The W term

Question - which two variable function W will make the following true:

$$W[\alpha_3, c[1|\alpha_3] + W[\alpha_4, c[1|\alpha_4] = W[\alpha_4 + \alpha_3(1 + c[1|\alpha_4]), c[1|(\alpha_4 + \alpha_3(1 + c[1|\alpha_4]))]$$

$$W(\alpha) + W(\beta) = W(\alpha + \beta + c\alpha\beta)$$

$$W\left(\alpha - \frac{1}{c}\right) + W\left(\beta - \frac{1}{c}\right) = W\left(\alpha - \frac{1}{c} + \beta - \frac{1}{c} + c\left(\alpha - \frac{1}{c}\right)\left(\beta - \frac{1}{c}\right)\right)$$

$$= W\left(c\alpha\beta - \frac{1}{c}\right) \qquad \overset{c\alpha\beta - \beta - \alpha + \frac{1}{c}}{}$$

$$W'(x) = W\left(x - \frac{1}{c}\right) \qquad\qquad W(x) = W'\left(x + \frac{1}{c}\right)$$

$$W'(\alpha) + W'(\beta) = W'(c\alpha\beta)$$

$$W'(x) = \log(cx) \qquad\qquad W(x) = \log(cx + 1)$$

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check: