Pensieve Header: Alexander blobs Results.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2011-08\\w-Computations"];
<< "AlexanderBlobs-Program.m"
```

```
b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]]
```

$-$Diag[h[1], ar[2, 3]] $+$ Diag[h[2], ar[1, 3]]

```
b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]] + b[r[1, 3], r[2, 3]]
```

0

```
ModDegree[4, R[1, 2]]
```

Diag[1] $+$ Diag[1, ar[1, 2]] $+$

$\frac{1}{2}$ Diag[1, ar[1, 2], ar[1, 2]] $+$ $\frac{1}{6}$ Diag[1, ar[1, 2], ar[1, 2], ar[1, 2]]

```
ModDegree[7, R[1, 2] ** R[1, 3] ** R[2, 3] - R[2, 3] ** R[1, 3] ** R[1, 2]]
```

0

```
v[0] = 0;
```

```
d = 1; ModDegree[d + 1,
 Print[
  v[d] = v[d - 1] + c1 Diag[1, ar[1, 2]] + c2 Diag[1, ar[2, 1]]
 ];
 V[d] = DExp[v[d]];
 {R4Eqn[V[d]], TwistEqn[V[d]]}
]
```

c1 Diag[1, ar[1, 2]] $+$ c2 Diag[1, ar[2, 1]]

$\Big\{0, -\frac{1}{2}$ Diag[1, ar[1, 2]] $-$ c1 Diag[1, ar[1, 2]] $+$ c2 Diag[1, ar[1, 2]] $+$

$\frac{1}{2}$ Diag[1, ar[2, 1]] $+$ c1 Diag[1, ar[2, 1]] $-$ c2 Diag[1, ar[2, 1]]$\Big\}$

```
Solve[{-1 / 2 - c1 + c2 == 0, 1 / 2 + c1 - c2 == 0}, {c1, c2}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. $\gg$

$\Big\{\Big\{c2 \to \frac{1}{2} + c1\Big\}\Big\}$

```
d = 1; ModDegree[d + 1,
 Print[
  v[d] = v[d - 1] + c1 Diag[1, ar[1, 2]] + (1 / 2 + c1) Diag[1, ar[2, 1]]
 ];
 V[d] = DExp[v[d]];
 {R4Eqn[V[d]], TwistEqn[V[d]]}
]
```

c1 Diag[1, ar[1, 2]] $+$ $\Big(\frac{1}{2} + c1\Big)$ Diag[1, ar[2, 1]]

{0, 0}

```
d = 2; ModDegree[d + 1,
 Print[
  v[d] = v[d - 1]
 ];
 V[d] = DExp[v[d]];
 {R4Eqn[V[d]], TwistEqn[V[d]]}
]
```

$$c1 \, Diag[1, ar[1, 2]] + \left(\frac{1}{2} + c1\right) Diag[1, ar[2, 1]]$$

$$\left\{0, \frac{1}{8} Diag[h[1], ar[2, 1]] + \frac{1}{2} c1 \, Diag[h[1], ar[2, 1]] + \right.$$

$$\frac{1}{8} Diag[h[1], ar[2, 2]] + \frac{1}{2} c1 \, Diag[h[1], ar[2, 2]] - \frac{1}{8} Diag[h[2], ar[1, 1]] -$$

$$\left. \frac{1}{2} c1 \, Diag[h[2], ar[1, 1]] - \frac{1}{8} Diag[h[2], ar[1, 2]] - \frac{1}{2} c1 \, Diag[h[2], ar[1, 2]]\right\}$$

```
d = 2; ModDegree[d + 1,
 Print[
  v[d] = (v[d - 1] /. c1 → -1 / 4)
 ];
 V[d] = DExp[v[d]];
 {R4Eqn[V[d]], TwistEqn[V[d]]}
]
```

$$-\frac{1}{4} Diag[1, ar[1, 2]] + \frac{1}{4} Diag[1, ar[2, 1]]$$

$$\{0, 0\}$$

```
d = 3; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1] + c21 r12 ** r12 + c22 r12 ** r21 +
     c23 r21 ** r12 + c24 r21 ** r21 + c25 Diag[h[1], ar[1, 2]] +
     c26 Diag[h[2], ar[1, 2]] + c27 Diag[h[1], ar[2, 1]] + c28 Diag[h[2], ar[2, 1]]
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
] // PullDiags
```

