BCH in the Blob Quotient

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10:37 AM

The "head merge" coefficients come from ordinary BCH:

$$\log\left(e^{\underline{\Upsilon}^x} e^{\underline{\Upsilon}^y}\right) = F(h_x, h_y)\,\underline{\Upsilon}^x + G(h_x, h_y)\,\underline{\Upsilon}^y$$

According to "BCH in Blobs.nb", $F \to \frac{(-1+e^x)(x+y)}{(-1+e^{x+y})x}$, $G \to \frac{e^x(-1+e^y)(x+y)}{(-1+e^{x+y})y}$, or, in human language:

$$F = \frac{x+y}{x}\,\frac{e^x-1}{e^{x+y}-1} \qquad G = \frac{x+y}{y}\,\frac{e^y-1}{e^{x+y}-1}\,e^x$$

$$= \frac{e^x-1}{x}\,\frac{x+y}{e^{x+y}-1} \qquad = \frac{e^y-1}{y}\,\frac{x+y}{e^{x+y}-1}\,e^x$$

The first thing to check would be associativity

This wants a story!

To degree 1, $F = 1 - \frac{y}{2}$, $G = 1 + \frac{x}{2}$, so

$$\log\left(e^{\underline{\Upsilon}^x} e^{\underline{\Upsilon}^y}\right) = x + y + \frac{[x,y]}{2} = x + y + \frac{c_x y - c_y x}{2}$$

In agreement with the above.

$$\frac{a-1}{ab-1} \longrightarrow \frac{b^{-1}-1}{a^{-1}b^{-1}-1} = \frac{a-ab}{1-ab} = a\left(\frac{b-1}{ab-1}\right)$$

$$\frac{a-1}{ab-1} = \frac{\sqrt{a}\left(\sqrt{a} - 1/\sqrt{a}\right)}{\sqrt{ab}\left(\sqrt{ab} - 1/\sqrt{ab}\right)} = \frac{1}{\sqrt{b}}\left(\frac{\sqrt{a} - 1/\sqrt{a}}{\sqrt{ab} - 1/\sqrt{ab}}\right)$$

$$\frac{a-1}{ab-1} = \frac{1-a^{-1}}{a^{-1}-b}$$

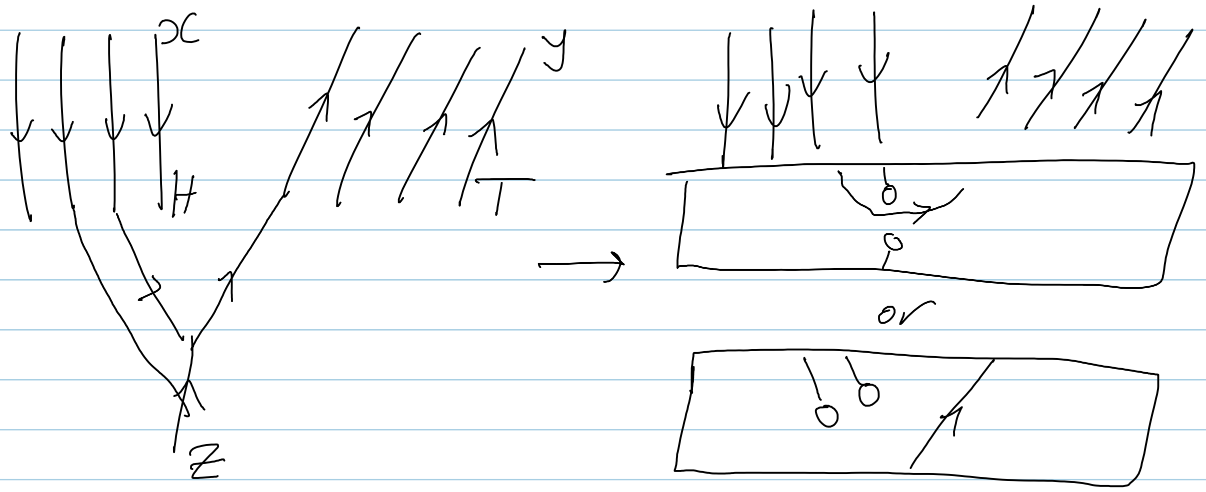
Aside. w/ $j_x = \frac{e^x-1}{x}$, $\quad j_x / j_{-x} = e^x$

Indeed, $\frac{e^x-1}{x} \Big/ \frac{e^{-x}-1}{-x} = \frac{e^x-1}{1-e^{-x}} = e^x\,\frac{1-e^{-x}}{1-e^{-x}} = e^x$

