15} f_8 l_1}{t^5}+\frac{e_1 f_{11} l_1}{t^4}-\frac{l_1^2}{t^4}+$$
$$\frac{e_1 f_1 l_8}{t^4}-\frac{e_{15} f_1 l_8}{t^4}+\frac{e_1 f_8 l_8}{t^4}-\frac{(-1+t)\, e_{15} f_8 l_8}{t^5}-\frac{l_1 l_8}{t^4}+\frac{e_9 f_{16} l_9}{t^4}-\frac{l_{10}}{t^4}-\frac{e_1 f_1 l_{11}}{t^4}-\frac{e_1 f_8 l_{11}}{t^4}+\frac{l_1 l_{11}}{t^4}+$$
$$\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-\frac{l_{12} l_{14}}{t^4}-\frac{e_1 f_1 l_{15}}{t^4}+\frac{e_{15} f_1 l_{15}}{t^4}-\frac{e_1 f_8 l_{15}}{t^4}+\frac{(-1+t)\, e_{15} f_8 l_{15}}{t^5}+\frac{l_1 l_{15}}{t^4}+\frac{l_9 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_1+l_{11}-l_{14}+l_{16}, -\frac{(1+t)\, e_1 f_1}{t^2}+\frac{e_{15} f_1}{t}+\frac{e_1 f_{11}}{t}-\frac{e_{12} f_{14}}{t^2}+\frac{e_9 f_{16}}{t},$$
$$-\frac{\left(5+4t+t^2\right) e_1^2 f_1^2}{4t^6}+\frac{e_{15}^2 f_1^2}{4t^4}-\frac{(1+t)\, e_1^2 f_1 f_{11}}{t^5}+\frac{e_1^2 f_{11}^2}{4t^4}-\frac{e_{12}^2 f_{14}^2}{4t^6}+\frac{e_9^2 f_{16}^2}{4t^4}+\frac{2\, l_1}{t^4}+\frac{(3+t)\, e_1 f_1 l_1}{t^5}+\frac{e_1 f_{11} l_1}{t^4}-\frac{2\, l_1^2}{t^4}+$$
$$\frac{e_9 f_{16} l_9}{t^4}-\frac{l_{10}}{t^4}-\frac{(1+t)\, e_1 f_1 l_{11}}{t^5}+\frac{l_1 l_{11}}{t^4}+\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-\frac{l_{12} l_{14}}{t^4}-\frac{(1+t)\, e_1 f_1 l_{15}}{t^5}+\frac{e_{15} f_1 l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}+\frac{l_9 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_1+l_{11}-l_{14}+l_{16}, -\frac{(1+t)\, e_1 f_1}{t^2}+\frac{e_{15} f_1}{t}+\frac{e_1 f_{11}}{t}-\frac{e_{12} f_{14}}{t^2}+e_1 f_{16}-\frac{(-1+t)\, e_{15} f_{16}}{t},$$
$$-\frac{\left(5+4t+t^2\right) e_1^2 f_1^2}{4t^6}+\frac{e_{15}^2 f_1^2}{4t^4}-\frac{(1+t)\, e_1^2 f_1 f_{11}}{t^5}+\frac{e_1^2 f_{11}^2}{4t^4}-\frac{e_{12}^2 f_{14}^2}{4t^6}-\frac{(1+t)\, e_1^2 f_1 f_{16}}{t^4}+\frac{(2+3t)\, e_1 e_{15} f_1 f_{16}}{t^4}-$$
$$\frac{(-1+2t)\, e_{15}^2 f_1 f_{16}}{t^4}+\frac{e_1^2 f_{11} f_{16}}{t^3}+\frac{e_1^2 f_{16}^2}{4t^2}-\frac{e_1 e_{15} f_{16}^2}{t^2}+\frac{(-1+t)\,(-1+3t)\, e_{15}^2 f_{16}^2}{4t^4}+\frac{2\, l_1}{t^4}+\frac{(3+t)\, e_1 f_1 l_1}{t^5}+$$
