

Pensieve header: NOE-1 demo for GWU-1612, using elf conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\GWU-1612"];
```

NOE-It

```
CF[ $\mathcal{E}$ ] := Module[{vars = Union@Cases[ $\mathcal{E}$ , e_ | l_ | f_,  $\infty$ ]},
  If[vars === {}, Factor[ $\mathcal{E}$ ],
    Total[CoefficientRules[ $\mathcal{E}$ , vars] /. (p_ -> c_) -> Factor[c] Times@@(vars^p)]
  ]];
```

```
Format[ $\mathcal{E}$ _E] := "E" @@ (CF /@  $\mathcal{E}$ );
Format[0[spec_List,  $\mathcal{E}$ _E]] := "0" [StringJoin@@((x -> ToString[x, StandardForm]) /@ spec),  $\mathcal{E}$ ];
```

$$\Delta[k_] := \left((t-1) (2(\alpha\beta + \delta\mu)^2 - \alpha^2\beta^2) - 4e_k l_k f_k \delta^2 \mu^2 - \delta(1+\mu)(f_k^2 \alpha^2 + e_k^2 \beta^2) - e_k^2 f_k^2 \delta^3 (1+3\mu) - 2(\alpha\beta + 2\delta\mu + e_k f_k \delta^2 (1+2\mu) + 2l_k \delta \mu^2)(f_k \alpha + e_k \beta) - 4(l_k \mu^2 + e_k f_k \delta(1+\mu))(\alpha\beta + \delta\mu)(1+t) \right) / 4;$$

$\Delta[k]$ // TeXForm

$$\frac{1}{4} (t+1) \left(-\delta (\mu+1) \left(\beta^2 e_k^2 + \alpha^2 f_k^2 \right) + \delta^3 (-3\mu - e_k^2 f_k^2 - 2 \left(\beta e_k + \alpha f_k \right) \delta (\mu + \delta^2 (2\mu - e_k f_k + 2 \delta \mu^2 l_k)) - 4 (\alpha \beta + \delta \mu) \delta (\mu + 1) e_k f_k + \mu l_k \right) - 4 \delta^2 \mu^2 e_k l_k + (t-1) \left(2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2 \right) \right) / 4$$

```
R_{i,j}^+ := E[1, l_j, e_i f_j, e_i l_i f_j + l_i l_j + e_i^2 f_j^2 / 4];
R_{i,j}^- := E[1, -l_j, -t^{-1} e_i f_j, t^{-1} e_i l_j f_j - l_i l_j - t^{-2} e_i^2 f_j^2 / 4];
(ur_{i_} := E[1, 0, 0, l_i]; nr_{i_} := E[1, 0, 0, -l_i];)
```

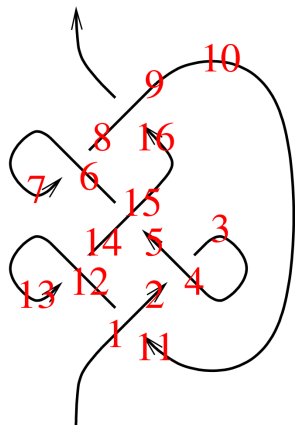
```
DP_{x->D_{\alpha}, y->D_{\beta}}[P_][f_] := (* means P[\partial_{\alpha}, \partial_{\beta}][f] *)
  Total[CoefficientRules[P, {x, y}] /. ({m_, n_} -> c_) -> c D[f, {\alpha, m}, {\beta, n}]]
```

```
CF[ $\mathcal{E}$ _E] := Expand /@ Together /@  $\mathcal{E}$ ;
E /: E[1, L1_, Q1_, P1_] E[1, L2_, Q2_, P2_] := CF@E[1, L1 + L2, Q1 + Q2, P1 + P2];
```

```
N_{f_i e_j -> k_}[E[\omega_, L_, Q_, P_]] := With[{q = ((1-t) \alpha \beta + \beta e_k + \alpha f_k + \delta e_k f_k) / \mu}, CF[
  E[\mu \omega, L, \mu \omega q + \mu (Q /. f_i | e_j -> \theta), \mu^4 e^{-q} DP_{f_i -> D_{\alpha}, e_j -> D_{\beta}}[P][e^q] + \omega^4 \Delta[k]] /. \mu -> 1 + (t-1) \delta /.
