

Pensieve Header: Alexander blobs Results.

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In[3]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations"];
<< "AlexanderBlobs-Program.m"

In[5]:= b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]]
Out[5]= -Diag[h[1], ar[2, 3]] + Diag[h[2], ar[1, 3]]

In[6]:= b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]] + b[r[1, 3], r[2, 3]]
Out[6]= 0

In[7]:= ModDegree[4, R[1, 2]]
Out[7]= Diag[1] + Diag[1, ar[1, 2]] +

$$\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] + \frac{1}{6} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]]$$

In[8]:= ModDegree[7, R[1, 2] ** R[1, 3] ** R[2, 3] - R[2, 3] ** R[1, 3] ** R[1, 2]]
Out[8]= 0

In[9]:= v[0] = 0;

In[10]:= d = 1; ModDegree[d + 1,
Print[
v[d] = v[d - 1] + c1 Diag[1, ar[1, 2]] + c2 Diag[1, ar[2, 1]]
];
V[d] = DExp[v[d]];
{R4Eqn[V[d]], TwistEqn[V[d]]}
]
c1 Diag[1, ar[1, 2]] + c2 Diag[1, ar[2, 1]]
Out[10]= {0, - $\frac{1}{2}$  Diag[1, ar[1, 2]] - c1 Diag[1, ar[1, 2]] + c2 Diag[1, ar[1, 2]] +
 $\frac{1}{2}$  Diag[1, ar[2, 1]] + c1 Diag[1, ar[2, 1]] - c2 Diag[1, ar[2, 1]]}
In[11]:= Solve[{-1/2 - c1 + c2 == 0, 1/2 + c1 - c2 == 0}, {c1, c2}]
Solve::svars : Equations may not give solutions for all "solve" variables. >>
Out[11]= {{c2 ->  $\frac{1}{2}$  + c1}}
In[12]:= d = 1; ModDegree[d + 1,
Print[
v[d] = v[d - 1] + c1 Diag[1, ar[1, 2]] + (1/2 + c1) Diag[1, ar[2, 1]]
];
V[d] = DExp[v[d]];
{R4Eqn[V[d]], TwistEqn[V[d]]}
]
c1 Diag[1, ar[1, 2]] +  $\left(\frac{1}{2} + c1\right)$  Diag[1, ar[2, 1]]
Out[12]= {0, 0}
```

```
In[13]:= d = 2; ModDegree[d + 1,
```

```
Print[
  v[d] = v[d - 1]
];
V[d] = DExp[v[d]];
{R4Eqn[V[d]], TwistEqn[V[d]]}
]
```

$$c1 \text{Diag}[1, \text{ar}[1, 2]] + \left(\frac{1}{2} + c1\right) \text{Diag}[1, \text{ar}[2, 1]]$$

$$\text{Out[13]}= \left\{0, \frac{1}{8} \text{Diag}[h[1], \text{ar}[2, 1]] + \frac{1}{2} c1 \text{Diag}[h[1], \text{ar}[2, 1]] + \frac{1}{8} \text{Diag}[h[1], \text{ar}[2, 2]] + \frac{1}{2} c1 \text{Diag}[h[1], \text{ar}[2, 2]] - \frac{1}{8} \text{Diag}[h[2], \text{ar}[1, 1]] - \frac{1}{2} c1 \text{Diag}[h[2], \text{ar}[1, 1]] - \frac{1}{8} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{2} c1 \text{Diag}[h[2], \text{ar}[1, 2]]\right\}$$

```
In[14]:= d = 2; ModDegree[d + 1,
```

```
Print[
  v[d] = (v[d - 1] /. c1 -> -1/4)
];
V[d] = DExp[v[d]];
{R4Eqn[V[d]], TwistEqn[V[d]]}
]
```

$$-\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]]$$

