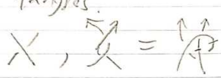
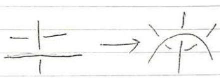
conversation-starter

Impromptu lecture on OU tangles ~~and~~ the Drinfel'd Double Formula, and quantization of Lie bialgebras.

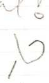
Define OU tangles.

Examples: 

<u>Thm</u> Every tangle is an OU tangle.

<u>proof</u> 

<u>Corollary</u> Knot theory is trivial!

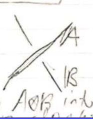
Too good to be true? What about  ?

Too good to be ignored? Perhaps true in some quotient/completion/image?

1. Quotient-completion: complete to virtual knots, divide by OC, get my paper "Knotted balloons & hoops".
   There may be more...
2. Completion like in $\mathbb{Q} \hookrightarrow \mathbb{A}$: See Dylan's talk.
3. Completion like in $\mathbb{Q}[x] \hookrightarrow \mathbb{Q}[[x]]$ or $K \to \varprojlim K/I^n$:

$$\boxed{\text{I wish I knew}}$$

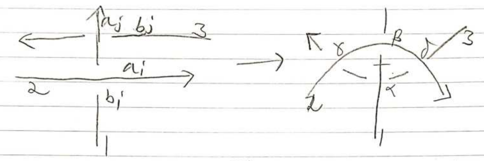
A "bake" formula.

4. represent!  $R \in \mathbb{A}\otimes\mathbb{B} = \sum a_i \otimes b_i$

need $b \cdot a := \sum a_i' b_i'$

(Need a rule making $A\otimes B$ into

Requirement on $\mathbb{A}, \mathbb{B}$, $R = \sum a_i \otimes b_i \in \mathbb{A}\otimes\mathbb{B}$

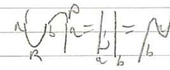


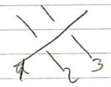
Fadd's Gold

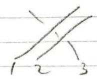
"FG relation"

1. $\mathbb{A}$ & $\mathbb{B}$ must be f.d. algebras. | $\mathbb{A}^* \to \mathbb{B}$ onto; $\mathbb{B}^* \to \mathbb{A}$ onto.

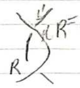
2. The b_i's must span all of $\mathbb{B}$; The a_i's must span all of $\mathbb{A}$ $\Rightarrow$ R is of max rank, $\dim\mathbb{A} = \dim\mathbb{B}$, $\exists$ pairing $\langle\,,\rangle \in \mathbb{B}^*\otimes\mathbb{A}^*$ "dual" to R:

 $\sum a_i \langle b_i, a\rangle = a$ $\quad \sum \langle b, a_i\rangle b_i = b$

3.  $\to \sum a_i a_j \otimes b_i \otimes b_j$ $\quad b \mapsto \sum \langle b, a_i a_j\rangle b_i \otimes b_j$
a co-product on $\mathbb{B}$ s.t. $(1\otimes\Delta)R = R^{12}R^{13}$

 $\to \sum a_i \otimes a_j \otimes b_j b_i$ $\quad a \mapsto \sum a_i \otimes a_j \langle b_j b_i, a\rangle$
a co-product on $\mathbb{A}$ s.t. $(\Delta\otimes 1)R = R^{23}R^{13}$

 $\rightsquigarrow$ $\mathbb{A}, \mathbb{B}$ are (dual) bi-algebras.

4.  $R^{-1} = \sum a_i' \otimes b_i'$
R is invertible in $\mathbb{A}\otimes\mathbb{B}$ $\quad$  $\to \sum a_i' \otimes b_i'$

$a \mapsto \sum a_i' \langle b_i', a\rangle$

$\to$ s.t. $(S\otimes 1)R = R^{-1}$ .. likewise for $\mathbb{B}$. $\quad$ an antipode for $\mathbb{A}$ $\to$

The FG relation:

$$\sum b_i \cdot a_j \otimes a_i \otimes b_j = \sum a_\alpha b_\beta \otimes a_\gamma a_\beta \bar{a}_\delta \otimes b_\delta b_\alpha b_\gamma$$

Pair w/ $b\otimes a$ on strands 2,3: $\quad \Delta(b) = b_1\otimes b_2 \otimes b_3$, $\quad \Delta(a) = a_1 \otimes a_2 \otimes a_3$

$$b \cdot a := \sum a_\alpha b_\beta \otimes \langle b_1, a_\gamma\rangle \langle b_2, a_\beta\rangle \langle b_3, \bar{a}_\delta\rangle$$

$$\langle b_\delta, a_1\rangle \langle b_\alpha, a_2\rangle \langle b_\gamma, a_3\rangle$$

$$= \langle b_1, a_3\rangle \langle b_3, \bar{a}_1\rangle a_2 b_2$$

<u>Example</u> $\mathbb{A} = \mathcal{U}(\langle a, x\rangle / [a,x] = x)$ $\quad \mathbb{B} = \mathcal{U}(\langle b, y\rangle / [b,y] = -\epsilon y)$

Pair w/ $\langle b, a\rangle = \hbar^{-1}$, $\langle y, x\rangle = \hbar^{-1}$

$\Delta(y, b, a, x) = y_1 + B_1 y_2,\ b_1 + b_2,\ a_1 + a_2,\ x_1 + A_1 x_2$

$B = e^{-\hbar b}$

$A = e^{-\hbar \epsilon a}$

$S(y\,b\,a\,x) = -B^{-1}y, -b, -a, -A^{-1}x$

$\Rightarrow \langle b^n, a^m\rangle = \hbar^{n}\delta_{nm} n!$ $\quad \langle y^n, x^m\rangle = \hbar^{-n}\delta_{nm}[n]_q!$

$q = e^{\hbar\epsilon}$

$$\Rightarrow R = \sum \frac{a^n b^n}{n!} \cdot \frac{x^m y^m}{[m]_q!}$$

$ba = ab$

$xy = qyx + (1 - AB)/\hbar \ \ldots$