  {\alpha -> \omega^{-1} (\partial_{f_i} Q /. e_j -> \theta), \beta -> \omega^{-1} (\partial_{e_j} Q /. f_i -> \theta), \delta -> \omega^{-1} \partial_{f_i, e_j} Q}]]];
```

```
N_{l_j (x:e|f)_i -> k_}[E[\omega_, L_, Q_, P_]] := With[{lambda = \partial_{l_j} L, \alpha = \partial_{x_i} Q, q = e^y \beta x_k + \gamma l_k}, CF[
  E[\omega, L /. l_j -> l_k, t^lambda \alpha x_k + (Q /. x_i -> \theta), e^{-q} DP_{l_j -> D_{\gamma}, x_i -> D_{\beta}}[P][e^q] /. {\beta -> \alpha / \omega, \gamma -> lambda Log[t]}]
  ]];
```

```
m_{i,j -> k_}[Z_E] := Module[{x, z},
  CF[(Z // N_{f_i e_j -> x} // N_{l_i e_x -> x} // N_{f_x l_j -> x}) /. Z_{-i|j|x} -> z_k]]
```



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$$z1 = 0 \left[\{e_1, l_1, f_1, e_2, l_2, f_2, e_3, l_3, f_3, e_4, l_4, f_4, e_5, l_5, f_5, e_6, l_6, f_6, e_7, l_7, f_7, e_8, l_8, f_8, e_9, l_9, f_9, e_{10}, l_{10}, f_{10}, e_{11}, l_{11}, f_{11}, e_{12}, l_{12}, f_{12}, e_{13}, l_{13}, f_{13}, e_{14}, l_{14}, f_{14}, e_{15}, l_{15}, f_{15}, e_{16}, l_{16}, f_{16}\}, R_{1,11}^+, R_{4,2}^-, nr_3, R_{15,5}^+, R_{6,8}^-, ur_7, R_{9,16}^+, nr_{10}, R_{12,14}^-, ur_{13} \right]$$

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$$0 \left[e_1 l_1 f_1 e_2 l_2 f_2 e_3 l_3 f_3 e_4 l_4 f_4 e_5 l_5 f_5 e_6 l_6 f_6 e_7 l_7 f_7 e_8 l_8 f_8 e_9 l_9 f_9 e_{10} l_{10} f_{10} e_{11} l_{11} f_{11} e_{12} l_{12} f_{12} e_{13} l_{13} f_{13} e_{14} l_{14} f_{14} e_{15} l_{15} f_{15} e_{16} l_{16} f_{16}, E \left[1, -l_2 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_2}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, -\frac{e_4^2 f_2^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + e_1 f_{11} l_1 + \frac{e_4 f_2 l_2}{t} - l_3 - l_2 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16} \right] \right]$$

$$z2 = \text{Last}[z1]$$

$$E \left[1, -l_2 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_2}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, -\frac{e_4^2 f_2^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + e_1 f_{11} l_1 + \frac{e_4 f_2 l_2}{t} - l_3 - l_2 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16} \right]$$

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$$(\text{Do}[z2 = \text{Echo}[z2 // m_{1,k+1}], \{k, 2, 16\}]; z2)$$

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$$\gg E \left[1, -l_1 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_1}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, -\frac{e_4^2 f_1^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 + \frac{e_4 f_1 l_1}{t} + e_1 f_{11} l_1 - l_3 - l_1 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16} \right]$$