```
Out[14]= {0, 0}
```

```
In[15]:= d = 3; ModDegree[d + 1,
```

```
Print[
  v[d] = v[d - 1] + c21 r12 ** r12 + c22 r12 ** r21 +
    c23 r21 ** r12 + c24 r21 ** r21 + c25 Diag[h[1], ar[1, 2]] +
    c26 Diag[h[2], ar[1, 2]] + c27 Diag[h[1], ar[2, 1]] + c28 Diag[h[2], ar[2, 1]]
];
V[d] = DExp[v[d]];
{R4Eqn[V[d]], TwistEqn[V[d]]}
] // PullDiags
```

$$-\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + c25 \text{Diag}[h[1], \text{ar}[1, 2]] + c27 \text{Diag}[h[1], \text{ar}[2, 1]] + c26 \text{Diag}[h[2], \text{ar}[1, 2]] + c28 \text{Diag}[h[2], \text{ar}[2, 1]] + c21 \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] + c22 \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1]] + c23 (\text{Diag}[h[1], \text{ar}[2, 1]] + \text{Diag}[h[1], \text{ar}[2, 2]] - \text{Diag}[h[2], \text{ar}[1, 1]] - \text{Diag}[h[2], \text{ar}[1, 2]] + \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1]]) + c24 \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1]]$$

$$\text{Out[15]}= \left\{\left(-\frac{1}{48} - c21 + c23 - c25 + c27\right) \text{Diag}[h[1]^2, \text{ar}[2, 3]] + \left(\frac{1}{48} + c21 - c23 + c25 - c27\right) \text{Diag}[h[1] h[2], \text{ar}[1, 3]] + \left(\frac{1}{48} + c23 - c24 - c26 + c28\right) \text{Diag}[h[1] h[2], \text{ar}[2, 3]] + \right\}$$

$$\begin{aligned}
& \left(-\frac{1}{48} - c_{23} + c_{24} + c_{26} - c_{28} \right) \text{Diag}[h[2]^2, \text{ar}[1, 3]] + \\
& (-2 c_{21} + c_{22} + c_{23}) \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 3]] + \\
& (-c_{22} - c_{23} + 2 c_{24}) \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 3]] + \\
& (2 c_{21} - c_{22} - c_{23}) \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[1, 3]] + \\
& (c_{22} + c_{23} - 2 c_{24}) \text{Diag}[h[2], \text{ar}[1, 3], \text{ar}[2, 1]], (-c_{25} + c_{28}) \text{Diag}[h[1], \text{ar}[1, 2]] + \\
& (c_{22} - c_{23} + c_{26} - c_{27}) \text{Diag}[h[1], \text{ar}[2, 1]] + (c_{22} - c_{23}) \text{Diag}[h[1], \text{ar}[2, 2]] + \\
& \left(-\frac{1}{32} - \frac{c_{21}}{8} + \frac{5 c_{22}}{8} + \frac{c_{23}}{8} - \frac{c_{24}}{8} - \frac{c_{25}}{8} + \frac{5 c_{26}}{8} + \frac{c_{27}}{8} - \frac{c_{28}}{8} \right) \text{Diag}[h[1]^2, \text{ar}[2, 1]] + \\
& \left(-\frac{1}{32} - \frac{c_{21}}{8} + \frac{5 c_{22}}{8} + \frac{c_{23}}{8} - \frac{c_{24}}{8} - \frac{c_{25}}{8} + \frac{5 c_{26}}{8} + \frac{c_{27}}{8} - \frac{c_{28}}{8} \right) \text{Diag}[h[1]^2, \text{ar}[2, 2]] + \\
& (-c_{22} + c_{23}) \text{Diag}[h[2], \text{ar}[1, 1]] + \\
& (-c_{22} + c_{23} - c_{26} + c_{27}) \text{Diag}[h[2], \text{ar}[1, 2]] + (c_{25} - c_{28}) \text{Diag}[h[2], \text{ar}[2, 1]] + \\
& \left(\frac{1}{32} + \frac{c_{21}}{8} - \frac{5 c_{22}}{8} - \frac{c_{23}}{8} + \frac{c_{24}}{8} + \frac{c_{25}}{8} - \frac{5 c_{26}}{8} - \frac{c_{27}}{8} + \frac{c_{28}}{8} \right) \text{Diag}[h[1] h[2], \text{ar}[1, 1]] + \\
& \left(\frac{1}{32} + \frac{c_{21}}{8} - \frac{5 c_{22}}{8} - \frac{c_{23}}{8} + \frac{c_{24}}{8} + \frac{c_{25}}{8} - \frac{5 c_{26}}{8} - \frac{c_{27}}{8} + \frac{c_{28}}{8} \right) \text{Diag}[h[1] h[2], \text{ar}[1, 2]] + \\
& \left(-\frac{1}{96} - \frac{5 c_{21}}{8} + \frac{c_{22}}{8} + \frac{c_{23}}{8} - \frac{c_{24}}{8} + \frac{5 c_{25}}{8} - \frac{c_{26}}{8} - \frac{c_{27}}{8} + \frac{c_{28}}{8} \right) \text{Diag}[h[1] h[2], \text{ar}[2, 1]] + \\
& \left(-\frac{1}{96} - \frac{5 c_{21}}{8} + \frac{c_{22}}{8} + \frac{c_{23}}{8} - \frac{c_{24}}{8} + \frac{5 c_{25}}{8} - \frac{c_{26}}{8} - \frac{c_{27}}{8} + \frac{c_{28}}{8} \right) \text{Diag}[h[1] h[2], \text{ar}[2, 2]] + \\
& \left(\frac{1}{96} + \frac{5 c_{21}}{8} - \frac{c_{22}}{8} - \frac{c_{23}}{8} + \frac{c_{24}}{8} - \frac{5 c_{25}}{8} + \frac{c_{26}}{8} + \frac{c_{27}}{8} - \frac{c_{28}}{8} \right) \text{Diag}[h[2]^2, \text{ar}[1, 1]] + \\