$$\frac{e_1 f_{11} l_1}{t^4}+\frac{e_1 f_{16} l_1}{t^3}-\frac{2\, e_{15} f_{16} l_1}{t^3}-\frac{2\, l_1^2}{t^4}-\frac{l_{10}}{t^4}-\frac{(1+t)\, e_1 f_1 l_{11}}{t^5}+\frac{e_1 f_{16} l_{11}}{t^3}+\frac{l_1 l_{11}}{t^4}+\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-\frac{l_{12} l_{14}}{t^4}-$$
$$\frac{(1+t)\, e_1 f_1 l_{15}}{t^5}+\frac{e_{15} f_1 l_{15}}{t^4}+\frac{e_1 f_{16} l_{15}}{t^3}-\frac{(-1+t)\, e_{15} f_{16} l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}-\frac{(1+t)\, e_1 f_1 l_{16}}{t^5}+\frac{e_{15} f_1 l_{16}}{t^4}+\frac{l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t}, -l_1+l_{11}-l_{14}+l_{16}, -\frac{(1+t)\, e_1 f_1}{t^2}+\frac{e_{15} f_1}{t}+\frac{e_1 f_{11}}{t}-\frac{e_{12} f_{14}}{t^2}+e_1 f_{16}-\frac{(-1+t)\, e_{15} f_{16}}{t},$$
$$\frac{(1+t)\, e_1 f_1}{t^5}-\frac{e_{15} f_1}{t^4}-\frac{\left(5+4t+t^2\right) e_1^2 f_1^2}{4t^6}+\frac{e_{15}^2 f_1^2}{4t^4}-\frac{(1+t)\, e_1^2 f_1 f_{11}}{t^5}+\frac{e_1^2 f_{11}^2}{4t^4}-\frac{e_{12}^2 f_{14}^2}{4t^6}-\frac{(1+t)\, e_1^2 f_1 f_{16}}{t^4}+$$
$$\frac{(2+3t)\, e_1 e_{15} f_1 f_{16}}{t^4}-\frac{(-1+2t)\, e_{15}^2 f_1 f_{16}}{t^4}+\frac{e_1^2 f_{11} f_{16}}{t^3}+\frac{e_1^2 f_{16}^2}{4t^2}-\frac{e_1 e_{15} f_{16}^2}{t^2}+\frac{(-1+t)\,(-1+3t)\, e_{15}^2 f_{16}^2}{4t^4}+\frac{l_1}{t^4}+$$
$$\frac{(3+t)\, e_1 f_1 l_1}{t^5}+\frac{e_1 f_{11} l_1}{t^4}+\frac{e_1 f_{16} l_1}{t^3}-\frac{2\, e_{15} f_{16} l_1}{t^3}-\frac{2\, l_1^2}{t^4}-\frac{(1+t)\, e_1 f_1 l_{11}}{t^5}+\frac{e_1 f_{16} l_{11}}{t^3}+\frac{l_1 l_{11}}{t^4}+\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-$$
$$\frac{l_{12} l_{14}}{t^4}-\frac{(1+t)\, e_1 f_1 l_{15}}{t^5}+\frac{e_{15} f_1 l_{15}}{t^4}+\frac{e_1 f_{16} l_{15}}{t^3}-\frac{(-1+t)\, e_{15} f_{16} l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}-\frac{(1+t)\, e_1 f_1 l_{16}}{t^5}+\frac{e_{15} f_1 l_{16}}{t^4}+\frac{l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t},\ -l_{14}+l_{16},\ -e_1 f_1+e_{15} f_1-\frac{e_{12} f_{14}}{t^2}+e_1 f_{16}-\frac{(-1+t)\ e_{15} f_{16}}{t},$$
$$-\frac{e_{15} f_1}{t^3}+\frac{\left(-4+3t^2\right) e_1^2 f_1^2}{4t^4}-\frac{(1+t)\ e_1 e_{15} f_1^2}{t^3}+\frac{e_{15}^2 f_1^2}{4t^2}-\frac{e_{12}^2 f_{14}^2}{4t^6}-\frac{(1+2t)\ e_1^2 f_1 f_{16}}{t^3}+$$