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$$\gg E \left[1, -l_1 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_4 f_1}{t} + e_{15} f_5 - \frac{e_6 f_8}{t} + e_1 f_{11} - \frac{e_{12} f_{14}}{t} + e_9 f_{16}, \frac{e_4 f_1}{t} - \frac{e_4^2 f_1^2}{4 t^2} + \frac{1}{4} e_{15}^2 f_5^2 - \frac{e_6^2 f_8^2}{4 t^2} + \frac{1}{4} e_1^2 f_{11}^2 - \frac{e_{12}^2 f_{14}^2}{4 t^2} + \frac{1}{4} e_9^2 f_{16}^2 - l_1 + \frac{e_4 f_1 l_1}{t} + e_1 f_{11} l_1 - l_1 l_4 + l_7 + \frac{e_6 f_8 l_8}{t} - l_6 l_8 + e_9 f_{16} l_9 - l_{10} + l_1 l_{11} + l_{13} + \frac{e_{12} f_{14} l_{14}}{t} - l_{12} l_{14} + e_{15} f_5 l_{15} + l_5 l_{15} + l_9 l_{16} \right]$$

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$$\gg E \left[\frac{1}{t}, -l_1 + l_5 - l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_1 f_1}{t^2} + \frac{e_{15} f_5}{t} - \frac{e_6 f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, -\frac{e_1^2 f_1^2}{4 t^6} + \frac{e_{15}^2 f_5^2}{4 t^4} - \frac{e_6^2 f_8^2}{4 t^6} - \frac{e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} + \frac{e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} + \frac{e_6 f_8 l_8}{t^5} - \frac{l_6 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^5} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} + \frac{e_{15} f_5 l_{15}}{t^4} + \frac{l_5 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right]$$

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$$\gg E \left[\frac{1}{t}, -l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_1 f_1}{t} + \frac{e_{15} f_1}{t} - \frac{e_6 f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, -\frac{e_1^2 f_1^2}{4 t^4} + \frac{e_{15}^2 f_1^2}{4 t^4} - \frac{e_6^2 f_8^2}{4 t^6} - \frac{e_1^2 f_1 f_{11}}{t^4} + \frac{e_1^2 f_{11}^2}{4 t^4} - \frac{e_{12}^2 f_{14}^2}{4 t^6} + \frac{e_9^2 f_{16}^2}{4 t^4} + \frac{l_1}{t^4} + \frac{e_1 f_1 l_1}{t^4} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} + \frac{e_6 f_8 l_8}{t^5} - \frac{l_6 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^4} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right]$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_1 f_1}{t} + \frac{e_{15} f_1}{t} - \frac{e_1 f_8}{t} + \frac{(-1+t) e_{15} f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & - \frac{e_1^2 f_1^2}{4t^4} + \frac{e_{15}^2 f_1^2}{4t^4} - \frac{(1+t) e_1 e_{15} f_1 f_8}{t^5} + \frac{e_{15}^2 f_1 f_8}{t^4} - \frac{e_1^2 f_8^2}{4t^4} - \frac{e_1 e_{15} f_8^2}{t^5} + \frac{(-1+t)(1+t) e_{15}^2 f_8^2}{4t^6} - \\ & \frac{e_1^2 f_1 f_{11}}{t^4} - \frac{e_1^2 f_8 f_{11}}{t^4} + \frac{e_1^2 f_{11}^2}{4t^4} - \frac{e_{12}^2 f_{14}^2}{4t^6} + \frac{e_9^2 f_{16}^2}{4t^4} + \frac{l_1}{t^4} + \frac{e_1 f_1 l_1}{t^4} + \frac{(1+t) e_{15} f_8 