& \left(\frac{1}{96} + \frac{5 c_{21}}{8} - \frac{c_{22}}{8} - \frac{c_{23}}{8} + \frac{c_{24}}{8} - \frac{5 c_{25}}{8} + \frac{c_{26}}{8} + \frac{c_{27}}{8} - \frac{c_{28}}{8} \right) \text{Diag}[h[2]^2, \text{ar}[1, 2]] + \\
& (-c_{21} + c_{24}) \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] + (c_{21} - c_{24}) \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1]] + \\
& \left(-\frac{3 c_{25}}{4} + \frac{3 c_{28}}{4} \right) \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[1, 2]] + \\
& \left(-\frac{c_{21}}{4} + \frac{3 c_{22}}{2} - \frac{c_{24}}{4} - \frac{c_{25}}{4} + \frac{3 c_{26}}{4} - \frac{3 c_{27}}{4} + \frac{c_{28}}{4} \right) \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 1]] + \\
& \left(-\frac{c_{21}}{4} + \frac{3 c_{22}}{2} - \frac{c_{24}}{4} \right) \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 2]] + \\
& \left(\frac{5 c_{21}}{4} - \frac{c_{23}}{2} + \frac{c_{24}}{4} + \frac{c_{26}}{4} - \frac{c_{27}}{4} \right) \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 1]] + \\
& \left(\frac{5 c_{21}}{4} - \frac{c_{23}}{2} + \frac{c_{24}}{4} \right) \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 2]] + \\
& \left(\frac{c_{21}}{4} - \frac{3 c_{22}}{2} + \frac{c_{24}}{4} \right) \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[1, 2]] + \\
& \left(-\frac{5 c_{21}}{4} + \frac{c_{23}}{2} - \frac{c_{24}}{4} \right) \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[2, 1]] + \\
& \left(\frac{c_{21}}{4} - \frac{3 c_{22}}{2} + \frac{c_{24}}{4} - \frac{3 c_{26}}{4} + \frac{3 c_{27}}{4} \right) \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[1, 2]] + \\
& \left(-\frac{5 c_{21}}{4} + \frac{c_{23}}{2} - \frac{c_{24}}{4} + \frac{3 c_{25}}{4} - \frac{c_{26}}{4} + \frac{c_{27}}{4} - \frac{3 c_{28}}{4} \right) \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[2, 1]] +
\end{aligned}$$

$$\left(\frac{c25}{4} - \frac{c28}{4}\right) \text{Diag}[h[2], \text{ar}[2, 1], \text{ar}[2, 1]] +$$

$$\left(-\frac{3c21}{4} + \frac{3c24}{4}\right) \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] +$$

$$\left(-\frac{c21}{4} + \frac{c24}{4}\right) \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[2, 1]] +$$

$$\left(\frac{3c21}{4} - \frac{3c24}{4}\right) \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1], \text{ar}[2, 1]] +$$

$$\left(\frac{c21}{4} - \frac{c24}{4}\right) \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1], \text{ar}[2, 1]]$$

In[16]:= $\text{Solve}\left[\left\{\left(-\frac{1}{48} - c21 + c23 - c25 + c27\right) = 0, \left(\frac{1}{48} + c23 - c24 - c26 + c28\right) = 0, \right.\right.$

$\left.(-2c21 + c22 + c23) = 0, (-c22 - c23 + 2c24) = 0, (-c25 + c28) = 0, \right.$

$\left.(c22 - c23 + c26 - c27) = 0, (c22 - c23) = 0, (c21 - c24) = 0\right\},$

$\{c21, c22, c23, c24, c25, c26, c27, c28\}$

Solve::svars : Equations may not give solutions for all "solve" variables. >>

Out[16]= $\left\{\{c22 \rightarrow c21, c23 \rightarrow c21, c24 \rightarrow c21, c26 \rightarrow \frac{1}{48} + c25, c27 \rightarrow \frac{1}{48} + c25, c28 \rightarrow c25\}\right\}$

In[17]:= $\mathbf{d = 3; ModDegree[d + 1,}$

Print[

$\mathbf{v[d] = v[d - 1] + 0 r12 ** r12 + 0 r12 ** r21 + 0 r21 ** r12 +}$

$\mathbf{0 r21 ** r21 + c25 Diag[h[1], ar[1, 2]] + \left(\frac{1}{48} + c25\right) Diag[h[2], ar[1, 2]] +}$

$\mathbf{\left(\frac{1}{48} + c25\right) Diag[h[1], ar[2, 1]] + c25 Diag[h[2], ar[2, 1]] /. c25 \rightarrow 1 / 32}$

$\mathbf{];}$

$\mathbf{V[d] = DExp[v[d]];}$

$\mathbf{\{R4Eqn[V[d]], TwistEqn[V[d]]\}}$

$\mathbf{] // PullDiags}$

$-\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[h[1], \text{ar}[1, 2]] +$

$\frac{5}{96} \text{Diag}[h[1], \text{ar}[2, 1]] + \frac{5}{96} \text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[h[2], \text{ar}[2, 1]]$

Out[17]= $\{0, 0\}$