$$\frac{2\ (1+2t)\ e_1 e_{15} f_1 f_{16}}{t^3}-\frac{(-1+2t)\ e_{15}^2 f_1 f_{16}}{t^3}+\frac{e_1^2 f_{16}^2}{4t^2}-\frac{e_1 e_{15} f_{16}^2}{t^2}+\frac{(-1+t)\ (-1+3t)\ e_{15}^2 f_{16}^2}{4t^4}+\frac{l_1}{t^4}-$$
$$\frac{(-2+t)\ e_1 f_1 l_1}{t^4}+\frac{e_{15} f_1 l_1}{t^3}+\frac{2\ e_1 f_{16} l_1}{t^3}-\frac{2\ e_{15} f_{16} l_1}{t^3}-\frac{l_1^2}{t^4}+\frac{l_{13}}{t^4}+\frac{e_{12} f_{14} l_{14}}{t^5}-\frac{l_{12} l_{14}}{t^4}-\frac{(1+t)\ e_1 f_1 l_{15}}{t^4}+$$
$$\frac{e_{15} f_1 l_{15}}{t^3}+\frac{e_1 f_{16} l_{15}}{t^3}-\frac{(-1+t)\ e_{15} f_{16} l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}-\frac{(1+t)\ e_1 f_1 l_{16}}{t^4}+\frac{e_{15} f_1 l_{16}}{t^3}+\frac{l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t},\ -l_{14}+l_{16},\ -e_1 f_1+e_{15} f_1-\frac{\left(1-t+t^2\right) e_1 f_{14}}{t^2}+\frac{(-1+t)\ e_{15} f_{14}}{t}+e_1 f_{16}-\frac{(-1+t)\ e_{15} f_{16}}{t},$$
$$-\frac{e_{15} f_1}{t^3}+\frac{\left(-4+3t^2\right) e_1^2 f_1^2}{4t^4}-\frac{(1+t)\ e_1 e_{15} f_1^2}{t^3}+\frac{e_{15}^2 f_1^2}{4t^2}-\frac{(-1+t)\ e_{15} f_{14}}{t^4}+\frac{(-1+t)\ (1+2t)\ e_1^2 f_1 f_{14}}{t^4}-$$
$$\frac{\left(-1+t+3t^2\right) e_1 e_{15} f_1 f_{14}}{t^4}+\frac{e_{15}^2 f_1 f_{14}}{t^2}+\frac{\left(-1+t^2-4t^3+3t^4\right) e_1^2 f_{14}^2}{4t^6}-\frac{\left(1-t+t^3\right) e_1 e_{15} f_{14}^2}{t^5}+\frac{(-1+t)\ (1+t)\ e_{15}^2 f_{14}^2}{4t^4}-$$
$$\frac{(1+2t)\ e_1^2 f_1 f_{16}}{t^3}+\frac{2\ (1+2t)\ e_1 e_{15} f_1 f_{16}}{t^3}-\frac{(-1+2t)\ e_{15}^2 f_1 f_{16}}{t^3}-\frac{\left(1-t+2t^2\right) e_1^2 f_{14} f_{16}}{t^4}+\frac{2\ (-1+2t)\ e_1 e_{15} f_{14} f_{16}}{t^3}-$$
$$\frac{(-1+t)\ (-1+2t)\ e_{15}^2 f_{14} f_{16}}{t^4}+\frac{e_1^2 f_{16}^2}{4t^2}-\frac{e_1 e_{15} f_{16}^2}{t^2}+\frac{(-1+t)\ (-1+3t)\ e_{15}^2 f_{16}^2}{4t^4}+\frac{l_1}{t^4}-\frac{(-2+t)\ e_1 f_1 l_1}{t^4}+$$
$$\frac{e_{15} f_1 l_1}{t^3}-\frac{2\ (-1+t)\ e_1 f_{14} l_1}{t^4}+\frac{2\ e_{15} f_{14} l_1}{t^3}+\frac{2\ e_1 f_{16} l_1}{t^3}-\frac{2\ e_{15} f_{16} l_1}{t^3}-\frac{l_1^2}{t^4}+\frac{l_{13}}{t^4}+\frac{e_1 f_1 l_{14}}{t^3}-\frac{e_{15} f_1 l_{14}}{t^3}+$$
$$\frac{\left(1-t+t^2\right) e_1 f_{14} l_{14}}{t^5}-\frac{(-1+t)\ e_{15} f_{14} l_{14}}{t^4}-\frac{l_1 l_{14}}{t^4}-\frac{(1+t)\ e_1 f_1 l_{15}}{t^4}+\frac{e_{15} f_1 l_{15}}{t^3}-\frac{e_1 f_{14} l_{15}}{t^3}+\frac{(-1+t)\ e_{15} f_{14} l_{15}}{t^4}+$$
