Everything around sl^{ϵ}_{2+} is **DoPeGDO**. So what?

Thanks for inviting me to Da Nang!
ωεβ:=http://drorbn.net/v19/
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Continues Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], joint with van der Veen.

Abstract. I'll explain what "everything around" means: classical and quantum m, Δ, S, tr, R, C, and θ, as well as P, Φ, J, $\mathbb{D}$, and more, and all of their compositions. What **DoPeGDO** means: the category of Docile Perturbed Gaussian Differential Operators. And what sl^{ϵ}_{2+} means: a solvable approximation of the semi-simple Lie algebra sl_2.

Knot theorists should rejoice because all this leads to very powerful and well-behaved poly-time-computable knot invariants. Quantum algebraists should rejoice because it's a realistic playground for testing complicated equations and theories.


$m\colon U\otimes U\to U$


$\Delta\colon U\to U\otimes U$

$S\colon U\to U$

$\mathrm{tr}\colon U\to U/xw{=}wx$

$R\in U\otimes U$

$C\in U$

$\Phi\in U^{\otimes 3}$

$J\in U\otimes U$

Less Abstract

With $\gamma\neq 0$, $sl^{\epsilon}_{2+} := L\langle y,b,a,x\rangle$ with $[a,x]=\gamma x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-\gamma y$, $[b,x]=\epsilon x$, and $[x,y]=\epsilon a+\gamma b$. So $t:=\epsilon a-\gamma b$ is central and $sl^{\epsilon}_{2+}/\langle t\rangle\cong sl_2$.

U is either $CU=\mathcal{U}(sl^{\epsilon}_{2+})$ or $QU=\mathcal{U}_\hbar(sl^{\epsilon}_{2+})=A\langle y,b,a,x\rangle$ with $[a,x]=\gamma x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-\gamma y$, $[b,x]=\epsilon x$, and $xy-qyx=(1-AB)/\hbar$, where $q=e^{\hbar\gamma\epsilon}$, $A=e^{-\hbar\epsilon a}$, and $B=e^{-\hbar\gamma b}$. Set $T=A^{-1}B=e^{\hbar t}$.

Conventions. For a set A, $z_A:=\{z_i\}_{i\in A}$ and $\zeta_A:=\{z_i^*=\zeta_i\}_{i\in A}$. Always, at least one of $\{z_i,\zeta_i\}$ is "heavy".

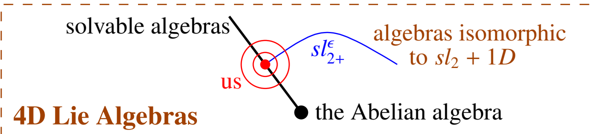
DoPeGDO$_2$:∼ The category with objects finite sets and mor($A\to B$):

$$\left\{F=\omega\exp\left(Q+\sum_{k\geq 1}\epsilon^k P^{(k)}\right)\right\}$$

• ω is a scalar. • Q is a quadratic in $\zeta_A\cup z_B$. • The P_k are "perturbation polynomials"; the "heavy degree" of P_k is $\leq k+1$. • Compositions:

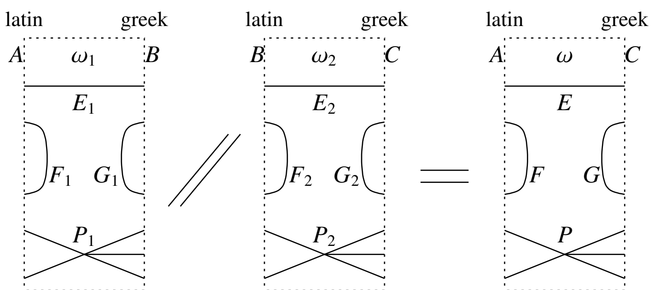
$$F/\!\!/G=G\circ F:=\left(G|_{\zeta_i\to\partial_{z_i}}F\right)_{z_i=0}.$$

Cool! $(V^*)^{\otimes A}\otimes V^{\otimes B}$ explodes; the ranks of quadratics and fixed-degree polynomials grow slowly!



Compositions.

Series always converge!

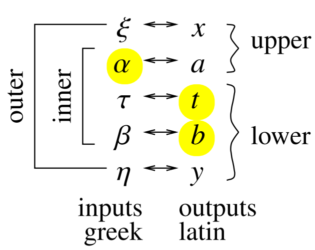


Where:

Variable Taxonomy

light

$t=\epsilon a-\gamma b$ is "central"



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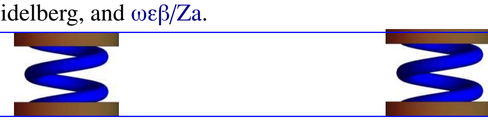