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \frac{l_7}{t^4} + \\ & \frac{e_1 f_1 l_8}{t^4} - \frac{e_{15} f_1 l_8}{t^4} + \frac{e_1 f_8 l_8}{t^4} - \frac{(-1+t) e_{15} f_8 l_8}{t^5} - \frac{l_1 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^4} - \frac{e_1 f_8 l_{11}}{t^4} + \frac{l_1 l_{11}}{t^4} + \\ & \left. \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^4} - \frac{e_1 f_8 l_{15}}{t^4} + \frac{(-1+t) e_{15} f_8 l_{15}}{t^5} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_8 + l_{11} - l_{14} + l_{16}, -\frac{e_1 f_1}{t} + \frac{e_{15} f_1}{t} - \frac{e_1 f_8}{t} + \frac{(-1+t) e_{15} f_8}{t^2} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & - \frac{e_1 f_1}{t^4} + \frac{e_{15} f_1}{t^4} - \frac{e_1^2 f_1^2}{4t^4} + \frac{e_{15}^2 f_1^2}{4t^4} - \frac{(1+t) e_1 e_{15} f_1 f_8}{t^5} + \frac{e_{15}^2 f_1 f_8}{t^4} - \frac{e_1^2 f_8^2}{4t^4} - \frac{e_1 e_{15} f_8^2}{t^5} + \frac{(-1+t)(1+t) e_{15}^2 f_8^2}{4t^6} - \\ & \frac{e_1^2 f_1 f_{11}}{t^4} - \frac{e_1^2 f_8 f_{11}}{t^4} + \frac{e_1^2 f_{11}^2}{4t^4} - \frac{e_{12}^2 f_{14}^2}{4t^6} + \frac{e_9^2 f_{16}^2}{4t^4} + \frac{2l_1}{t^4} + \frac{e_1 f_1 l_1}{t^4} + \frac{(1+t) e_{15} f_8 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{l_1^2}{t^4} + \\ & \frac{e_1 f_1 l_8}{t^4} - \frac{e_{15} f_1 l_8}{t^4} + \frac{e_1 f_8 l_8}{t^4} - \frac{(-1+t) e_{15} f_8 l_8}{t^5} - \frac{l_1 l_8}{t^4} + \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{e_1 f_1 l_{11}}{t^4} - \frac{e_1 f_8 l_{11}}{t^4} + \frac{l_1 l_{11}}{t^4} + \\ & \left. \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^4} - \frac{e_1 f_8 l_{15}}{t^4} + \frac{(-1+t) e_{15} f_8 l_{15}}{t^5} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_1 + l_{11} - l_{14} + l_{16}, -\frac{(1+t) e_1 f_1}{t^2} + \frac{e_{15} f_1}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + \frac{e_9 f_{16}}{t}, \right. \\ & - \frac{(5+4t+t^2) e_1^2 f_1^2}{4t^6} + \frac{e_{15}^2 f_1^2}{4t^4} - \frac{(1+t) e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4t^4} - \frac{e_{12}^2 f_{14}^2}{4t^6} + \frac{e_9^2 f_{16}^2}{4t^4} + \frac{2l_1}{t^4} + \frac{(3+t) e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} - \frac{2l_1^2}{t^4} + \\ & \left. \frac{e_9 f_{16} l_9}{t^4} - \frac{l_{10}}{t^4} - \frac{(1+t) e_1 f_1 l_{11}}{t^5} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^5} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} + \frac{l_9 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_1 + l_{11} - l_{14} + l_{16}, -\frac{(1+t) e_1 f_1}{t^2} + \frac{e_{15} f_1}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & - \frac{(5+4t+t^2) e_1^2 f_1^2}{4t^6} + \frac{e_{15}^2 