$$\frac{e_1 f_{16} l_{15}}{t^3}-\frac{(-1+t)\ e_{15} f_{16} l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}-\frac{(1+t)\ e_1 f_1 l_{16}}{t^4}+\frac{e_{15} f_1 l_{16}}{t^3}-\frac{e_1 f_{14} l_{16}}{t^3}+\frac{(-1+t)\ e_{15} f_{14} l_{16}}{t^4}+\frac{l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t},\ -l_{14}+l_{16},\ -e_1 f_1+e_{15} f_1-\frac{\left(1-t+t^2\right) e_1 f_{14}}{t^2}+\frac{(-1+t)\ e_{15} f_{14}}{t}+e_1 f_{16}-\frac{(-1+t)\ e_{15} f_{16}}{t},$$
$$-\frac{e_1 f_1}{t^3}+\frac{\left(-4+3t^2\right) e_1^2 f_1^2}{4t^4}-\frac{(1+t)\ e_1 e_{15} f_1^2}{t^3}+\frac{e_{15}^2 f_1^2}{4t^2}-\frac{(-1+t)\ e_{15} f_{14}}{t^4}+\frac{(-1+t)\ (1+2t)\ e_1^2 f_1 f_{14}}{t^4}-$$
$$\frac{\left(-1+t+3t^2\right) e_1 e_{15} f_1 f_{14}}{t^4}+\frac{e_{15}^2 f_1 f_{14}}{t^2}+\frac{\left(-1+t^2-4t^3+3t^4\right) e_1^2 f_{14}^2}{4t^6}-\frac{\left(1-t+t^3\right) e_1 e_{15} f_{14}^2}{t^5}+\frac{(-1+t)\ (1+t)\ e_{15}^2 f_{14}^2}{4t^4}-$$
$$\frac{(1+2t)\ e_1^2 f_1 f_{16}}{t^3}+\frac{2\ (1+2t)\ e_1 e_{15} f_1 f_{16}}{t^3}-\frac{(-1+2t)\ e_{15}^2 f_1 f_{16}}{t^3}-\frac{\left(1-t+2t^2\right) e_1^2 f_{14} f_{16}}{t^4}+\frac{2\ (-1+2t)\ e_1 e_{15} f_{14} f_{16}}{t^3}-$$
$$\frac{(-1+t)\ (-1+2t)\ e_{15}^2 f_{14} f_{16}}{t^4}+\frac{e_1^2 f_{16}^2}{4t^2}-\frac{e_1 e_{15} f_{16}^2}{t^2}+\frac{(-1+t)\ (-1+3t)\ e_{15}^2 f_{16}^2}{4t^4}+\frac{2\ l_1}{t^4}-\frac{(-2+t)\ e_1 f_1 l_1}{t^4}+\frac{e_{15} f_1 l_1}{t^3}-$$
$$\frac{2\ (-1+t)\ e_1 f_{14} l_1}{t^4}+\frac{2\ e_{15} f_{14} l_1}{t^3}+\frac{2\ e_1 f_{16} l_1}{t^3}-\frac{2\ e_{15} f_{16} l_1}{t^3}-\frac{l_1^2}{t^4}+\frac{e_1 f_1 l_{14}}{t^3}-\frac{e_{15} f_1 l_{14}}{t^3}+\frac{\left(1-t+t^2\right) e_1 f_{14} l_{14}}{t^5}-$$
$$\frac{(-1+t)\ e_{15} f_{14} l_{14}}{t^4}-\frac{l_1 l_{14}}{t^4}-\frac{(1+t)\ e_1 f_1 l_{15}}{t^4}+\frac{e_{15} f_1 l_{15}}{t^3}-\frac{e_1 f_{14} l_{15}}{t^3}+\frac{(-1+t)\ e_{15} f_{14} l_{15}}{t^4}+\frac{e_1 f_{16} l_{15}}{t^3}-$$
$$\frac{(-1+t)\ e_{15} f_{16} l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}-\frac{(1+t)\ e_1 f_1 l_{16}}{t^4}+\frac{e_{15} f_1 l_{16}}{t^3}-\frac{e_1 f_{14} l_{16}}{t^3}+\frac{(-1+t)\ e_{15} f_{14} l_{16}}{t^4}+\frac{l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1}{t},\ -l_1+l_{16},\ -\frac{\left(1+t^2\right) e_1 f_1}{t^2}+e_{15} f_1+e_1 f_{16}-\frac{(-1+t)\ e_{15} f_{16}}{t},$$