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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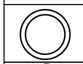
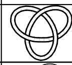
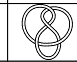
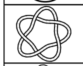
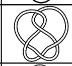
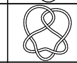
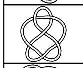
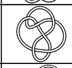
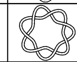
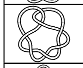
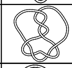
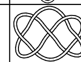
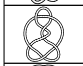
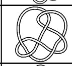
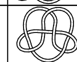
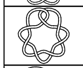
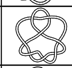
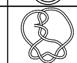
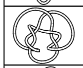
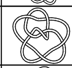
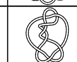
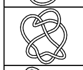
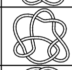
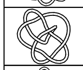
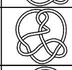
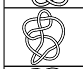
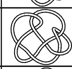
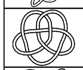
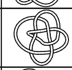
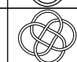
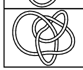
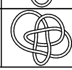
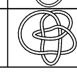
Reserved

diagram	n_k^t Today's ρ_1^+	Alexander's ω^+	genus / ribbon unknotting # / amphi?	diagram	n_k^t Today's ρ_1^+	Alexander's ω^+	genus / ribbon unknotting # / amphi?	diagram	n_k^t Today's ρ_1^+	Alexander's ω^+	genus / ribbon unknotting # / amphi?
	0_1^a 0	1	0 / ✔ 0 / ✔		3_1^a t	$t-1$	1 / ✘ 1 / ✘		4_1^a 0	$3-t$	1 / ✘ 1 / ✔
	5_1^a $2t^3+3t$	t^2-t+1	2 / ✘ 2 / ✘		5_2^a $5t-4$	$2t-3$	1 / ✘ 1 / ✘		6_1^a $t-4$	$5-2t$	1 / ✔ 1 / ✘
	6_2^a t^3-4t^2+4t-4	$-t^2+3t-3$	2 / ✘ 1 / ✘		6_3^a 0	t^2-3t+5	2 / ✘ 1 / ✔		7_1^a $3t^5+5t^3+6t$	t^3-t^2+t-1	3 / ✘ 3 / ✘
	7_2^a $14t-16$	$3t-5$	1 / ✘ 1 / ✘		7_3^a $-9t^3+8t^2-16t+12$	$2t^2-3t+3$	2 / ✘ 2 / ✘		7_4^a $32-24t$	$4t-7$	1 / ✘ 2 / ✘
	7_5^a $9t^3-16t^2+29t-28$	$2t^2-4t+5$	2 / ✘ 2 / ✘		7_6^a $t^3-8t^2+19t-20$	$-t^2+5t-7$	2 / ✘ 1 / ✘		7_7^a $8-3t$	t^2-5t+9	2 / ✘ 1 / ✘
	8_1^a $5t-16$	$7-3t$	1 / ✘ 1 / ✘		8_2^a $2t^5-8t^4+10t^3-12t^2+13t-12$	$-t^3+3t^2-3t+3$	3 / ✘ 2 / ✘		8_3^a 0	$9-4t$	1 / ✘ 2 / ✔
	8_4^a $3t^3-8t^2+6t-4$	$-2t^2+5t-5$	2 / ✘ 2 / ✘		8_5^a $-2t^5+8t^4-13t^3+20t^2-22t+24$	$-t^3+3t^2-4t+5$	3 / ✘ 2 / ✘		8_6^a $5t^3-20t^2+28t-32$	$-2t^2+6t-7$	2 / ✘ 2 / ✘
	8_7^a $-t^5+4t^4-10t^3+12t^2-13t+12$	t^3-3t^2+5t-5	3 / ✘ 1 / ✘		8_8^a $-t^3+4t^2-12t+16$	$2t^2-6t+9$	2 / ✔ 2 / ✘		8_9^a 0	$-t^3+3t^2-5t+7$	3 / ✔ 1 / ✔
	8_{10}^a $-t^5+4t^4-11t^3+16t^2-21t+20$	t^3-3t^2+6t-7	3 / ✘ 2 / ✘		8_{11}^a $5t^3-24t^2+39t-44$	$-2t^2+7t-9$	2 / ✘ 1 / ✘		8_{12}^a 0	$t^2-7t+13$	2 / ✘ 2 / ✔
	8_{13}^a $-t^3+4t^2-14t+20$	$2t^2-7t+11$	2 / ✘ 1 / ✘		8_{14}^a $5t^3-28t^2+57t-68$	$-2t^2+8t-11$	2 / ✘ 1 / ✘		8_{15}^a $21t^3-64t^2+120t-140$	$3t^2-8t+11$	2 / ✘ 2 / ✘
	8_{16}^a $t^5-6t^4+17t^3-28t^2+35t-36$	t^3-4t^2+8t-9	3 / ✘ 2 / ✘		8_{17}^a 0	$-t^3+4t^2-8t+11$	3 / ✘ 1 / ✔		8_{18}^a 0	$-t^3+5t^2-10t+13$	3 / ✘ 2 / ✔
	8_{19}^n $-3t^5-4t^2-3t$	t^3-t^2+1	3 / ✘ 3 / ✘		8_{20}^n $4t-4$	t^2-2t+3	2 / ✔ 1 / ✘		8_{21}^n $t^3-8t^2+16t-20$	$-t^2+4t-5$	2 / ✘ 1 / ✘