$$-\frac{1}{4} Diag[1, ar[1, 2]] + \frac{1}{4} Diag[1, ar[2, 1]] + c25 \, Diag[h[1], ar[1, 2]] +$$
$$c27 \, Diag[h[1], ar[2, 1]] + c26 \, Diag[h[2], ar[1, 2]] + c28 \, Diag[h[2], ar[2, 1]] +$$
$$c21 \, Diag[1, ar[1, 2], ar[1, 2]] + c22 \, Diag[1, ar[1, 2], ar[2, 1]] +$$
$$c23 \, (Diag[h[1], ar[2, 1]] + Diag[h[1], ar[2, 2]] - Diag[h[2], ar[1, 1]] -$$
$$Diag[h[2], ar[1, 2]] + Diag[1, ar[1, 2], ar[2, 1]]) + c24 \, Diag[1, ar[2, 1], ar[2, 1]]$$

$$\left\{\left(-\frac{1}{48} - c21 + c23 - c25 + c27\right) Diag\left[h[1]^2, ar[2, 3]\right] +\right.$$

$$\left(\frac{1}{48} + c21 - c23 + c25 - c27\right) Diag[h[1] \, h[2], ar[1, 3]] +$$

$$\left(\frac{1}{48} + c23 - c24 - c26 + c28\right) Diag[h[1] \, h[2], ar[2, 3]] +$$

$$\left(-\frac{1}{48} - c23 + c24 + c26 - c28\right) \text{Diag}\left[h[2]^2, ar[1, 3]\right] +$$

$$(-2\,c21 + c22 + c23)\,\text{Diag}[h[1], ar[1, 2], ar[2, 3]] +$$
$$(-c22 - c23 + 2\,c24)\,\text{Diag}[h[1], ar[2, 1], ar[2, 3]] +$$
$$(2\,c21 - c22 - c23)\,\text{Diag}[h[2], ar[1, 2], ar[1, 3]] +$$
$$(c22 + c23 - 2\,c24)\,\text{Diag}[h[2], ar[1, 3], ar[2, 1]], \quad (-c25 + c28)\,\text{Diag}[h[1], ar[1, 2]] +$$
$$(c22 - c23 + c26 - c27)\,\text{Diag}[h[1], ar[2, 1]] + (c22 - c23)\,\text{Diag}[h[1], ar[2, 2]] +$$

$$\left(-\frac{1}{32} - \frac{c21}{8} + \frac{5\,c22}{8} + \frac{c23}{8} - \frac{c24}{8} - \frac{c25}{8} + \frac{5\,c26}{8} + \frac{c27}{8} - \frac{c28}{8}\right) \text{Diag}\left[h[1]^2, ar[2, 1]\right] +$$

$$\left(-\frac{1}{32} - \frac{c21}{8} + \frac{5\,c22}{8} + \frac{c23}{8} - \frac{c24}{8} - \frac{c25}{8} + \frac{5\,c26}{8} + \frac{c27}{8} - \frac{c28}{8}\right) \text{Diag}\left[h[1]^2, ar[2, 2]\right] +$$

$$(-c22 + c23)\,\text{Diag}[h[2], ar[1, 1]] +$$
$$(-c22 + c23 - c26 + c27)\,\text{Diag}[h[2], ar[1, 2]] + (c25 - c28)\,\text{Diag}[h[2], ar[2, 1]] +$$

$$\left(\frac{1}{32} + \frac{c21}{8} - \frac{5\,c22}{8} - \frac{c23}{8} + \frac{c24}{8} + \frac{c25}{8} - \frac{5\,c26}{8} - \frac{c27}{8} + \frac{c28}{8}\right) \text{Diag}[h[1]\,h[2], ar[1, 1]] +$$

$$\left(\frac{1}{32} + \frac{c21}{8} - \frac{5\,c22}{8} - \frac{c23}{8} + \frac{c24}{8} + \frac{c25}{8} - \frac{5\,c26}{8} - \frac{c27}{8} + \frac{c28}{8}\right) \text{Diag}[h[1]\,h[2], ar[1, 2]] +$$