f_1^2}{4t^4} - \frac{(1+t) e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4t^4} - \frac{e_{12}^2 f_{14}^2}{4t^6} - \frac{(1+t) e_1^2 f_1 f_{16}}{t^4} + \frac{(2+3t) e_1 e_{15} f_1 f_{16}}{t^4} - \\ & \frac{(-1+2t) e_{15}^2 f_1 f_{16}}{t^4} + \frac{e_1^2 f_{11} f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t)(-1+3t) e_{15}^2 f_{16}^2}{4t^4} + \frac{2l_1}{t^4} + \frac{(3+t) e_1 f_1 l_1}{t^5} + \\ & \frac{e_1 f_{11} l_1}{t^4} + \frac{e_1 f_{16} l_1}{t^3} - \frac{2e_{15} f_{16} l_1}{t^3} - \frac{2l_1^2}{t^4} - \frac{l_{10}}{t^4} - \frac{(1+t) e_1 f_1 l_{11}}{t^5} + \frac{e_1 f_{16} l_{11}}{t^3} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \\ & \left. \frac{(1+t) e_1 f_1 l_{15}}{t^5} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_1 + l_{11} - l_{14} + l_{16}, -\frac{(1+t) e_1 f_1}{t^2} + \frac{e_{15} f_1}{t} + \frac{e_1 f_{11}}{t} - \frac{e_{12} f_{14}}{t^2} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & \frac{(1+t) e_1 f_1}{t^5} - \frac{e_{15} f_1}{t^4} - \frac{(5+4t+t^2) e_1^2 f_1^2}{4t^6} + \frac{e_{15}^2 f_1^2}{4t^4} - \frac{(1+t) e_1^2 f_1 f_{11}}{t^5} + \frac{e_1^2 f_{11}^2}{4t^4} - \frac{e_{12}^2 f_{14}^2}{4t^6} - \frac{(1+t) e_1^2 f_1 f_{16}}{t^4} + \\ & \frac{(2+3t) e_1 e_{15} f_1 f_{16}}{t^4} - \frac{(-1+2t) e_{15}^2 f_1 f_{16}}{t^4} + \frac{e_1^2 f_{11} f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t)(-1+3t) e_{15}^2 f_{16}^2}{4t^4} + \frac{l_1}{t^4} + \\ & \frac{(3+t) e_1 f_1 l_1}{t^5} + \frac{e_1 f_{11} l_1}{t^4} + \frac{e_1 f_{16} l_1}{t^3} - \frac{2e_{15} f_{16} l_1}{t^3} - \frac{2l_1^2}{t^4} - \frac{(1+t) e_1 f_1 l_{11}}{t^5} + \frac{e_1 f_{16} l_{11}}{t^3} + \frac{l_1 l_{11}}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \\ & \left. \frac{l_{12} l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^5} + \frac{e_{15} f_1 l_{15}}{t^4} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_{14} + l_{16}, -e_1 f_1 + e_{15} f_1 - \frac{e_{12} f_{14}}{t^2} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & - \frac{e_{15} f_1}{t^3} + \frac{(-4+3t^2) e_1^2 f_1^2}{4t^4} - \frac{(1+t) e_1 e_{15} f_1^2}{t^3} + \frac{e_{15}^2 f_1^2}{4t^2} - \frac{e_{12}^2 f_{14}^2}{4t^6} - \frac{(1+2t) e_1^2 f_1 f_{16}}{t^3} + \\ & \frac{2(1+2t) e_1 e_{15} f_1 f_{16}}{t^3} - \frac{(-1+2t) e_{15}^2 f_1 f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t)(-1+3t) e_{15}^2 f_{16}^2}{4t^4} + \frac{l_1}{t^4} - \\ & \frac{(-2+t) e_1 f_1 l_1}{t^4} + \frac{e_{15} f_1 l_1}{t^3} + \frac{2e_1 f_{16} l_1}{t^3} - \frac{2e_{15} f_{16} l_1}{t^3} - \frac{l_1^2}{t^4} + \frac{l_{13}}{t^4} + \frac{e_{12} f_{14} l_{14}}{t^5} - \frac{l_{12} l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^4} + \\ & \left. \frac{e_{15} f_1 