```

In[18]:= While[True,
  Print["At d=", ++d];
  ModDegree[d + 1,
    v[d] = v[d - 1] + Sum[c[2 k] Diag[h[1] ^ k h[2] ^ (d - 2 - k), ar[1, 2]] +
      c[2 k + 1] Diag[h[1] ^ k h[2] ^ (d - 2 - k), ar[2, 1]], {k, 0, d - 2}];
  V[d] = DExp[v[d]];
  eq1 = R4Eqn[V[d]];
  diags = Union[Cases[eq1, _Diag, Infinity]];
  eqns = Union[Coefficient[eq1, #] & /@ diags];
  eq2 = TwistEqn[V[d]];
  diags = Union[Cases[eq2, _Diag, Infinity]];
  eqns = Union[eqns, Coefficient[eq2, #] & /@ diags];
  Print[eqns];
  {sol} = Solve[({# == 0} & /@ eqns, c /@ Range[0, 2 d - 3]);
  Print["sol is ", sol];
  v[d] = (v[d] /. sol);
  Print["v is ", v[d]];
  Print["pol is ", Series[
    -4 v[d] /. {Diag[_ , ar[2, 1]] -> 0, Diag[hs_ , ar[1, 2]] :-> hs} /.
    {h[1] -> h x, h[2] -> h y},
    {h, 0, d - 2}
  ]];
  V[d] = DExp[v[d]];
  Print["{R4Eqn[V[d]], TwistEqn[V[d]]}=", {R4Eqn[V[d]], TwistEqn[V[d]]}];
]
]

```

At d=4

$$\left\{ c[0] - c[1], -c[0] + c[1], -\frac{c[2]}{2}, \frac{c[2]}{2}, c[2] - c[3], \frac{c[2]}{2} - \frac{c[3]}{2}, \frac{c[2]}{4} - \frac{c[3]}{4}, \right. \\
-\frac{3c[2]}{4} + \frac{3c[3]}{4}, -c[2] + c[3], c[1] - c[4], \frac{3c[1]}{4} - \frac{3c[4]}{4}, -\frac{c[1]}{4} + \frac{c[4]}{4}, \\
-c[1] + c[4], c[0] - c[5], c[4] - c[5], \frac{3c[0]}{4} + \frac{c[1]}{4} - \frac{c[4]}{4} - \frac{3c[5]}{4}, \\
\frac{c[0]}{4} - \frac{c[5]}{4}, -\frac{5c[0]}{8} + \frac{c[1]}{8} + \frac{c[4]}{8} - \frac{c[5]}{8}, -\frac{c[0]}{8} + \frac{c[1]}{8} + \frac{5c[4]}{8} - \frac{c[5]}{8}, \\
\frac{c[0]}{8} - \frac{c[1]}{8} - \frac{5c[4]}{8} + \frac{c[5]}{8}, \frac{5c[0]}{8} - \frac{c[1]}{8} - \frac{c[4]}{8} + \frac{c[5]}{8}, \\
-\frac{c[0]}{4} - \frac{3c[1]}{4} + \frac{3c[4]}{4} + \frac{c[5]}{4}, -\frac{3c[0]}{4} + \frac{3c[5]}{4}, -c[0] + c[5], -c[4] + c[5] \left. \right\}$$

sol is {c[0] -> 0, c[1] -> 0, c[2] -> 0, c[3] -> 0, c[4] -> 0, c[5] -> 0}

$$\begin{aligned}
v \text{ is } & -\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[h[1], \text{ar}[1, 2]] + \\
& \frac{5}{96} \text{Diag}[h[1], \text{ar}[2, 1]] + \frac{5}{96} \text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[h[2], \text{ar}[2, 1]]
\end{aligned}$$

$$\text{pol is } 1 + \left(-\frac{x}{8} - \frac{5y}{24} \right) h + O[h]^3$$

{R4Eqn[V[d]], TwistEqn[V[d]]}={0, 0}

At d=5

$$\left\{ \frac{7}{23040} + c[0] - c[1], -\frac{7}{23040} - c[0] + c[1], \frac{1}{7680} + c[2] - c[3], -\frac{1}{7680} - c[2] + c[3], \right.$$

$$c[3] - c[4], \frac{3c[3]}{4} - \frac{3c[4]}{4}, -\frac{c[3]}{4} + \frac{c[4]}{4}, -c[3] + c[4], c[2] - c[5], -\frac{1}{7680} + c[4] - c[5],$$

$$\frac{3c[2]}{4} + \frac{c[3]}{4} - \frac{c[4]}{4} - \frac{3c[5]}{4}, \frac{c[2]}{4} - \frac{c[5]}{4}, -\frac{49}{69120} - \frac{5c[2]}{8} + \frac{c[3]}{8} + \frac{c[4]}{8} - \frac{c[5]}{8},$$

$$\frac{1}{1728} - \frac{c[2]}{8} + \frac{c[3]}{8} + \frac{5c[4]}{8} - \frac{c[5]}{8}, -\frac{1}{1728} + \frac{c[2]}{8} - \frac{c[3]}{8} - \frac{5c[4]}{8} + \frac{c[5]}{8},$$

$$\frac{49}{69120} + \frac{5c[2]}{8} - \frac{c[3]}{8} - \frac{c[4]}{8} + \frac{c[5]}{8}, -\frac{c[2]}{4} - \frac{3c[3]}{4} + \frac{3c[4]}{4} + \frac{c[5]}{4}, -\frac{3c[2]}{4} + \frac{3c[5]}{4},$$

$$-c[2] + c[5], \frac{1}{7680} - c[4] + c[5], c[1] - c[6], \frac{3c[1]}{4} - \frac{3c[6]}{4}, -\frac{c[1]}{4} + \frac{c[6]}{4},$$

$$-c[1] + c[6], c[0] - c[7], -\frac{7}{23040} + c[6] - c[7], \frac{3c[0]}{4} + \frac{c[1]}{4} - \frac{c[6]}{4} - \frac{3c[7]}{4},$$

$$\frac{c[0]}{4} - \frac{c[7]}{4}, -\frac{1}{2304} - \frac{5c[0]}{8} + \frac{c[1]}{8} + \frac{c[6]}{8} - \frac{c[7]}{8}, \frac{1}{7680} - \frac{c[0]}{8} + \frac{c[1]}{8} + \frac{5c[6]}{8} - \frac{c[7]}{8},$$

$$-\frac{1}{7680} + \frac{c[0]}{8} - \frac{c[1]}{8} - \frac{5c[6]}{8} + \frac{c[7]}{8}, \frac{1}{2304} + \frac{5c[0]}{8} - \frac{c[1]}{8} - \frac{c[6]}{8} + \frac{c[7]}{8},$$

$$-\frac{c[0]}{4} - \frac{3c[1]}{4} + \frac{3c[6]}{4} + \frac{c[7]}{4}, -\frac{3c[0]}{4} + \frac{3c[7]}{4}, -c[0] + c[7], \frac{7}{23040} - c[6] + c[7] \left. \right\}$$

$$\text{sol is } \left\{ c[0] \rightarrow -\frac{11}{15360}, c[1] \rightarrow -\frac{19}{46080}, c[2] \rightarrow -\frac{187}{138240}, c[3] \rightarrow -\frac{169}{138240}, \right.$$

$$\left. c[4] \rightarrow -\frac{169}{138240}, c[5] \rightarrow -\frac{187}{138240}, c[6] \rightarrow -\frac{19}{46080}, c[7] \rightarrow -\frac{11}{15360} \right\}$$

$$\text{v is } -\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[\text{h}[1], \text{ar}[1, 2]] +$$