$$-\frac{e_{15} f_1}{t^3}+\frac{\left(-5-8t+3t^4\right) e_1^2 f_1^2}{4t^6}-\frac{\left(1+t+t^2\right) e_1 e_{15} f_1^2}{t^4}+\frac{e_{15}^2 f_1^2}{4t^2}-\frac{\left(2+t+2t^2\right) e_1^2 f_1 f_{16}}{t^4}+$$
$$\frac{2\left(1+t+2t^2\right) e_1 e_{15} f_1 f_{16}}{t^4}-\frac{(-1+2t)\ e_{15}^2 f_1 f_{16}}{t^3}+\frac{e_1^2 f_{16}^2}{4t^2}-\frac{e_1 e_{15} f_{16}^2}{t^2}+\frac{(-1+t)\ (-1+3t)\ e_{15}^2 f_{16}^2}{4t^4}+$$
$$\frac{2\ l_1}{t^4}-\frac{(-3+t)\ (1+t)\ e_1 f_1 l_1}{t^5}+\frac{e_{15} f_1 l_1}{t^3}+\frac{2\ e_1 f_{16} l_1}{t^3}-\frac{2\ e_{15} f_{16} l_1}{t^3}-\frac{2\ l_1^2}{t^4}-\frac{\left(1+t+t^2\right) e_1 f_1 l_{15}}{t^5}+$$
$$\frac{e_{15} f_1 l_{15}}{t^3}+\frac{e_1 f_{16} l_{15}}{t^3}-\frac{(-1+t)\ e_{15} f_{16} l_{15}}{t^4}+\frac{l_1 l_{15}}{t^4}-\frac{\left(1+t+t^2\right) e_1 f_1 l_{16}}{t^5}+\frac{e_{15} f_1 l_{16}}{t^3}+\frac{l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1-t+t^2}{t},\ -l_1+l_{16},\ -\frac{\left(1-t+t^2\right)e_1 f_1}{t^2}+\frac{\left(1-t+t^2\right)e_1 f_{16}}{t},$$
$$\frac{(-1+t)\left(1-t+t^2\right)^2\left(1-t+2t^2\right)}{t^3}+\frac{\left(-1-t+t^2\right)\left(1-t+t^2\right)^3 e_1 f_1}{t^5}-\frac{\left(1-t+t^2\right)^3\left(1+3t+t^2\right)e_1^2 f_1^2}{4t^6}-$$
$$\frac{2\left(1-t+t^2\right)^3 e_1 f_{16}}{t^2}+\frac{(-1+2t)\left(1-t+t^2\right)^3 e_1^2 f_1 f_{16}}{t^5}-\frac{\left(1-t+t^2\right)^3\left(-1+t+3t^2\right)e_1^2 f_{16}^2}{4t^4}-$$
$$\frac{(-1+t)(2+t)\left(1-t+t^2\right)^3 l_1}{t^4}+\frac{\left(1-t+t^2\right)^3 e_1 f_1 l_1}{t^5}+\frac{\left(1-t+t^2\right)^3\left(1-2t+2t^2\right)e_1 f_{16} l_1}{t^4}-\frac{\left(1-t+t^2\right)^4 l_1^2}{t^4}-$$
$$\frac{(-1+t)\left(1-t+t^2\right)^3 l_{16}}{t^3}-\frac{\left(1-t+t^2\right)^3 e_1 f_1 l_{16}}{t^5}-\frac{(-1+t)\left(1-t+t^2\right)^3 e_1 f_{16} l_{16}}{t^3}+\frac{\left(1-t+t^2\right)^4 l_1 l_{16}}{t^4}\Big]$$

131

» $$\mathbb{E}\Big[\frac{1-t+t^2}{t},\ 0,\ 0,\ \frac{(-1+t)\left(1-t+t^2\right)^2\left(1-t+2t^2\right)}{t^3}-\frac{2(1+t)\left(1-t+t^2\right)^3 e_1 f_1}{t^4}-\frac{2(-1+t)(1+t)\left(1-t+t^2\right)^3 l_1}{t^4}\Big]$$

131