l_{15}}{t^3} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^4} + \frac{e_{15} f_1 l_{16}}{t^3} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_{14} + l_{16}, -e_1 f_1 + e_{15} f_1 - \frac{(1-t+t^2) e_1 f_{14}}{t^2} + \frac{(-1+t) e_{15} f_{14}}{t} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & - \frac{e_{15} f_1}{t^3} + \frac{(-4+3t^2) e_1^2 f_1^2}{4t^4} - \frac{(1+t) e_1 e_{15} f_1^2}{t^3} + \frac{e_{15}^2 f_1^2}{4t^2} - \frac{(-1+t) e_{15} f_{14}}{t^4} + \frac{(-1+t)(1+2t) e_1^2 f_1 f_{14}}{t^4} - \\ & \frac{(-1+t+3t^2) e_1 e_{15} f_1 f_{14}}{t^4} + \frac{e_{15}^2 f_1 f_{14}}{t^2} + \frac{(-1+t^2-4t^3+3t^4) e_1^2 f_{14}^2}{4t^6} - \frac{(1-t+t^3) e_1 e_{15} f_{14}^2}{t^5} + \frac{(-1+t)(1+t) e_{15}^2 f_{14}^2}{4t^4} - \\ & \frac{(1+2t) e_1^2 f_1 f_{16}}{t^3} + \frac{2(1+2t) e_1 e_{15} f_1 f_{16}}{t^3} - \frac{(-1+2t) e_{15}^2 f_1 f_{16}}{t^3} - \frac{(1-t+2t^2) e_1^2 f_{14} f_{16}}{t^4} + \frac{2(-1+2t) e_1 e_{15} f_{14} f_{16}}{t^3} - \\ & \frac{(-1+t)(-1+2t) e_{15}^2 f_{14} f_{16}}{t^4} + \frac{e_1^2 f_{16}^2}{4t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t)(-1+3t) e_{15}^2 f_{16}^2}{4t^4} + \frac{l_1}{t^4} - \frac{(-2+t) e_1 f_1 l_1}{t^4} + \\ & \frac{e_{15} f_1 l_1}{t^3} - \frac{2(-1+t) e_1 f_{14} l_1}{t^4} + \frac{2e_{15} f_{14} l_1}{t^3} + \frac{2e_1 f_{16} l_1}{t^3} - \frac{2e_{15} f_{16} l_1}{t^3} - \frac{l_1^2}{t^4} + \frac{l_{13}}{t^4} + \frac{e_1 f_1 l_{14}}{t^3} - \frac{e_{15} f_1 l_{14}}{t^3} + \\ & \frac{(1-t+t^2) e_1 f_{14} l_{14}}{t^5} - \frac{(-1+t) e_{15} f_{14} l_{14}}{t^4} - \frac{l_1 l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^3} - \frac{e_1 f_{14} l_{15}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{15}}{t^4} + \\ & \left. \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^4} + \frac{e_{15} f_1 l_{16}}{t^3} - \frac{e_1 f_{14} l_{16}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_{14} + l_{16}, -e_1 f_1 + e_{15} f_1 - \frac{(1-t+t^2) e_1 f_{14}}{t^2} + \frac{(-1+t) e_{15} f_{14}}{t} + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & - \frac{e_1 f_1}{t^3} + \frac{(-4+3t^2) e_1^2 f_1^2}{4t^4} - \frac{(1+t) e_1 e_{15} f_1^2}{t^3} + \frac{e_{15}^2 f_1^2}{4t^2} - \frac{(-1+t) e_{15} f_{14}}{t^4} + \frac{(-1+t)(1+2t) e_1^2 f_1 f_{14}}{t^4} - \\ & \frac{(-1+t+3t^2) e_1 e_{15} f_1 f_{14}}{t^4} + \frac{e_{15}^2 f_1 f_{14}}{t^2} + \frac{(-1+t^2-4t^3+3t^4) e_1^2 f_{14}^2}{4t^6} - \frac{(1-t+t^3) e_1 e_{15} f_{14}^2}{t^5} + \frac{(-1+t)(1+t) e_{15}^2 f_{14}^2}{4t^4} - \\ & \frac{(1+2t) e_1^2 f_1 f_{16}}{t^3} + \frac{2(1+2t) e_1 e_{15} f_1 f_{16}}{t^3} - \frac{(-1+2t) e_{15}^2 f_1 f_{16}}{t^3} - \frac{(1-t+2t^2) e_1^2 f_{14} f_{16}}{t^4} + \frac{2(-1+2t) e_1 e_{15} f_{14} f_{16}}{t^3} - \\ & \frac{(-1+t)(-1+2t) e_{15}^2 