$$\left(-\frac{1}{96} - \frac{5\,c21}{8} + \frac{c22}{8} + \frac{c23}{8} - \frac{c24}{8} + \frac{5\,c25}{8} - \frac{c26}{8} - \frac{c27}{8} + \frac{c28}{8}\right) \text{Diag}[h[1]\,h[2], ar[2, 1]] +$$

$$\left(-\frac{1}{96} - \frac{5\,c21}{8} + \frac{c22}{8} + \frac{c23}{8} - \frac{c24}{8} + \frac{5\,c25}{8} - \frac{c26}{8} - \frac{c27}{8} + \frac{c28}{8}\right) \text{Diag}[h[1]\,h[2], ar[2, 2]] +$$

$$\left(\frac{1}{96} + \frac{5\,c21}{8} - \frac{c22}{8} - \frac{c23}{8} + \frac{c24}{8} - \frac{5\,c25}{8} + \frac{c26}{8} + \frac{c27}{8} - \frac{c28}{8}\right) \text{Diag}\left[h[2]^2, ar[1, 1]\right] +$$

$$\left(\frac{1}{96} + \frac{5\,c21}{8} - \frac{c22}{8} - \frac{c23}{8} + \frac{c24}{8} - \frac{5\,c25}{8} + \frac{c26}{8} + \frac{c27}{8} - \frac{c28}{8}\right) \text{Diag}\left[h[2]^2, ar[1, 2]\right] +$$

$$(-c21 + c24)\,\text{Diag}[1, ar[1, 2], ar[1, 2]] + (c21 - c24)\,\text{Diag}[1, ar[2, 1], ar[2, 1]] +$$

$$\left(-\frac{3\,c25}{4} + \frac{3\,c28}{4}\right) \text{Diag}[h[1], ar[1, 2], ar[1, 2]] +$$

$$\left(-\frac{c21}{4} + \frac{3\,c22}{2} - \frac{c24}{4} - \frac{c25}{4} + \frac{3\,c26}{4} - \frac{3\,c27}{4} + \frac{c28}{4}\right) \text{Diag}[h[1], ar[1, 2], ar[2, 1]] +$$

$$\left(-\frac{c21}{4} + \frac{3\,c22}{2} - \frac{c24}{4}\right) \text{Diag}[h[1], ar[1, 2], ar[2, 2]] +$$

$$\left(\frac{5\,c21}{4} - \frac{c23}{2} + \frac{c24}{4} + \frac{c26}{4} - \frac{c27}{4}\right) \text{Diag}[h[1], ar[2, 1], ar[2, 1]] +$$

$$\left(\frac{5\,c21}{4} - \frac{c23}{2} + \frac{c24}{4}\right) \text{Diag}[h[1], ar[2, 1], ar[2, 2]] +$$

$$\left(\frac{c21}{4} - \frac{3\,c22}{2} + \frac{c24}{4}\right) \text{Diag}[h[2], ar[1, 1], ar[1, 2]] +$$

$$\left(-\frac{5\,c21}{4} + \frac{c23}{2} - \frac{c24}{4}\right) \text{Diag}[h[2], ar[1, 1], ar[2, 1]] +$$

$$\left(\frac{c21}{4} - \frac{3\,c22}{2} + \frac{c24}{4} - \frac{3\,c26}{4} + \frac{3\,c27}{4}\right) \text{Diag}[h[2], ar[1, 2], ar[1, 2]] +$$

$$\left(-\frac{5\,c21}{4} + \frac{c23}{2} - \frac{c24}{4} + \frac{3\,c25}{4} - \frac{c26}{4} + \frac{c27}{4} - \frac{3\,c28}{4}\right) \text{Diag}[h[2], ar[1, 2], ar[2, 1]] +$$

$$\left(\frac{c25}{4} - \frac{c28}{4}\right) \text{Diag}[h[2], ar[2, 1], ar[2, 1]] +$$

$$\left(-\frac{3\,c21}{4} + \frac{3\,c24}{4}\right) \text{Diag}[1, ar[1, 2], ar[1, 2], ar[1, 2]] +$$

$$\left(-\frac{c21}{4} + \frac{c24}{4}\right) \text{Diag}[1, ar[1, 2], ar[1, 2], ar[2, 1]] +$$