$$\frac{5}{96} \text{Diag}[\text{h}[1], \text{ar}[2, 1]] - \frac{19 \text{Diag}[\text{h}[1]^3, \text{ar}[1, 2]]}{46080} - \frac{11 \text{Diag}[\text{h}[1]^3, \text{ar}[2, 1]]}{15360} +$$

$$\frac{5}{96} \text{Diag}[\text{h}[2], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[\text{h}[2], \text{ar}[2, 1]] - \frac{169 \text{Diag}[\text{h}[1]^2 \text{h}[2], \text{ar}[1, 2]]}{138240} -$$

$$\frac{187 \text{Diag}[\text{h}[1]^2 \text{h}[2], \text{ar}[2, 1]]}{138240} - \frac{187 \text{Diag}[\text{h}[1] \text{h}[2]^2, \text{ar}[1, 2]]}{138240} -$$

$$\frac{169 \text{Diag}[\text{h}[1] \text{h}[2]^2, \text{ar}[2, 1]]}{138240} - \frac{11 \text{Diag}[\text{h}[2]^3, \text{ar}[1, 2]]}{15360} - \frac{19 \text{Diag}[\text{h}[2]^3, \text{ar}[2, 1]]}{46080}$$

$$\text{pol is } 1 + \left(-\frac{x}{8} - \frac{5y}{24} \right) h + \frac{(57x^3 + 169x^2y + 187xy^2 + 99y^3)h^3}{34560} + O[h]^4$$

$$\{\text{R4Eqn}[\text{V}[\text{d}]], \text{TwistEqn}[\text{V}[\text{d}]]\} = \{0, 0\}$$

At d=6

$$\left\{ c[0] - c[1], -c[0] + c[1], c[2] - c[3], -c[2] + c[3], -\frac{c[4]}{2}, \frac{c[4]}{2}, c[4] - c[5], \frac{c[4]}{2} - \frac{c[5]}{2}, \right. \\ \left. \frac{c[4]}{4} - \frac{c[5]}{4}, -\frac{3c[4]}{4} + \frac{3c[5]}{4}, -c[4] + c[5], c[3] - c[6], \frac{3c[3]}{4} - \frac{3c[6]}{4}, -\frac{c[3]}{4} + \frac{c[6]}{4}, \right. \\ \left. -c[3] + c[6], c[2] - c[7], c[6] - c[7], \frac{3c[2]}{4} + \frac{c[3]}{4} - \frac{c[6]}{4} - \frac{3c[7]}{4}, \frac{c[2]}{4} - \frac{c[7]}{4}, \right. \\ \left. -\frac{5c[2]}{8} + \frac{c[3]}{8} + \frac{c[6]}{8} - \frac{c[7]}{8}, -\frac{c[2]}{8} + \frac{c[3]}{8} + \frac{5c[6]}{8} - \frac{c[7]}{8}, \frac{c[2]}{8} - \frac{c[3]}{8} - \frac{5c[6]}{8} + \frac{c[7]}{8}, \right. \\ \left. \frac{5c[2]}{8} - \frac{c[3]}{8} - \frac{c[6]}{8} + \frac{c[7]}{8}, -\frac{c[2]}{4} - \frac{3c[3]}{4} + \frac{3c[6]}{4} + \frac{c[7]}{4}, -\frac{3c[2]}{4} + \frac{3c[7]}{4}, -c[2] + c[7], \right. \\ \left. -c[6] + c[7], c[1] - c[8], \frac{3c[1]}{4} - \frac{3c[8]}{4}, -\frac{c[1]}{4} + \frac{c[8]}{4}, -c[1] + c[8], c[0] - c[9], \right. \\ \left. c[8] - c[9], \frac{3c[0]}{4} + \frac{c[1]}{4} - \frac{c[8]}{4} - \frac{3c[9]}{4}, \frac{c[0]}{4} - \frac{c[9]}{4}, -\frac{5c[0]}{8} + \frac{c[1]}{8} + \frac{c[8]}{8} - \frac{c[9]}{8}, \right. \\ \left. -\frac{c[0]}{8} + \frac{c[1]}{8} + \frac{5c[8]}{8} - \frac{c[9]}{8}, \frac{c[0]}{8} - \frac{c[1]}{8} - \frac{5c[8]}{8} + \frac{c[9]}{8}, \frac{5c[0]}{8} - \frac{c[1]}{8} - \frac{c[8]}{8} + \frac{c[9]}{8}, \right. \\ \left. -\frac{c[0]}{4} - \frac{3c[1]}{4} + \frac{3c[8]}{4} + \frac{c[9]}{4}, -\frac{3c[0]}{4} + \frac{3c[9]}{4}, -c[0] + c[9], -c[8] + c[9] \right\}$$

sol is {c[0] → 0, c[1] → 0, c[2] → 0, c[3] → 0,
c[4] → 0, c[5] → 0, c[6] → 0, c[7] → 0, c[8] → 0, c[9] → 0}

$$v \text{ is } -\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[h[1], \text{ar}[1, 2]] + \\ \frac{5}{96} \text{Diag}[h[1], \text{ar}[2, 1]] - \frac{19 \text{Diag}[h[1]^3, \text{ar}[1, 2]]}{46080} - \frac{11 \text{Diag}[h[1]^3, \text{ar}[2, 1]]}{15360} + \\ \frac{5}{96} \text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[h[2], \text{ar}[2, 1]] - \frac{169 \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]]}{138240} - \\ \frac{187 \text{Diag}[h[1]^2 h[2], \text{ar}[2, 1]]}{138240} - \frac{187 \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]]}{138240} - \\ \frac{169 \text{Diag}[h[1] h[2]^2, \text{ar}[2, 1]]}{138240} - \frac{11 \text{Diag}[h[2]^3, \text{ar}[1, 2]]}{15360} - \frac{19 \text{Diag}[h[2]^3, \text{ar}[2, 1]]}{46080}$$

$$\text{pol is } 1 + \left(-\frac{x}{8} - \frac{5y}{24} \right) h + \frac{(57x^3 + 169x^2y + 187xy^2 + 99y^3)h^3}{34560} + O[h]^5$$

{R4Eqn[V[d]], TwistEqn[V[d]]}={0, 0}

At d=7

$$\left\{ -\frac{5}{1548288} + c[0] - c[1], \frac{5}{1548288} - c[0] + c[1], -\frac{37}{23224320} + c[2] - c[3], \right.$$

$$\frac{37}{23224320} - c[2] + c[3], -\frac{13}{11612160} + c[4] - c[5], \frac{13}{11612160} - c[4] + c[5], c[5] - c[6],$$

$$\frac{3c[5]}{4} - \frac{3c[6]}{4}, -\frac{c[5]}{4} + \frac{c[6]}{4}, -c[5] + c[6], c[4] - c[7], \frac{13}{11612160} + c[6] - c[7],$$

$$\frac{3c[4]}{4} + \frac{c[5]}{4} - \frac{c[6]}{4} - \frac{3c[7]}{4}, \frac{c[4]}{4} - \frac{c[7]}{4}, \frac{731}{19906560} - \frac{5c[4]}{8} + \frac{c[5]}{8} + \frac{c[6]}{8} - \frac{c[7]}{8},$$

$$-\frac{4961}{139345920} - \frac{c[4]}{8} + \frac{c[5]}{8} + \frac{5c[6]}{8} - \frac{c[7]}{8}, \frac{4961}{139345920} + \frac{c[4]}{8} - \frac{c[5]}{8} - \frac{5c[6]}{8} + \frac{c[7]}{8},$$