f_{14} f_{16}}{t^4} + \frac{e_1^2 f_{16}^2}{4t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t)(-1+3t) e_{15}^2 f_{16}^2}{4t^4} + \frac{2l_1}{t^4} - \frac{(-2+t) e_1 f_1 l_1}{t^4} + \frac{e_{15} f_1 l_1}{t^3} - \\ & \frac{2(-1+t) e_1 f_{14} l_1}{t^4} + \frac{2e_{15} f_{14} l_1}{t^3} + \frac{2e_1 f_{16} l_1}{t^3} - \frac{2e_{15} f_{16} l_1}{t^3} - \frac{l_1^2}{t^4} + \frac{e_1 f_1 l_{14}}{t^3} - \frac{e_{15} f_1 l_{14}}{t^3} + \frac{(1-t+t^2) e_1 f_{14} l_{14}}{t^5} - \\ & \frac{(-1+t) e_{15} f_{14} l_{14}}{t^4} - \frac{l_1 l_{14}}{t^4} - \frac{(1+t) e_1 f_1 l_{15}}{t^4} + \frac{e_{15} f_1 l_{15}}{t^3} - \frac{e_1 f_{14} l_{15}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{15}}{t^4} + \frac{e_1 f_{16} l_{15}}{t^3} - \\ & \left. \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t) e_1 f_1 l_{16}}{t^4} + \frac{e_{15} f_1 l_{16}}{t^3} - \frac{e_1 f_{14} l_{16}}{t^3} + \frac{(-1+t) e_{15} f_{14} l_{16}}{t^4} + \frac{l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1}{t}, -l_1 + l_{16}, -\frac{(1+t^2) e_1 f_1}{t^2} + e_{15} f_1 + e_1 f_{16} - \frac{(-1+t) e_{15} f_{16}}{t}, \right. \\ & - \frac{e_{15} f_1}{t^3} + \frac{(-5-8t+3t^4) e_1^2 f_1^2}{4t^6} - \frac{(1+t+t^2) e_1 e_{15} f_1^2}{t^4} + \frac{e_{15}^2 f_1^2}{4t^2} - \frac{(2+t+2t^2) e_1^2 f_1 f_{16}}{t^4} + \\ & \frac{2(1+t+2t^2) e_1 e_{15} f_1 f_{16}}{t^4} - \frac{(-1+2t) e_{15}^2 f_1 f_{16}}{t^3} + \frac{e_1^2 f_{16}^2}{4t^2} - \frac{e_1 e_{15} f_{16}^2}{t^2} + \frac{(-1+t)(-1+3t) e_{15}^2 f_{16}^2}{4t^4} + \\ & \frac{2l_1}{t^4} - \frac{(-3+t)(1+t) e_1 f_1 l_1}{t^5} + \frac{e_{15} f_1 l_1}{t^3} + \frac{2e_1 f_{16} l_1}{t^3} - \frac{2e_{15} f_{16} l_1}{t^3} - \frac{2l_1^2}{t^4} - \frac{(1+t+t^2) e_1 f_1 l_{15}}{t^5} + \\ & \frac{e_{15} f_1 l_{15}}{t^3} + \frac{e_1 f_{16} l_{15}}{t^3} - \frac{(-1+t) e_{15} f_{16} l_{15}}{t^4} + \frac{l_1 l_{15}}{t^4} - \frac{(1+t+t^2) e_1 f_1 l_{16}}{t^5} + \frac{e_{15} f_1 l_{16}}{t^3} + \frac{l_1 l_{16}}{t^4} \left. \right] \end{aligned}$$

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$$\begin{aligned} & \gg E \left[\frac{1-t+t^2}{t}, -l_1 + l_{16}, -\frac{(1-t+t^2) e_1 f_1}{t^2} + \frac{(1-t+t^2) e_1 f_{16}}{t}, \right. \\ & \frac{(-1+t)(1-t+t^2)^2(1-t+2t^2)}{t^3} + \frac{(-1-t+t^2)(1-t+t^2)^3 e_1 f_1}{t^5} - \frac{(1-t+t^2)^3(1+3t+t^2) e_1^2 f_1^2}{4t^6} - \\ & \frac{2(1-t+t^2)^3 e_1 f_{16}}{t^2} + \frac{(-1+2t)(1-t+t^2)^3 e_1^2 f_1 f_{16}}{t^5} - \frac{(1-t+t^2)^3(-1+t+3t^2) e_1^2 f_{16}^2}{4t^4} - \\ & \frac{(-1+t)(2+t)(1-t+t^2)^3 l_1}{t^4} + \frac{(1-t+t^2)^3 e_1 f_1 l_1}{t^5} + \frac{(1-t+t^2)^3(1-2t+2t^2) e_1 f_{16} l_1}{t^4} - \frac{(1-t+t^2)^4 l_1^2}{t^4} - \\ & \left. \frac{(-1+t)(1-t+t^2)^3 l_{16}}{t^3} - \frac{(1-t+t^2)^3 e_1 f_1 l_{16}}{t^5} - \frac{(-1+t)(1-t+t^2)^3 e_1 f_{16} l_{16}}{t^3} + \frac{(1-t+t^2)^4 l_1 l_{16}}{t^4} \right] \end{aligned}$$