$$\left(\frac{3\,c21}{4} - \frac{3\,c24}{4}\right) \text{Diag}[1, ar[1, 2], ar[2, 1], ar[2, 1]] +$$

$$\left(\frac{c21}{4} - \frac{c24}{4}\right) \text{Diag}[1, ar[2, 1], ar[2, 1], ar[2, 1]]\}$$

```
Solve[{(-1/48 - c21 + c23 - c25 + c27) == 0, (1/48 + c23 - c24 - c26 + c28) == 0,
  (-2 c21 + c22 + c23) == 0, (-c22 - c23 + 2 c24) == 0, (-c25 + c28) == 0,
  (c22 - c23 + c26 - c27) == 0, (c22 - c23) == 0, (c21 - c24) == 0},
 {c21, c22, c23, c24, c25, c26, c27, c28}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{\left\{c22 \to c21, c23 \to c21, c24 \to c21, c26 \to \frac{1}{48} + c25, c27 \to \frac{1}{48} + c25, c28 \to c25\right\}\right\}$$

```
d = 3; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1] + 0 r12 ** r12 + 0 r12 ** r21 + 0 r21 ** r12 +
     0 r21 ** r21 + c25 Diag[h[1], ar[1, 2]] + (1/48 + c25) Diag[h[2], ar[1, 2]] +
     (1/48 + c25) Diag[h[1], ar[2, 1]] + c25 Diag[h[2], ar[2, 1]] /. c25 → 1 / 32
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$$-\frac{1}{4} \text{Diag}[1, ar[1, 2]] + \frac{1}{4} \text{Diag}[1, ar[2, 1]] + \frac{1}{32} \text{Diag}[h[1], ar[1, 2]] +$$

$$\frac{5}{96} \text{Diag}[h[1], ar[2, 1]] + \frac{5}{96} \text{Diag}[h[2], ar[1, 2]] + \frac{1}{32} \text{Diag}[h[2], ar[2, 1]]$$

{0, 0}

```
d = 4; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1]
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$-\dfrac{1}{4}$ Diag[1, ar[1, 2]] $+\dfrac{1}{4}$ Diag[1, ar[2, 1]] $+\dfrac{1}{32}$ Diag[h[1], ar[1, 2]] +

$\dfrac{5}{96}$ Diag[h[1], ar[2, 1]] $+\dfrac{5}{96}$ Diag[h[2], ar[1, 2]] $+\dfrac{1}{32}$ Diag[h[2], ar[2, 1]]

{0, 0}

```
d = 5; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1] + Sum[c[2 k] Diag[h[1]^k h[2]^(d - 2 - k), ar[1, 2]] +
       c[2 k + 1] Diag[h[1]^k h[2]^(d - 2 - k), ar[2, 1]], {k, 0, d - 2}]
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$-\dfrac{1}{4}$ Diag[1, ar[1, 2]] $+\dfrac{1}{4}$ Diag[1, ar[2, 1]] $+\dfrac{1}{32}$ Diag[h[1], ar[1, 2]] +

$\dfrac{5}{96}$ Diag[h[1], ar[2, 1]] $+$ c[6] Diag$\left[h[1]^3, ar[1, 2]\right]$ $+$ c[7] Diag$\left[h[1]^3, ar[2, 1]\right]$ +

$\dfrac{5}{96}$ Diag[h[2], ar[1, 2]] $+\dfrac{1}{32}$ Diag[h[2], ar[2, 1]] $+$ c[4] Diag$\left[h[1]^2 h[2], ar[1, 2]\right]$ +

c[5] Diag$\left[h[1]^2 h[2], ar[2, 1]\right]$ $+$ c[2] Diag$\left[h[1] h[2]^2, ar[1, 2]\right]$ +

c[3] Diag$\left[h[1] h[2]^2, ar[2, 1]\right]$ $+$ c[0] Diag$\left[h[2]^3, ar[1, 2]\right]$ $+$ c[1] Diag$\left[h[2]^3, ar[2, 1]\right]$

$\{\left(\dfrac{7}{23\,040} - c[6] + c[7]\right)$ Diag$\left[h[1]^4, ar[2, 3]\right]$ +

