Pensieve header: The main program and demo. Continues pensieve://Talks/ICERM-2305

```
In[∘]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Budapest-2311"];
```

tex

{\bf\red Implementation} (sources: \url{http://drorbn.net/bu23/ap}). I like it most when the implementa-

pdf

```
In[∘]:= Once[<< KnotTheory`];
```

pdf

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

tex

\par{\bf\red Utilities } The step function, algebraic numbers, canonical forms

pdf

```
In[∘]:= θ[x_] /; NumericQ[x] := UnitStep[x]
```

pdf

```
In[∘]:= ω2[v_][p_] := Module[{q = Expand[p], n, c},
           If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]]];
```

pdf

```
In[∘]:= sign[ℰ_] := Module[{n, d, v, p, rs, e, k},
          {n, d} = NumeratorDenominator[ℰ];
          {n, d} /= ω^(Exponent[n,ω]/2+Exponent[n,ω,Min]/2);
          p = Factor[ω2[v]@n * ω2[v]@d /. v → 4 u^2 - 2];
          rs = Solve[p == 0, u, Reals];
          If[rs === {}, Sign[p /. u → 0],
           rs = Union@(u /. rs);
           Sign[(-1)^(e=Exponent[p,u]) Coefficient[p, u, e]] + Sum[
```

pdf

```
In[∘]:= SetAttributes[B, Orderless];
```

pdf

```
In[∘]:= CF[ℰ_] := Module[{γs = Union@Cases[ℰ, γ_ | γ̄_, ∞]},
```

pdf

```
In[∘]:= CF[{}] = {};
        CF[𝒞_List] := Module[{γs = Union@Cases[𝒞, γ_, ∞], γ},
```

pdf

```
In[∘]:= (ℰ_)* := ℰ /. {γ̄ → γ, γ → γ̄, ω → ω^-1, c_Complex :→ c*};
```

pdf

```
In[∘]:= RulesOf[γi_ + rest_.] := (γi → -rest)+;
        CF[PQ[𝒞_, q_]] := Module[{n𝒞 = CF[𝒞]},
```

pdf

```
In[∘]:= CF[Σb_[σ_, pq_]] := ΣCF[b][σ, CF[pq]]
```

tex

\needspace{32mm}

pdf

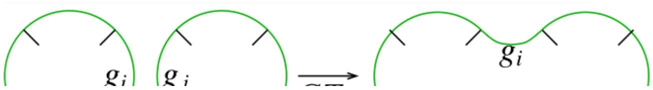
```
In[∘]:= Format[Σb_B[σ_, PQ[𝒞_, q_]]] := Module[{γs},
           γs = γ# & /@ Join @@ b;
           Column[{TraditionalForm@σ,
             TableForm[Join[
               Prepend[""] /@ Table[TraditionalForm[∂c r], {r, 𝒞}, {c, γs}],
               {Prepend[""][
```

tex

\par{\bf\red The Face-Centric Core }

pdf

In[∘]:=
```
Σb1_[σ1_, PQ[C1_, q1_]] ⊕ Σb2_[σ2_, PQ[C2_, q2_]] ^:=
```



tex

\par GT for Gap Touch: \hfill \import{../ICERM-2305}{figs/GT.pdf_t}

pdf

In[∘]:=
```
GTi_,j_@ΣB[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[C_, q_]] :=
```

cor·don (kôr′dn)

THE FREE DICTIONARY BY FARLEX

n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon.*

tex

\par\vskip 1mm\par\Cordon

pdf

In[∘]:=
```
Cordoni_@ΣB[{li___,i_,ri___},bs___][σ_, PQ[C_, q_]] :=
 Module[{ϕ = ∂γi C, λ = ∂γ̄i,γi q, nσ = σ, nC, nq, p},
  {p} = FirstPosition[(# =!= 0) & /@ ϕ, True, {0}];
  {nC, nq} = Which[
    p > 0, {C, q} /. (γi → −C⟦p⟧ / ϕ⟦p⟧)+ /. (γi → 0)+,
```

tex

\par\needspace{20mm}

pdf

In[∘]:=
```
ci_,j_@t : ΣB[{li___,i_,ri___},{___,j_,___},___][__] := t // GTj,First@{ri,li} // Cordonj
```

