

```

cs = Last /@ {c[0] -> - $\frac{11}{15360}$ , c[1] -> - $\frac{19}{46080}$ , c[2] -> - $\frac{187}{138240}$ , c[3] -> - $\frac{169}{138240}$ ,
  c[4] -> - $\frac{169}{138240}$ , c[5] -> - $\frac{187}{138240}$ , c[6] -> - $\frac{19}{46080}$ , c[7] -> - $\frac{11}{15360}$ }
{- $\frac{11}{15360}$ , - $\frac{19}{46080}$ , - $\frac{187}{138240}$ , - $\frac{169}{138240}$ , - $\frac{169}{138240}$ , - $\frac{187}{138240}$ , - $\frac{19}{46080}$ , - $\frac{11}{15360}$ }
-138240 * cs
{99, 57, 187, 169, 169, 187, 57, 99}

```

```

pol = - $\frac{1}{4}$  Diag[1, ar[1, 2]] +  $\frac{1}{4}$  Diag[1, ar[2, 1]] +  $\frac{1}{32}$  Diag[h[1], ar[1, 2]] +
 $\frac{5}{96}$  Diag[h[1], ar[2, 1]] -  $\frac{19 \text{ Diag}[h[1]^3, ar[1, 2]]}{46080}$  -  $\frac{11 \text{ Diag}[h[1]^3, ar[2, 1]]}{15360}$  +
 $\frac{37 \text{ Diag}[h[1]^5, ar[1, 2]]}{4644864}$  +  $\frac{13 \text{ Diag}[h[1]^5, ar[2, 1]]}{1161216}$  +  $\frac{5}{96}$  Diag[h[2], ar[1, 2]] +
 $\frac{1}{32}$  Diag[h[2], ar[2, 1]] -  $\frac{169 \text{ Diag}[h[1]^2 h[2], ar[1, 2]]}{138240}$  -
 $\frac{187 \text{ Diag}[h[1]^2 h[2], ar[2, 1]]}{138240}$  +  $\frac{1279 \text{ Diag}[h[1]^4 h[2], ar[1, 2]]}{34836480}$  +
 $\frac{2669 \text{ Diag}[h[1]^4 h[2], ar[2, 1]]}{69672960}$  -  $\frac{187 \text{ Diag}[h[1] h[2]^2, ar[1, 2]]}{138240}$  -
 $\frac{169 \text{ Diag}[h[1] h[2]^2, ar[2, 1]]}{138240}$  +  $\frac{125 \text{ Diag}[h[1]^3 h[2]^2, ar[1, 2]]}{1741824}$  +
 $\frac{2539 \text{ Diag}[h[1]^3 h[2]^2, ar[2, 1]]}{34836480}$  -  $\frac{11 \text{ Diag}[h[2]^3, ar[1, 2]]}{15360}$  -  $\frac{19 \text{ Diag}[h[2]^3, ar[2, 1]]}{46080}$  +
 $\frac{2539 \text{ Diag}[h[1]^2 h[2]^3, ar[1, 2]]}{34836480}$  +  $\frac{125 \text{ Diag}[h[1]^2 h[2]^3, ar[2, 1]]}{1741824}$  +
 $\frac{2669 \text{ Diag}[h[1] h[2]^4, ar[1, 2]]}{69672960}$  +  $\frac{1279 \text{ Diag}[h[1] h[2]^4, ar[2, 1]]}{34836480}$  +
 $\frac{13 \text{ Diag}[h[2]^5, ar[1, 2]]}{1161216}$  +  $\frac{37 \text{ Diag}[h[2]^5, ar[2, 1]]}{4644864}$  /. {
  Diag[_, ar[2, 1]] -> 0,
  Diag[hs_, ar[1, 2]] -> hs
}

```

```

 $-\frac{1}{4} + \frac{h[1]}{32} - \frac{19 h[1]^3}{46080} + \frac{37 h[1]^5}{4644864} + \frac{5 h[2]}{96} - \frac{169 h[1]^2 h[2]}{138240} + \frac{1279 h[1]^4 h[2]}{34836480} - \frac{187 h[1] h[2]^2}{138240} +$ 
 $\frac{125 h[1]^3 h[2]^2}{1741824} - \frac{11 h[2]^3}{15360} + \frac{2539 h[1]^2 h[2]^3}{34836480} + \frac{2669 h[1] h[2]^4}{69672960} + \frac{13 h[2]^5}{1161216}$ 

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cs = {19 / 46080, 169 / 138240, 187 / 138240, 11 / 15360}

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{ $\frac{19}{46080}$ ,  $\frac{169}{138240}$ ,  $\frac{187}{138240}$ ,  $\frac{11}{15360}$ }

```

138 240 cs

{57, 169, 187, 99}

GCD@@ (138 240 cs)

1

Expand[4 pol]

$$-1 + \frac{h[1]}{8} - \frac{19 h[1]^3}{11520} + \frac{37 h[1]^5}{1161216} + \frac{5 h[2]}{24} - \frac{169 h[1]^2 h[2]}{34560} + \frac{1279 h[1]^4 h[2]}{8709120} - \frac{187 h[1] h[2]^2}{34560} +$$

$$\frac{125 h[1]^3 h[2]^2}{435456} - \frac{11 h[2]^3}{3840} + \frac{2539 h[1]^2 h[2]^3}{8709120} + \frac{2669 h[1] h[2]^4}{17418240} + \frac{13 h[2]^5}{290304}$$

cs1 = Table[Coefficient[4 * 17418240 * pol, h[1]^(5 - k) h[2]^k], {k, 0, 5}]