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$$\gg E \left[\frac{1-t+t^2}{t}, 0, 0, \frac{(-1+t)(1-t+t^2)^2(1-t+2t^2)}{t^3} - \frac{2(1+t)(1-t+t^2)^3 e_1 f_1}{t^4} - \frac{2(-1+t)(1+t)(1-t+t^2)^3 l_1}{t^4} \right]$$

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$$E \left[\frac{1-t+t^2}{t}, 0, 0, \frac{(-1+t)(1-t+t^2)^2(1-t+2t^2)}{t^3} - \frac{2(1+t)(1-t+t^2)^3 e_1 f_1}{t^4} - \frac{2(-1+t)(1+t)(1-t+t^2)^3 l_1}{t^4} \right]$$

Earliest computation of the above (from NOE-Demo.nb, on this folder) gives:

$$E\left[\frac{1-t+t^2}{t}, 0, 0, \frac{(-1+t)(1-t+t^2)^2(1-t+2t^2)}{t^3} - \frac{2(1+t)(1-t+t^2)^3 e_1 f_1}{t^4} - \frac{2(-1+t)(1+t)(1-t+t^2)^3 l_1}{t^4}\right]$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
    DeleteFile[temp],
    (* else *) Print["Exporting " <> fname <> "..."];
    If[exists, DeleteFile[fname]];
    RenameFile[temp, fname]
  ];
  fname
];

SetOptions[$FrontEndSession, PrintingStyleEnvironment -> "Working"];
TagProperties[_] := {};
TagProperties["131"] = {PageWidth -> 3.2/0.66};
Options[CellExport] = {
  PageWidth -> 4/0.66, CellFilter -> Identity, ExportDirectory -> "Snips",
  ExportBaseFilename -> Automatic, ExportFormat -> ".pdf", ExportOptions -> {}, Split -> False
};
CellExport[tag_String, opts___Rule] := CellExport[
  NotebookGet[EvaluationNotebook[]],
  tag, opts
];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{
    OptionValue[ExportDirectory] /. Automatic -> Directory[],
    OptionValue[ExportBaseFilename] /. Automatic -> tag
  }];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
    nb, c_Cell /; FreeQ[List@@c, Cell] && !FreeQ[c, CellTags -> tag],
    Infinity
  ]];
  If[!OptionValue[Split],
    If[Length[cells] >= 1,
      If[Length[cells] == 1,
        cells = Join[First[cells],
          Cell[PageWidth -> 1.2 * 72 OptionValue[PageWidth], Background -> {White, Opacity[0]}]],
        cells = Cell[CellGroup[cells], PageWidth -> 72 OptionValue[PageWidth]]
      ];
    ConditionalExport[
      filename <> format, cells,
      ImageResolution -> 300,
      OptionValue[ExportOptions]
    ]
  ]
];
```

```
],
k = 0;
Table[
  ++k;
  ConditionalExport[
    filename <> "-" <> ToString[k] <> format,
    Append[cell, PageWidth → 72 OptionValue[PageWidth]],
    ImageResolution → 300,
    OptionValue[ExportOptions]
  ],
  {cell, cells}
]
];

ExportCells := (
  nb = NotebookGet[EvaluationNotebook[]];
  tags = Cases[nb, (CellTags → tag_String) ⇒ tag, Infinity] // Union;
  Print[tags];
  CellExport /@ tags;
  Print["Done."]
);
```

(*ExportCells*)