$$-\frac{731}{19906560} + \frac{5c[4]}{8} - \frac{c[5]}{8} - \frac{c[6]}{8} + \frac{c[7]}{8}, -\frac{c[4]}{4} - \frac{3c[5]}{4} + \frac{3c[6]}{4} + \frac{c[7]}{4},$$

$$-\frac{3c[4]}{4} + \frac{3c[7]}{4}, -c[4] + c[7], -\frac{13}{11612160} - c[6] + c[7], c[3] - c[8],$$

$$\frac{3c[3]}{4} - \frac{3c[8]}{4}, -\frac{c[3]}{4} + \frac{c[8]}{4}, -c[3] + c[8], c[2] - c[9], \frac{37}{23224320} + c[8] - c[9],$$

$$\frac{3c[2]}{4} + \frac{c[3]}{4} - \frac{c[8]}{4} - \frac{3c[9]}{4}, \frac{c[2]}{4} - \frac{c[9]}{4}, \frac{5449}{278691840} - \frac{5c[2]}{8} + \frac{c[3]}{8} + \frac{c[8]}{8} - \frac{c[9]}{8},$$

$$-\frac{143}{7962624} - \frac{c[2]}{8} + \frac{c[3]}{8} + \frac{5c[8]}{8} - \frac{c[9]}{8}, \frac{143}{7962624} + \frac{c[2]}{8} - \frac{c[3]}{8} - \frac{5c[8]}{8} + \frac{c[9]}{8},$$

$$-\frac{5449}{278691840} + \frac{5c[2]}{8} - \frac{c[3]}{8} - \frac{c[8]}{8} + \frac{c[9]}{8}, -\frac{c[2]}{4} - \frac{3c[3]}{4} + \frac{3c[8]}{4} + \frac{c[9]}{4},$$

$$\frac{3c[2]}{4} + \frac{3c[9]}{4}, -c[2] + c[9], -\frac{37}{23224320} - c[8] + c[9], c[1] - c[10],$$

$$\frac{3c[1]}{4} - \frac{3c[10]}{4}, -\frac{c[1]}{4} + \frac{c[10]}{4}, -c[1] + c[10], c[0] - c[11], \frac{5}{1548288} + c[10] - c[11],$$

$$\frac{3c[0]}{4} + \frac{c[1]}{4} - \frac{c[10]}{4} - \frac{3c[11]}{4}, \frac{c[0]}{4} - \frac{c[11]}{4}, \frac{17}{2654208} - \frac{5c[0]}{8} + \frac{c[1]}{8} + \frac{c[10]}{8} - \frac{c[11]}{8},$$

$$-\frac{59}{18579456} - \frac{c[0]}{8} + \frac{c[1]}{8} + \frac{5c[10]}{8} - \frac{c[11]}{8}, \frac{59}{18579456} + \frac{c[0]}{8} - \frac{c[1]}{8} - \frac{5c[10]}{8} + \frac{c[11]}{8},$$

$$-\frac{17}{2654208} + \frac{5c[0]}{8} - \frac{c[1]}{8} - \frac{c[10]}{8} + \frac{c[11]}{8}, -\frac{c[0]}{4} - \frac{3c[1]}{4} + \frac{3c[10]}{4} + \frac{c[11]}{4},$$

$$-\frac{3c[0]}{4} + \frac{3c[11]}{4}, -c[0] + c[11], -\frac{5}{1548288} - c[10] + c[11] \}$$

$$\text{sol is } \left\{ c[0] \rightarrow \frac{13}{1161216}, c[1] \rightarrow \frac{37}{4644864}, c[2] \rightarrow \frac{2669}{69672960}, c[3] \rightarrow \frac{1279}{34836480}, \right.$$

$$c[4] \rightarrow \frac{2539}{34836480}, c[5] \rightarrow \frac{125}{1741824}, c[6] \rightarrow \frac{125}{1741824}, c[7] \rightarrow \frac{2539}{34836480},$$

$$c[8] \rightarrow \frac{1279}{34836480}, c[9] \rightarrow \frac{2669}{69672960}, c[10] \rightarrow \frac{37}{4644864}, c[11] \rightarrow \frac{13}{1161216} \}$$

$$\begin{aligned}
v \text{ is } & -\frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[h[1], \text{ar}[1, 2]] + \\
& \frac{5}{96} \text{Diag}[h[1], \text{ar}[2, 1]] - \frac{19 \text{Diag}[h[1]^3, \text{ar}[1, 2]]}{46080} - \frac{11 \text{Diag}[h[1]^3, \text{ar}[2, 1]]}{15360} + \\
& \frac{37 \text{Diag}[h[1]^5, \text{ar}[1, 2]]}{4644864} + \frac{13 \text{Diag}[h[1]^5, \text{ar}[2, 1]]}{1161216} + \frac{5}{96} \text{Diag}[h[2], \text{ar}[1, 2]] + \\
& \frac{1}{32} \text{Diag}[h[2], \text{ar}[2, 1]] - \frac{169 \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]]}{138240} - \\
& \frac{187 \text{Diag}[h[1]^2 h[2], \text{ar}[2, 1]]}{138240} + \frac{1279 \text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{34836480} + \\
& \frac{2669 \text{Diag}[h[1]^4 h[2], \text{ar}[2, 1]]}{69672960} - \frac{187 \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]]}{138240} - \\
& \frac{169 \text{Diag}[h[1] h[2]^2, \text{ar}[2, 1]]}{138240} + \frac{125 \text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{1741824} + \\
& \frac{2539 \text{Diag}[h[1]^3 h[2]^2, \text{ar}[2, 1]]}{34836480} - \frac{11 \text{Diag}[h[2]^3, \text{ar}[1, 2]]}{15360} - \\
& \frac{19 \text{Diag}[h[2]^3, \text{ar}[2, 1]]}{46080} + \frac{2539 \text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{34836480} + \\
& \frac{125 \text{Diag}[h[1]^2 h[2]^3, \text{ar}[2, 1]]}{1741824} + \frac{2669 \text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{69672960} + \\
& \frac{1279 \text{Diag}[h[1] h[2]^4, \text{ar}[2, 1]]}{34836480} + \frac{13 \text{Diag}[h[2]^5, \text{ar}[1, 2]]}{1161216} + \frac{37 \text{Diag}[h[2]^5, \text{ar}[2, 1]]}{4644864} \\
\text{pol is } & 1 + \left(-\frac{x}{8} - \frac{5y}{24} \right) h + \frac{(57x^3 + 169x^2y + 187xy^2 + 99y^3) h^3}{34560} + \\
& \frac{(-555x^5 - 2558x^4y - 5000x^3y^2 - 5078x^2y^3 - 2669xy^4 - 780y^5) h^5}{17418240} + O[h]^6
\end{aligned}$$

{R4Eqn[V[d]], TwistEqn[V[d]]}={0, 0}

At d=8

Out[18]= \$Aborted