$$\mathbb{E}\Big[\frac{1-t+t^2}{t},\ 0,\ 0,\ \frac{\left(-1+t\right)\left(1-t+t^2\right)^2\left(1-t+2t^2\right)}{t^3}-\frac{2\left(1+t\right)\left(1-t+t^2\right)^3 e_1 f_1}{t^4}-\frac{2\left(-1+t\right)\left(1+t\right)\left(1-t+t^2\right)^3 l_1}{t^4}\Big]$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
   DeleteFile[temp],
   (* else *) Print["Exporting " <> fname <> "..."];
   If[exists, DeleteFile[fname]];
   RenameFile[temp, fname]
  ];
  fname
 ]

SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
TagProperties[_] := {};
TagProperties["131"] = {PageWidth → 3.2/0.66};
Options[CellExport] = {
   PageWidth → 4/0.66, CellFilter → Identity, ExportDirectory → "Snips",
   ExportBaseFilename → Automatic, ExportFormat → ".pdf", ExportOptions → {}, Split → False
  };
CellExport[tag_String, opts___Rule] := CellExport[
   NotebookGet[EvaluationNotebook[]],
   tag, opts
  ];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
   {cells, cell, filename, format},
   filename = FileNameJoin[{
      OptionValue[ExportDirectory] /. Automatic → Directory[],
      OptionValue[ExportBaseFilename] /. Automatic → tag
     }];
   format = OptionValue[ExportFormat];
   cells = OptionValue[CellFilter][Cases[
      nb, c_Cell /; FreeQ[List @@ c, Cell] && ! FreeQ[c, CellTags → tag],
      Infinity
     ]];
```

```
    If[! OptionValue[Split],
     If[Length[cells] ≥ 1,
      If[Length[cells] == 1,
       cells = Join[First[cells],
         Cell[PageWidth → 1.2 × 72 OptionValue[PageWidth], Background → {White, Opacity[0]}]],
       cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
      ];
      ConditionalExport[
       filename <> format, cells,
       ImageResolution → 300,
       OptionValue[ExportOptions]
      ]
     ],
     k = 0;
     Table[
      ++k;
      ConditionalExport[
       filename <> "-" <> ToString[k] <> format,
       Append[cell, PageWidth → 72 OptionValue[PageWidth]],
       ImageResolution → 300,
       OptionValue[ExportOptions]
      ],
      {cell, cells}
     ]
    ]
   ];

ExportCells := (
    nb = NotebookGet[EvaluationNotebook[]];
    tags = Cases[nb, (CellTags → tag_String) :↦ tag, Infinity] // Union;
    Print[tags];
    CellExport /@ tags;
    Print["Done."]
   );
```

```
(*ExportCells*)
```