pdf

In[∘]:=
```
ci_,j_@t : ΣB[{___,i_,j_,___},___][__] := Cordonj@t
ci_,j_@t : ΣB[{j_,___,i_},___][__] := Cordonj@t
```

pdf

In[∘]:=
```
mc[ℰ_] := ℰ //.
  t : ΣB[{___,i_,___},{___,i_,___},___][__] | ΣB[{___,i_,i_,___},___][__] | ΣB[{i_,___,i_},___][__] /;
```

tex

\par{\bf\red The Crossings} (and empty strands)

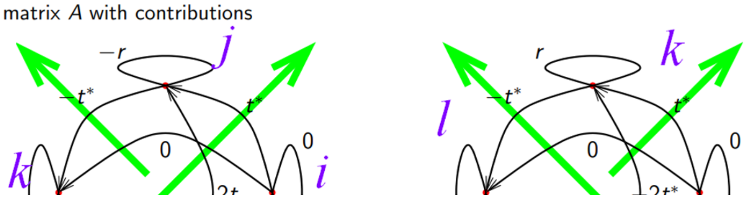
pdf

In[∘]:=
```
TL@Pi_,j_ := CF@ΣB[{i,j}][0, PQ[{}, 0]]
```

http://drorbn.net/cms21

Bedlewo for Mathematicians.

For a knot K and a complex unit ω set $t = 1 - \omega$, $r = 2\Re(t)$, make an $F \times F$ matrix A with contributions



pdf

In[∘]:=
```
TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x], X-i,j,k,-l, X̄-j,k,l,-i];
TL[(x : X | X̄)fs__] := Module[{t = 1 - ω, r, γs, m},
  r = t + t*; γs = γ# & /@ {fs};
  m = If[x === X,
     ( -r  -t  2 t  t* )  (  r   -t  -2 t*  t* )
```

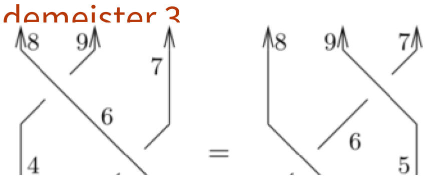
tex

\par{\bf\red Evaluation on Tangles and Knots.}

pdf

In[∘]:=
```
TL[K_] := Fold[mc[#1 ⊕ #2] &, ΣB[] [0, PQ[{}, 0]], List @@ (TL /@ PD@K)] /.
   θ[c_ + u] /; Abs[c] ≥ 1 ⧴ θ[c];
```

Reidemeister 3



tex

\par\needspace{20mm}
\parpic[r]{\import{../ICERM-2305}{figs/R3.pdf_t}}

pdf

In[∘]:=
```
R3L = PD[X-2,5,4,-1, X-3,7,6,-5,
    X-6,9,8,-4];
R3R = PD[X-3,5,4,-2, X-4,6,8,-1,
```

Out[∘]=

True

tex

\needspace{15mm}

pdf

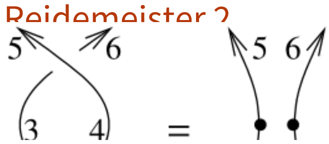
In[∘]:= TL@R3L

Out[∘]=

-1

	$(\gamma_{-3}$	γ_7	γ_9	γ_8	γ_{-1}	$\gamma_{-2})$
$\bar\gamma_{-3}$	$\frac{\omega^2+1}{\omega}$	$\omega-1$	$-2\,\omega$	2	0	$-\frac{\omega+1}{\omega}$
$\bar\gamma_7$	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	0	0
$\bar\gamma_9$	$-\frac{2}{\omega}$	$1-\omega$	$\frac{\omega^2+1}{\omega}$	$-\frac{\omega+1}{\omega}$	0	$\frac{2}{\omega}$

Reidemeister 2



tex

\par
\needspace{20mm}

pdf

In[∘]:= TL@PD[X-3,4,3,-1, X̄-4,6,5,-3]