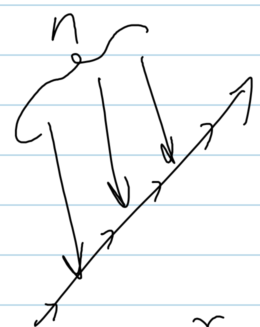
Indeed, $\frac{e^x-1}{x} \Big/ \frac{e^{-x}-1}{-x} = \frac{e^x-1}{1-e^{-x}} = e^x \frac{1-e^{-x}}{1-e^{-x}} = e^x$

Aside^2: $j_x \cdot j_{-x} = \frac{(e^x-1)(e^{-x}-1)}{-x^2} = \frac{e^x-2+e^{-x}}{x^2}$ not illuminating.

I also need to know the crossover coefficients.

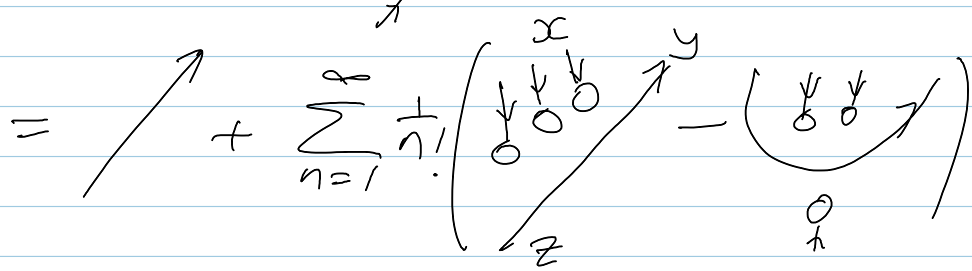


$$e^{\operatorname{ad}H}(e^T) = e^H e^T e^{-H} \Rightarrow e^H e^T = e^{\operatorname{ad}H}(e^T)\, e^H$$

$e^{\operatorname{ad}H}(T) = \sum \frac{1}{n!}$  $=$

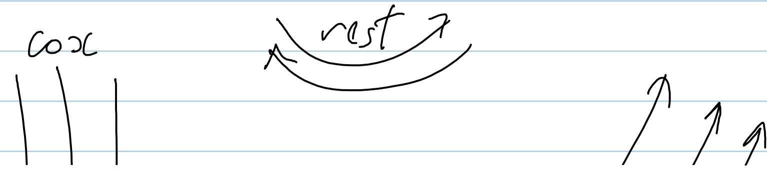
A formalism would be better than an intuition.

I need better bulk management!



$$= e^{c_x} a_{zy} + \frac{1-e^{c_x}}{c_x} c_z a_{xy}$$

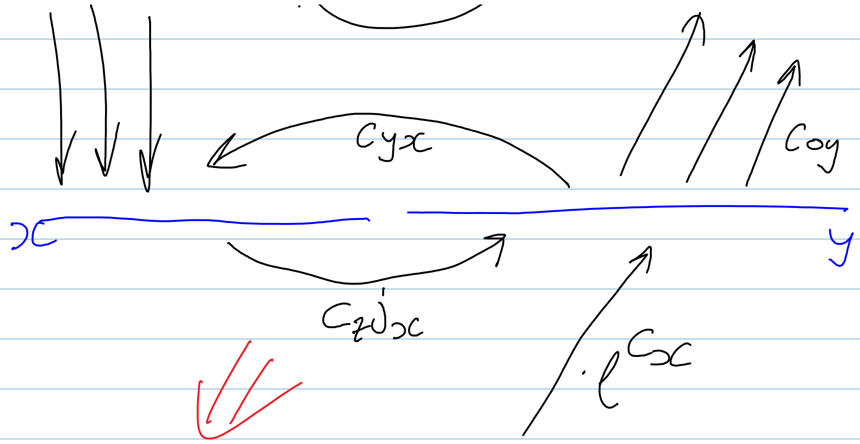
$\left(1-\frac{1-e^x}{x}a\right)^{-1} =$



```
cox = D[mix, h[x]] /. t[y] -> 0;
coy = D[mix, t[y]] /. h[x] -> 0;
cyx = Coefficient[mix, t[y] h[x]];
```

```
cox = D[mix, h[x]] /. t[y] -> 0;
coy = D[mix, t[y]] /. h[x] -> 0;
cyx = Coefficient[mix, t[y] h[x]];
rest = mix /. {h[x] -> 0, t[y] -> 0};
```

$jx = If[cox === 0, 1, \frac{(-1 + e^{cox})}{\text{cox}}$



rest +