```

In[19]:= Clear[Phi];
Phi[d_] := ModDegree[d+1,
  V[d] = DExp[v[d]];
  Phi[d] = PutOn[{1, 2}, 3, Adjoint[V[d]]] **
    Adjoint[V[d]] ** PutOn[2, 3, V[d]] ** PutOn[1, {2, 3}, V[d]]
]

```

In[21]:= Phi[2]

$$\begin{aligned}
\text{Out[21]} = & \text{Diag}[1] + \frac{1}{16} \text{Diag}[h[1], \text{ar}[2, 3]] - \frac{1}{16} \text{Diag}[h[2], \text{ar}[1, 3]] + \\
& \frac{1}{16} \text{Diag}[h[2], \text{ar}[3, 1]] - \frac{1}{16} \text{Diag}[h[3], \text{ar}[2, 1]]
\end{aligned}$$

```
In[22]:= Pentagon[Phi_] := Phi ** PutOn[1, {2, 3}, 4, Phi] ** PutOn[2, 3, 4, Phi] -
      PutOn[{1, 2}, 3, 4, Phi] ** PutOn[1, 2, {3, 4}, Phi];
RR[d_] := ModDegree[d + 1, DExp[1 / 2 (r[1, 2] + r[2, 1])]];
Hexagon[Phi_, RR_] := PutOn[{1, 2}, 3, RR] - Phi ** PutOn[2, 3, RR] **
      PutOn[1, 3, 2, DInvert[Phi]] ** PutOn[1, 3, RR] ** PutOn[3, 1, 2, Phi];
```

```
In[25]:= ModDegree[3, Pentagon[Phi[2]]]
```

```
Out[25]= 0
```

```
In[26]:= ModDegree[3, Hexagon[Phi[2], RR[2]]]
```

```
Out[26]= 0
```

```
In[27]:= Phi[3]
```

```
Out[27]= Diag[1] +  $\frac{1}{32}$  Diag[h[1], ar[2, 3]] -  $\frac{5}{96}$  Diag[h[1], ar[3, 2]] -  $\frac{1}{24}$  Diag[h[2], ar[1, 3]] +
 $\frac{1}{24}$  Diag[h[2], ar[3, 1]] +  $\frac{5}{96}$  Diag[h[3], ar[1, 2]] -  $\frac{1}{32}$  Diag[h[3], ar[2, 1]]
```