Out[∘]=

0

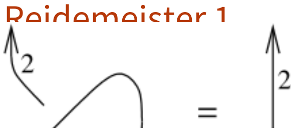
	1	0	-1	0
	$(\gamma_{-2}$	γ_6	γ_5	$\gamma_{-1})$
$\bar\gamma_{-2}$	0	0	0	0

pdf

In[∘]:= TL@PD[$X_{2,4,3,1}$, $\overline{X}_{4,6,5,3}$] == $GT_{5,2}$@TL@PD[$P_{1,5}$, $P_{2,6}$]

Out[∘]= True

Reidemeister 1



tex

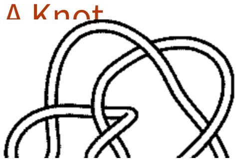
\par
\parpic[r]{\import{../ICERM-2305}{figs/R1.pdf_t}}

pdf

In[∘]:= TL@PD[$X_{3,3,2,1}$] == TL@$P_{1,2}$

Out[∘]= True

A Knot



tex

\par
\parpic[r]{\includegraphics[width=1in]{../ICERM-2305/8_5.png}}

pdf

In[∘]:= f = TLSig[Knot[8, 5]]

pdf

KnotTheory: Loading precomputed data in PD4Knots`

Out[∘]= $2\,\theta\left[-\frac{\sqrt{3}}{\ldots}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{\ldots}+u\right]-2\,\theta\left[u-0.630\ldots\right]+2\,\theta\left[u-0.630\ldots\right]$

tex

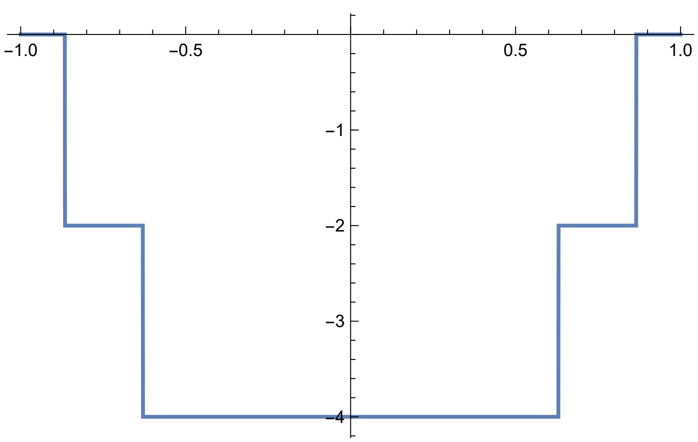
\par\picskip{0}{
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}

pdf

In[∘]:= Plot[f, {u, -1, 1}]

Out[∘]=

pdf



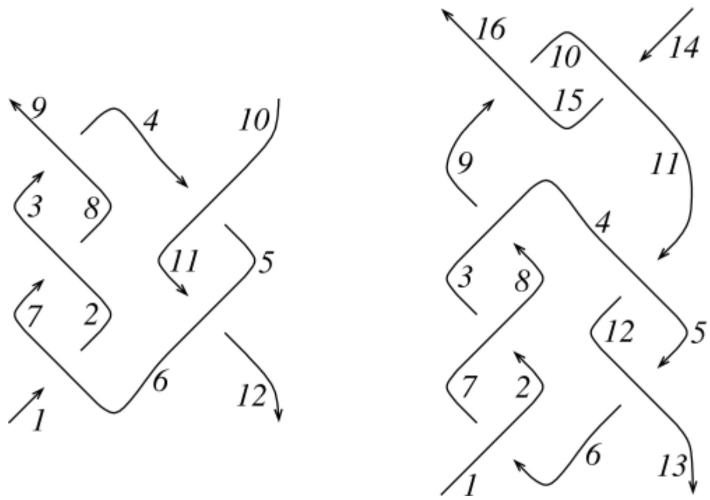
tex

}

Some Tangles

tex

\needspace{30mm}
\par\parpic[r]{\includegraphics[width=1.88in]{../ICERM-2305/figs/CKT.pdf}}
{\bf\red The Conway-Kinoshita-Terasaka Tangles.}