$\left(-\dfrac{7}{23\,040} + c[6] - c[7]\right)$ Diag$\left[h[1]^3 h[2], ar[1, 3]\right]$ +

$\left(\dfrac{1}{7680} - c[4] + c[5]\right)$ Diag$\left[h[1]^3 h[2], ar[2, 3]\right]$ +

$\left(-\dfrac{1}{7680} + c[4] - c[5]\right)$ Diag$\left[h[1]^2 h[2]^2, ar[1, 3]\right]$ +

$\left(-\dfrac{1}{7680} - c[2] + c[3]\right)$ Diag$\left[h[1]^2 h[2]^2, ar[2, 3]\right]$ +

$\left(\dfrac{1}{7680} + c[2] - c[3]\right)$ Diag$\left[h[1] h[2]^3, ar[1, 3]\right]$ +

$\left(-\dfrac{7}{23\,040} - c[0] + c[1]\right)$ Diag$\left[h[1] h[2]^3, ar[2, 3]\right]$ +

$\left(\dfrac{7}{23\,040} + c[0] - c[1]\right)$ Diag$\left[h[2]^4, ar[1, 3]\right]$,

(c[1] - c[6]) Diag$\left[h[1]^3, ar[1, 2]\right]$ $+$ (c[0] - c[7]) Diag$\left[h[1]^3, ar[2, 1]\right]$ +

$$\left(\frac{1}{2304} + \frac{5\,c[0]}{8} - \frac{c[1]}{8} - \frac{c[6]}{8} + \frac{c[7]}{8}\right) \text{Diag}\left[h[1]^4,\ ar[2,1]\right] +$$

$$\left(\frac{1}{2304} + \frac{5\,c[0]}{8} - \frac{c[1]}{8} - \frac{c[6]}{8} + \frac{c[7]}{8}\right) \text{Diag}\left[h[1]^4,\ ar[2,2]\right] +$$

$$(c[3] - c[4])\ \text{Diag}\left[h[1]^2\,h[2],\ ar[1,2]\right] + (c[2] - c[5])\ \text{Diag}\left[h[1]^2\,h[2],\ ar[2,1]\right] +$$

$$\left(-\frac{1}{2304} - \frac{5\,c[0]}{8} + \frac{c[1]}{8} + \frac{c[6]}{8} - \frac{c[7]}{8}\right) \text{Diag}\left[h[1]^3\,h[2],\ ar[1,1]\right] +$$

$$\left(-\frac{1}{2304} - \frac{5\,c[0]}{8} + \frac{c[1]}{8} + \frac{c[6]}{8} - \frac{c[7]}{8}\right) \text{Diag}\left[h[1]^3\,h[2],\ ar[1,2]\right] +$$

$$\left(\frac{49}{69120} + \frac{5\,c[2]}{8} - \frac{c[3]}{8} - \frac{c[4]}{8} + \frac{c[5]}{8}\right) \text{Diag}\left[h[1]^3\,h[2],\ ar[2,1]\right] +$$

$$\left(\frac{49}{69120} + \frac{5\,c[2]}{8} - \frac{c[3]}{8} - \frac{c[4]}{8} + \frac{c[5]}{8}\right) \text{Diag}\left[h[1]^3\,h[2],\ ar[2,2]\right] +$$

$$(-c[2] + c[5])\ \text{Diag}\left[h[1]\,h[2]^2,\ ar[1,2]\right] + (-c[3] + c[4])\ \text{Diag}\left[h[1]\,h[2]^2,\ ar[2,1]\right] +$$

$$\left(-\frac{49}{69120} - \frac{5\,c[2]}{8} + \frac{c[3]}{8} + \frac{c[4]}{8} - \frac{c[5]}{8}\right) \text{Diag}\left[h[1]^2\,h[2]^2,\ ar[1,1]\right] +$$

$$\left(-\frac{49}{69120} - \frac{5\,c[2]}{8} + \frac{c[3]}{8} + \frac{c[4]}{8} - \frac{c[5]}{8}\right) \text{Diag}\left[h[1]^2\,h[2]^2,\ ar[1,2]\right] +$$