{555, 2558, 5000, 5078, 2669, 780}

$$\text{(Alt) In[1]:= ser} = 1 + \left(-\frac{x}{8} - \frac{5y}{24}\right) h + \frac{(57x^3 + 169x^2y + 187xy^2 + 99y^3)h^3}{34560} +$$

$$\frac{1}{17418240} (-555x^5 - 2558x^4y - 5000x^3y^2 - 5078x^2y^3 - 2669xy^4 - 780y^5)h^5 +$$

$$\frac{1}{50164531200} (31989x^7 + 205357x^6y + 592097x^5y^2 +$$

$$964985x^4y^3 + 949967x^3y^4 + 565943x^2y^5 + 186139xy^6 + 34275y^7)h^7 +$$

$$\frac{1}{397303087104000} (-5104911x^9 - 42750073x^8y - 165124408x^7y^2 -$$

$$376113312x^6y^3 - 552802446x^5y^4 - 542218746x^4y^5 - 352835712x^3y^6 -$$

$$146191048x^2y^7 - 33851563xy^8 - 4104741y^9)h^9 + O[h]^{10}$$

$$\text{(Alt) Out[1]=} 1 + \left(-\frac{x}{8} - \frac{5y}{24}\right) h + \frac{(57x^3 + 169x^2y + 187xy^2 + 99y^3)h^3}{34560} + \frac{1}{17418240}$$

$$(-555x^5 - 2558x^4y - 5000x^3y^2 - 5078x^2y^3 - 2669xy^4 - 780y^5)h^5 +$$

$$\frac{1}{50164531200} (31989x^7 + 205357x^6y + 592097x^5y^2 +$$

$$964985x^4y^3 + 949967x^3y^4 + 565943x^2y^5 + 186139xy^6 + 34275y^7)h^7 +$$

$$\frac{1}{397303087104000} (-5104911x^9 - 42750073x^8y - 165124408x^7y^2 -$$

$$376113312x^6y^3 - 552802446x^5y^4 - 542218746x^4y^5 - 352835712x^3y^6 -$$

$$146191048x^2y^7 - 33851563xy^8 - 4104741y^9)h^9 + O[h]^{10}$$

FindSequenceFunction[{24, 34560, 17418240}]

FindSequenceFunction[{24, 34560, 17418240}]

(Alt) In[2]:= ser /. {x | y -> 1}

$$\text{(Alt) Out[2]=} 1 - \frac{h}{3} + \frac{2h^3}{135} - \frac{13h^5}{13608} + \frac{431h^7}{6123600} - \frac{27113h^9}{4849891200} + O[h]^{10}$$

(Alt) In[3]:= FindSequenceFunction[{1/3, -2/135, 13/13608, -431/6123600, 27113/4849891200}]

$$\text{(Alt) Out[3]=} \text{FindSequenceFunction}\left[\left\{\frac{1}{3}, -\frac{2}{135}, \frac{13}{13608}, -\frac{431}{6123600}, \frac{27113}{4849891200}\right\}\right]$$

(Alt) In[4]:= **ser /. {x → 1, y → -1}**

$$(Alt) Out[4]= 1 + \frac{h}{12} - \frac{h^3}{1440} + \frac{h^5}{90720} - \frac{h^7}{4838400} + \frac{h^9}{239500800} + O[h]^{10}$$

(Alt) In[5]:= **FindSequenceFunction[{12, 1440, 90720, 4838400, 239500800}]**

(Alt) Out[5]= **FindSequenceFunction[{12, 1440, 90720, 4838400, 239500800}]**

Google finds 12, 1440, 90720, 4838400, 239500800 !

(Alt) In[9]:= **{24 / 12, 34560 / 1440, 17418240 / 90720, 50164531200 / 4838400, 397303087104000 / 239500800}**

(Alt) Out[9]= **{2, 24, 192, 10368, 1658880}**

(Alt) In[10]:= **FactorInteger[{2, 24, 192, 10368, 1658880}]**

(Alt) Out[10]= **{{{2, 1}}, {{2, 3}, {3, 1}}, {{2, 6}, {3, 1}}, {{2, 7}, {3, 4}}, {{2, 12}, {3, 4}, {5, 1}}}**

(Alt) In[12]:= **FactorInteger[{2, 24, 192, 10368, 1658880} / Range[5] !]**

(Alt) Out[12]= **{{{2, 1}}, {{2, 2}, {3, 1}}, {{2, 5}}, {{2, 4}, {3, 3}}, {{2, 9}, {3, 3}}}**

(Alt) In[14]:= **Simplify[ser /. {x → (a + b) / 2, y → (a - b) / 2}]**

$$(Alt) Out[14]= 1 + \frac{1}{24} (-4a + b)h + \frac{(64a^3 - 18a^2b + 14ab^2 - 3b^3)h^3}{34560} + \frac{(-1040a^5 + 96a^4b - 228a^3b^2 + 117a^2b^3 - 67ab^4 + 12b^5)h^5}{34836480} + \frac{1}{50164531200} (27584a^7 + 1356a^6b + 2564a^5b^2 - 1248a^4b^3 + 2472a^3b^4 - 1170a^2b^5 + 512ab^6 - 81b^7)h^7 + \frac{1}{397303087104000} (-4338080a^9 - 481200a^8b + 50744a^7b^2 - 204330a^6b^3 - 138390a^5b^4 + 118080a^4b^5 - 156782a^3b^6 + 64125a^2b^7 - 22318ab^8 + 3240b^9)h^9 + O[h]^{10}$$