De-Symmetrization

(Following "BCH in Blobs.nb")

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9:46 AM

$$\left\{\left\{\xi \to \frac{-\mathrm{Log}\left[e^{-x\alpha - y\beta}\right] + \mathrm{Log}\left[\frac{x\alpha + e^{-x\alpha - y\beta}\, y\beta}{x\alpha + y\beta}\right]}{x},\ \eta \to -\frac{\mathrm{Log}\left[\frac{x\alpha + e^{-x\alpha - y\beta}\, y\beta}{x\alpha + y\beta}\right]}{y}\right\}\right\}$$

Claim. Under $[x,y] = C_x y - C_y x$,

$$e^{\alpha x + \beta y} = e^{\xi x} e^{\eta y}, \quad \text{where} \quad (\text{with } \gamma = \alpha C_x + \beta C_y):$$

$$\xi = \frac{\log \frac{\alpha C_x e^{\alpha C_x + \beta C_y} + \beta C_y}{\alpha C_x + \beta C_y}}{C_x} = \frac{1}{C_x} \log \frac{1}{\gamma}\left(e^{\gamma} \alpha C_x + \beta C_y\right)$$

$$\eta = \frac{-\log \frac{\alpha C_x + e^{-\alpha C_x - \beta C_y} \cdot \beta C_y}{\alpha C_x + \beta C_y}}{C_y} = -\frac{1}{C_y} \log \frac{1}{\gamma}\left(\alpha C_x + e^{-\gamma} \beta C_y\right)$$

So I'm good at copying eqn's.
Now what?

$$[x, y+z] = C_x y - C_y x + C_x z - C_z x$$
$$= C_x(y+z) - (C_y + C_z)x$$