pdf

In[∘]:=
$$T1 = PD\left[\overline{X}_{-6,2,7,-1},\ \overline{X}_{-2,8,3,-7},\ \overline{X}_{-8,4,9,-3},\ X_{-11,6,12,-5},\ X_{-4,11,5,-10}\right];$$
$$T2 = PD\left[X_{-6,2,7,-1},\ X_{-2,8,3,-7},\ X_{-8,4,9,-3},\ \overline{X}_{-12,6,13,-5},\ \overline{X}_{-4,12,5,-11},\ \overline{X}_{-10,15,11,-14},\ \overline{X}_{-15,10,16,-9}\right];$$

tex

\par\needspace{10mm}

pdf

In[∘]:= $TL[T1]$

Out[∘]=
pdf

$$-2\,\Theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\Theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

	Υ_{-10}	Υ_{9}	Υ_{-1}	Υ_{12}
$\overline{\Upsilon}_{-10}$	0	$1-\omega$	0	$\omega-1$
$\overline{\Upsilon}_{9}$	$\frac{\omega-1}{\omega}$	$\frac{2\omega}{\omega^2-\omega+1}$	$-\frac{\omega-1}{\omega}$	$-\frac{2\omega}{\omega^2-\omega+1}$
$\overline{\Upsilon}_{-1}$	0	$\omega-1$	0	$1-\omega$
$\overline{\Upsilon}_{12}$	$-\frac{\omega-1}{\omega}$	$-\frac{2\omega}{\omega^2-\omega+1}$	$\frac{\omega-1}{\omega}$	$\frac{2\omega}{\omega^2-\omega+1}$

pdf

In[∘]:=
```
TL[T2]
```

Out[∘]=
pdf

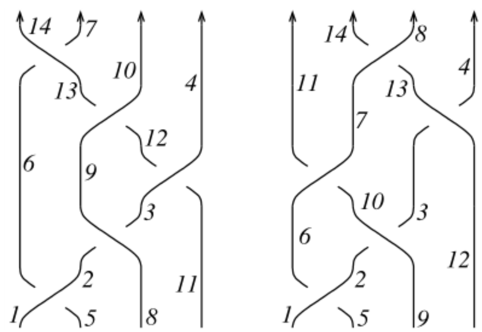
$$0$$

	$(\Upsilon_{-14}$	Υ_{16}	Υ_{-1}	$\Upsilon_{13})$
$\overline{\Upsilon}_{-14}$	0	$1-\omega$	0	$\omega-1$
$\overline{\Upsilon}_{16}$	$\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$
$\overline{\Upsilon}_{-1}$	0	$\omega-1$	0	$1-\omega$
$\overline{\Upsilon}_{13}$	$-\frac{\omega-1}{\omega}$	$\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$	$\frac{\omega-1}{\omega}$	$-\frac{2(\omega-1)^2\omega}{\omega^4-3\omega^3+5\omega^2-3\omega+1}$

Some Braids

tex

\parpic[r]{\includegraphics[width=1.88in]{../ICERM-2305/figs/B1B2.pdf}}
{\bf\red Examples with non-trivial codimension.}



In[∘]:=
```
PD[X[5, 2, 6, 1], X[2, 9, 3, 10], X[10, 7, 11, 6], X[3, 12, 4, 13], X[13, 8, 14, 7]] /.
 x : X[i_, j_, k_, l_] :> If[PositiveQ@x, X_{-i,j,k,-l}, X̄_{-j,k,l,-i}]
```

Out[∘]=

$\mathrm{PD}\left[X_{-5,2,6,-1}, \overline{X}_{-9,3,10,-2}, X_{-10,7,11,-6}, \overline{X}_{-12,4,13,-3}, X_{-13,8,14,-7}\right]$

pdf

In[∘]:=

$B1 = \mathrm{PD}\left[X_{-5,2,6,-1}, \overline{X}_{-8,3,9,-2}, X_{-11,4,12,-3}, X_{-12,10,13,-9}, \overline{X}_{-13,7,14,-6}\right];$