$$\left(\frac{1}{1728} - \frac{c[2]}{8} + \frac{c[3]}{8} + \frac{5\,c[4]}{8} - \frac{c[5]}{8}\right) \text{Diag}\left[h[1]^2\,h[2]^2,\ ar[2,1]\right] +$$

$$\left(\frac{1}{1728} - \frac{c[2]}{8} + \frac{c[3]}{8} + \frac{5\,c[4]}{8} - \frac{c[5]}{8}\right) \text{Diag}\left[h[1]^2\,h[2]^2,\ ar[2,2]\right] +$$

$$(-c[0] + c[7])\ \text{Diag}\left[h[2]^3,\ ar[1,2]\right] + (-c[1] + c[6])\ \text{Diag}\left[h[2]^3,\ ar[2,1]\right] +$$

$$\left(-\frac{1}{1728} + \frac{c[2]}{8} - \frac{c[3]}{8} - \frac{5\,c[4]}{8} + \frac{c[5]}{8}\right) \text{Diag}\left[h[1]\,h[2]^3,\ ar[1,1]\right] +$$

$$\left(-\frac{1}{1728} + \frac{c[2]}{8} - \frac{c[3]}{8} - \frac{5\,c[4]}{8} + \frac{c[5]}{8}\right) \text{Diag}\left[h[1]\,h[2]^3,\ ar[1,2]\right] +$$

$$\left(\frac{1}{7680} - \frac{c[0]}{8} + \frac{c[1]}{8} + \frac{5\,c[6]}{8} - \frac{c[7]}{8}\right) \text{Diag}\left[h[1]\,h[2]^3,\ ar[2,1]\right] +$$

$$\left(\frac{1}{7680} - \frac{c[0]}{8} + \frac{c[1]}{8} + \frac{5\,c[6]}{8} - \frac{c[7]}{8}\right) \text{Diag}\left[h[1]\,h[2]^3,\ ar[2,2]\right] +$$

$$\left(-\frac{1}{7680} + \frac{c[0]}{8} - \frac{c[1]}{8} - \frac{5\,c[6]}{8} + \frac{c[7]}{8}\right) \text{Diag}\left[h[2]^4,\ ar[1,1]\right] +$$

$$\left(-\frac{1}{7680} + \frac{c[0]}{8} - \frac{c[1]}{8} - \frac{5\,c[6]}{8} + \frac{c[7]}{8}\right) \text{Diag}\left[h[2]^4,\ ar[1,2]\right] +$$

$$\left(\frac{3\,c[1]}{4} - \frac{3\,c[6]}{4}\right) \text{Diag}\left[h[1]^3,\ ar[1,2],\ ar[1,2]\right] +$$

$$\left(\frac{3\,c[0]}{4} + \frac{c[1]}{4} - \frac{c[6]}{4} - \frac{3\,c[7]}{4}\right) \text{Diag}\left[h[1]^3,\ ar[1,2],\ ar[2,1]\right] +$$

$$\left(\frac{c[0]}{4} - \frac{c[7]}{4}\right) \text{Diag}\left[h[1]^3,\ ar[2,1],\ ar[2,1]\right] +$$

$$\left(\frac{3\,c[3]}{4} - \frac{3\,c[4]}{4}\right) \text{Diag}\left[h[1]^2\,h[2],\ ar[1,2],\ ar[1,2]\right] +$$

$$\left(\frac{3\,c[2]}{4} + \frac{c[3]}{4} - \frac{c[4]}{4} - \frac{3\,c[5]}{4}\right) \text{Diag}\left[h[1]^2\,h[2], ar[1, 2], ar[2, 1]\right] +$$

$$\left(\frac{c[2]}{4} - \frac{c[5]}{4}\right) \text{Diag}\left[h[1]^2\,h[2], ar[2, 1], ar[2, 1]\right] +$$

$$\left(-\frac{3\,c[2]}{4} + \frac{3\,c[5]}{4}\right) \text{Diag}\left[h[1]\,h[2]^2, ar[1, 2], ar[1, 2]\right] +$$

$$\left(-\frac{c[2]}{4} - \frac{3\,c[3]}{4} + \frac{3\,c[4]}{4} + \frac{c[5]}{4}\right) \text{Diag}\left[h[1]\,h[2]^2, ar[1, 2], ar[2, 1]\right] +$$