```
In[28]:= ModDegree[4, Pentagon[Phi[3]]]
```

```
Out[28]= 0
```

```
In[29]:= v[3]
```

```
Out[29]= - $\frac{1}{4}$  Diag[1, ar[1, 2]] +  $\frac{1}{4}$  Diag[1, ar[2, 1]] +  $\frac{1}{32}$  Diag[h[1], ar[1, 2]] +
 $\frac{5}{96}$  Diag[h[1], ar[2, 1]] +  $\frac{5}{96}$  Diag[h[2], ar[1, 2]] +  $\frac{1}{32}$  Diag[h[2], ar[2, 1]]
```

In[30]:= **V[3]**

Out[30]=
$$\begin{aligned} & \text{Diag}[1] - \frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] + \\ & \frac{1}{32} \text{Diag}[h[1], \text{ar}[1, 2]] + \frac{1}{48} \text{Diag}[h[1], \text{ar}[2, 1]] - \frac{1}{32} \text{Diag}[h[1], \text{ar}[2, 2]] + \\ & \frac{1}{32} \text{Diag}[h[2], \text{ar}[1, 1]] + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[h[2], \text{ar}[2, 1]] + \\ & \frac{1}{192} \text{Diag}[h[1] h[2], \text{ar}[2, 1]] + \frac{1}{192} \text{Diag}[h[1] h[2], \text{ar}[2, 2]] - \\ & \frac{1}{192} \text{Diag}[h[2]^2, \text{ar}[1, 1]] - \frac{1}{192} \text{Diag}[h[2]^2, \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] - \\ & \frac{1}{16} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1]] - \\ & \frac{1}{128} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[1, 2]] + \frac{1}{384} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 1]] + \\ & \frac{1}{128} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 2]] + \frac{1}{192} \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 1]] - \\ & \frac{1}{128} \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 2]] - \frac{1}{128} \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[1, 2]] + \\ & \frac{1}{128} \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[2, 1]] - \frac{1}{48} \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[1, 2]] + \\ & \frac{5}{384} \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{1}{128} \text{Diag}[h[2], \text{ar}[2, 1], \text{ar}[2, 1]] - \\ & \frac{1}{384} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] + \frac{1}{128} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[2, 1]] - \\ & \frac{1}{128} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1], \text{ar}[2, 1]] + \frac{1}{384} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1], \text{ar}[2, 1]] \end{aligned}$$

In[31]:= **ModDegree[4, V[3] ** Adjoint[V[3]]]**

Out[31]= **Diag[1]**

In[32]:= **ModDegree[4, Hexagon[Phi[3], RR[3]]]**

Out[32]= 0

Phi[4]

\$Aborted

ModDegree[5, Pentagon[Phi[4]]]

ModDegree[5, Hexagon[Phi[4], RR[4]]]

Exploration - can the inverse of V be written in terms of V?

```
In[51]:= ModDegree[4, PutOn[2, 1, V[3]] ** DExp[r[1, 2] / 2] - Adjoint[V[3]]]
```

$$\begin{aligned}
\text{Out[51]} = & \frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{16} \text{Diag}[\text{h}[1], \text{ar}[1, 2]] - \frac{1}{12} \text{Diag}[\text{h}[1], \text{ar}[2, 1]] - \\
& \frac{3}{16} \text{Diag}[\text{h}[1], \text{ar}[2, 2]] - \frac{1}{48} \text{Diag}[\text{h}[1]^2, \text{ar}[2, 1]] - \frac{1}{48} \text{Diag}[\text{h}[1]^2, \text{ar}[2, 2]] + \\
& \frac{3}{16} \text{Diag}[\text{h}[2], \text{ar}[1, 1]] + \frac{7}{24} \text{Diag}[\text{h}[2], \text{ar}[1, 2]] + \frac{1}{16} \text{Diag}[\text{h}[2], \text{ar}[2, 1]] + \\
& \frac{1}{48} \text{Diag}[\text{h}[1] \text{h}[2], \text{ar}[1, 1]] + \frac{1}{48} \text{Diag}[\text{h}[1] \text{h}[2], \text{ar}[1, 2]] + \\
& \frac{1}{4} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{3}{16} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1]] + \\
& \frac{1}{16} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[\text{h}[1], \text{ar}[1, 2], \text{ar}[1, 2]] - \\
& \frac{11}{128} \text{Diag}[\text{h}[1], \text{ar}[1, 2], \text{ar}[2, 1]] - \frac{15}{128} \text{Diag}[\text{h}[1], \text{ar}[1, 2], \text{ar}[2, 2]] - \\
& \frac{1}{384} \text{Diag}[\text{h}[1], \text{ar}[2, 1], \text{ar}[1, 2]] + \frac{1}{48} \text{Diag}[\text{h}[1], \text{ar}[2, 1], \text{ar}[2, 1]] + \\
& \frac{5}{128} \text{Diag}[\text{h}[1], \text{ar}[2, 1], \text{ar}[2, 2]] - \frac{1}{128} \text{Diag}[\text{h}[1], \text{ar}[2, 2], \text{ar}[1, 2]] + \\
& \frac{1}{128} \text{Diag}[\text{h}[1], \text{ar}[2, 2], \text{ar}[2, 1]] + \frac{15}{128} \text{Diag}[\text{h}[2], \text{ar}[1, 1], \text{ar}[1, 2]] - \\
& \frac{5}{128} \text{Diag}[\text{h}[2], \text{ar}[1, 1], \text{ar}[2, 1]] + \frac{1}{128} \text{Diag}[\text{h}[2], \text{ar}[1, 2], \text{ar}[1, 1]] + \\
& \frac{17}{96} \text{Diag}[\text{h}[2], \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{11}{384} \text{Diag}[\text{h}[2], \text{ar}[1, 2], \text{ar}[2, 1]] - \\
& \frac{1}{128} \text{Diag}[\text{h}[2], \text{ar}[2, 1], \text{ar}[1, 1]] - \frac{5}{384} \text{Diag}[\text{h}[2], \text{ar}[2, 1], \text{ar}[1, 2]] - \\
& \frac{1}{64} \text{Diag}[\text{h}[2], \text{ar}[2, 1], \text{ar}[2, 1]] + \frac{13}{192} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] - \\
& \frac{9}{128} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{3}{128} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1], \text{ar}[2, 1]] + \\
& \frac{1}{128} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{1}{128} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1], \text{ar}[1, 2]]
\end{aligned}$$