Pensieve header: Compositions in generic DoPeGDO.

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In[•]:= RM_{A_→B_} := Module[{vs},
          vs = Table[ζ_i, {i, A}] ⋃ Table[z_i, {i, B}];
          M_{A→B}[1, Sum[RandomInteger[{-3, 3}] vi vj, {vi, vs}, {vj, vs}], 0]
        ];
M_{A_→B_}[ω1_, Q1_, P1_] ≡ M_{A_→B_}[ω2_, Q2_, P2_] := Simplify[ω1 == ω2 ∧ Q1 == Q2 ∧ P1 == P2];
M_{A_→B_}[ω1_, Q1_, P1_] // M_{B_→C_}[ω2_, Q2_, P2_] :=
 Module[{E1, F1, G1, E2, F2, G2, ζA, zC, I},
  E1 = Table[∂_{ζ_i,z_j} Q1, {i, A}, {j, B}];
  F1 = Table[∂_{ζ_i,ζ_j} Q1, {i, A}, {j, A}];
  G1 = Table[∂_{z_i,z_j} Q1, {i, B}, {j, B}];
  E2 = Table[∂_{ζ_i,z_j} Q2, {i, B}, {j, C}];
  F2 = Table[∂_{ζ_i,ζ_j} Q2, {i, B}, {j, B}];
  G2 = Table[∂_{z_i,z_j} Q2, {i, C}, {j, C}];
  ζA = Table[ζ_i, {i, A}]; zC = Table[z_i, {i, C}];
  I = IdentityMatrix@Length@B;
  Expand /@ M_{A→C}[ω1 ω2 Det[I - F2.G1]^{-1},
    ζA.E1.Inverse[I - F2.G1].E2.zC + 1/2 ζA.(F1 + E1.F2.Inverse[I - G1.F2].E1^T).ζA +
     1/2 zC.(G2 + E2^T.G1.Inverse[I - F2.G1].E2).zC, 0]
 ]
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In[•]:= M1 = RM_{{1,2}→{1,2,3}}
        M2 = RM_{{1,2,3}→{1,2,3}}
        M3 = RM_{{1,2,3}→{1,2}}
```

Out[•]= $\mathbb{M}_{\{1,2\}\to\{1,2,3\}}\left[1,\ -z_1^2+z_1 z_2-2 z_1 z_3+2 z_3^2+5 z_1 \zeta_1-z_2 \zeta_1-2 z_3 \zeta_1+3 \zeta_1^2+z_1 \zeta_2+3 \zeta_2^2,\ 0\right]$

Out[•]= $\mathbb{M}_{\{1,2,3\}\to\{1,2,3\}}\left[1,\ 3 z_1^2+z_2^2+4 z_1 z_3+z_2 z_3+z_3^2-2 z_1 \zeta_1-z_2 \zeta_1-z_3 \zeta_1+\right.$
$\left.\zeta_1^2+z_2 \zeta_2+4 z_3 \zeta_2-\zeta_1 \zeta_2-2 z_1 \zeta_3+2 z_2 \zeta_3+z_3 \zeta_3-2 \zeta_1 \zeta_3-2 \zeta_2 \zeta_3+2 \zeta_3^2,\ 0\right]$

Out[•]= $\mathbb{M}_{\{1,2,3\}\to\{1,2\}}\left[1,\right.$
$\left.-3 z_1^2+5 z_1 z_2-2 z_2^2+z_1 \zeta_1-z_2 \zeta_1+\zeta_1^2+4 z_1 \zeta_2-z_2 \zeta_2-6 \zeta_1 \zeta_2+6 z_1 \zeta_3+3 z_2 \zeta_3-3 \zeta_1 \zeta_3+3 \zeta_2 \zeta_3,\ 0\right]$

```
In[•]:= (M1 // M2) // M3
```

Out[•]= $\mathbb{M}_{\{1,2\}\to\{1,2\}}\left[\frac{1}{177\,405},\ \frac{378\,368 z_1^2}{177\,405}+\frac{1\,420\,781 z_1 z_2}{177\,405}-\frac{175\,612 z_2^2}{177\,405}-\frac{91\,128 z_1 \zeta_1}{59\,135}-\right.$
$\left.\frac{41\,793 z_2 \zeta_1}{59\,135}+\frac{224\,281 \zeta_1^2}{59\,135}+\frac{7144 z_1 \zeta_2}{177\,405}+\frac{34\,814 z_2 \zeta_2}{177\,405}+\frac{31\,853 \zeta_1 \zeta_2}{59\,135}+\frac{519\,548 \zeta_2^2}{177\,405},\ 0\right]$

```
In[•]:= M1 // (M2 // M3)
```

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$\left.\frac{41\,793 z_2 \zeta_1}{59\,135}+\frac{224\,281 \zeta_1^2}{59\,135}+\frac{7144 z_1 \zeta_2}{177\,405}+\frac{34\,814 z_2 \zeta_2}{177\,405}+\frac{31\,853 \zeta_1 \zeta_2}{59\,135}+\frac{519\,548 \zeta_2^2}{177\,405},\ 0\right]$

In[•]:=
```
((M1 // M2) // M3) ≡ (M1 // (M2 // M3))
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Out[•]=
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True
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