$B2 = \mathrm{PD}\left[X_{-5,2,6,-1}, \overline{X}_{-9,3,10,-2}, X_{-10,7,11,-6}, \overline{X}_{-12,4,13,-3}, X_{-13,8,14,-7}\right];$

pdf

In[∘]:= **TL[B1]**

Out[∘]=
pdf

0

	1	0	-1	0	$\frac{1}{\omega}$	0	$-\frac{1}{\omega}$	0
	0	0	0	-1	$\frac{1}{\omega}$	0	$-\frac{1}{\omega}$	1
	$(\gamma_{-11}$	γ_{4}	γ_{10}	γ_{7}	γ_{14}	γ_{-1}	γ_{-5}	$\gamma_{-8})$
$\overline{\gamma}_{-11}$	0	0	0	0	0	0	0	0
$\overline{\gamma}_{4}$	0	0	0	0	$\frac{\omega-1}{\omega^2}$	0	$-\frac{\omega-1}{\omega^2}$	0
$\overline{\gamma}_{10}$	0	0	0	0	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0
$\overline{\gamma}_{7}$	0	0	0	0	$\frac{(\omega-1)^2}{\omega^2}$	0	$-\frac{(\omega-1)^2}{\omega^2}$	0
$\overline{\gamma}_{14}$	0	$-((\omega-1)\,\omega)$	$\omega-1$	$(\omega-1)^2$	0	$-\frac{\omega-1}{\omega}$	$\frac{\omega-1}{\omega}$	0
$\overline{\gamma}_{-1}$	0	0	0	0	$\omega-1$	0	$1-\omega$	0
$\overline{\gamma}_{-5}$	0	$(\omega-1)\,\omega$	$1-\omega$	$-(\omega-1)^2$	$1-\omega$	$\frac{\omega-1}{\omega}$	$\frac{(\omega-1)^2}{\omega}$	0
$\overline{\gamma}_{-8}$	0	0	0	0	0	0	0	0

tex

\par\needspace{10mm}

pdf

In[∘]:= **TL[B2]**

Out[∘]=
pdf

0

	$(\gamma_{-12}$	γ_{4}	γ_{8}	γ_{14}	γ_{11}	γ_{-1}	
$\overline{\gamma}_{-12}$	$\frac{(\omega-1)^2}{\omega}$	$\omega-1$	$-2\,(\omega-1)$	$\frac{2\,(\omega-1)^2}{\omega}$	$\frac{2\,(\omega-1)}{\omega^2}$	0	[illegible]
$\overline{\gamma}_{4}$	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	0	0	
$\overline{\gamma}_{8}$	$\frac{2\,(\omega-1)}{\omega}$	$1-\omega$	$\frac{(\omega-1)^2}{\omega}$	$-\frac{(\omega-1)\,(2\,\omega-3)}{\omega}$	$-\frac{2\,(\omega-1)}{\omega^2}$	0	[illegible]
$\overline{\gamma}_{14}$	$\frac{2\,(\omega-1)^2}{\omega}$	0	$-\frac{(\omega-1)\,(3\,\omega-2)}{\omega}$	$\frac{3\,(\omega-1)^2}{\omega}$	$-\frac{(\omega-2)\,(\omega-1)}{\omega^2}$	0	[illegible]
$\overline{\gamma}_{11}$	$-2\,(\omega-1)\,\omega$	0	$2\,(\omega-1)\,\omega$	$-((\omega-1)\,(2\,\omega-1))$	$\frac{(\omega-1)^2}{\omega}$	$-\frac{\omega-1}{\omega}$	[illegible]
$\overline{\gamma}_{-1}$	0	0	0	0	$\omega-1$	0	[illegible]
$\overline{\gamma}_{-5}$	$2\,(\omega-1)\,\omega$	0	$-2\,(\omega-1)\,\omega$	$2\,(\omega-1)\,\omega$	$-2\,(\omega-1)$	$\frac{\omega-1}{\omega}$	[illegible]
$\overline{\gamma}_{-9}$	$-\frac{(\omega-1)\,(3\,\omega-2)}{\omega}$	0	$\frac{2\,(\omega-1)\,(2\,\omega-1)}{\omega}$	$-\frac{2\,(\omega-1)\,(2\,\omega-1)}{\omega}$	$\frac{2\,(\omega-1)^2}{\omega^2}$	0	[illegible]