$$\left(-\frac{c[3]}{4} + \frac{c[4]}{4}\right) \text{Diag}\left[h[1]\,h[2]^2, ar[2, 1], ar[2, 1]\right] +$$

$$\left(-\frac{3\,c[0]}{4} + \frac{3\,c[7]}{4}\right) \text{Diag}\left[h[2]^3, ar[1, 2], ar[1, 2]\right] +$$

$$\left(-\frac{c[0]}{4} - \frac{3\,c[1]}{4} + \frac{3\,c[6]}{4} + \frac{c[7]}{4}\right) \text{Diag}\left[h[2]^3, ar[1, 2], ar[2, 1]\right] +$$

$$\left(-\frac{c[1]}{4} + \frac{c[6]}{4}\right) \text{Diag}\left[h[2]^3, ar[2, 1], ar[2, 1]\right]\Big\}$$

Solve$\Big[$ (# == 0) & /@ $\left\{\left(\dfrac{7}{23\,040} - c[6] + c[7]\right),\ \dfrac{1}{7680} - c[4] + c[5],\right.$

$\dfrac{1}{7680} + c[2] - c[3],\ \dfrac{7}{23\,040} + c[0] - c[1],\ c[1] - c[6],\ c[0] - c[7],$

$\left.c[3] - c[4],\ c[2] - c[5],\ \dfrac{11}{30\,720} + \dfrac{c[0]}{2},\ -\dfrac{187}{276\,480} - \dfrac{c[2]}{2}\right\}$, **c** /@ **Range[0, 7]**$\Big]$

$$\left\{\left\{c[0] \to -\frac{11}{15\,360},\ c[1] \to -\frac{19}{46\,080},\ c[2] \to -\frac{187}{138\,240},\ c[3] \to -\frac{169}{138\,240},\right.\right.$$

$$\left.\left.c[4] \to -\frac{169}{138\,240},\ c[5] \to -\frac{187}{138\,240},\ c[6] \to -\frac{19}{46\,080},\ c[7] \to -\frac{11}{15\,360}\right\}\right\}$$

LCM[23 040, 7680, 2304, 69 120, 1728] / 720 / 24 / 2

2

```
d = 5; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1] + Sum[c[2 k] Diag[h[1] ^ k h[2] ^ (d - 2 - k), ar[1, 2]] +
       c[2 k + 1] Diag[h[1] ^ k h[2] ^ (d - 2 - k), ar[2, 1]], {k, 0, d - 2}] /.
```

$$\left\{c[0] \to -\frac{11}{15\,360}, c[1] \to -\frac{19}{46\,080}, c[2] \to -\frac{187}{138\,240}, c[3] \to -\frac{169}{138\,240},\right.$$

$$\left. c[4] \to -\frac{169}{138\,240}, c[5] \to -\frac{187}{138\,240}, c[6] \to -\frac{19}{46\,080}, c[7] \to -\frac{11}{15\,360}\right\}$$

```
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$$-\frac{1}{4}\,\text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4}\,\text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32}\,\text{Diag}[h[1], \text{ar}[1, 2]] +$$

$$\frac{5}{96}\,\text{Diag}[h[1], \text{ar}[2, 1]] - \frac{19\,\text{Diag}\left[h[1]^3, \text{ar}[1, 2]\right]}{46\,080} - \frac{11\,\text{Diag}\left[h[1]^3, \text{ar}[2, 1]\right]}{15\,360} +$$

$$\frac{5}{96}\,\text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32}\,\text{Diag}[h[2], \text{ar}[2, 1]] - \frac{169\,\text{Diag}\left[h[1]^2\,h[2], \text{ar}[1, 2]\right]}{138\,240} -$$

$$\frac{187\,\text{Diag}\left[h[1]^2\,h[2], \text{ar}[2, 1]\right]}{138\,240} - \frac{187\,\text{Diag}\left[h[1]\,h[2]^2, \text{ar}[1, 2]\right]}{138\,240} -$$

$$\frac{169\,\text{Diag}\left[h[1]\,h[2]^2, \text{ar}[2, 1]\right]}{138\,240} - \frac{11\,\text{Diag}\left[h[2]^3, \text{ar}[1, 2]\right]}{15\,360} - \frac{19\,\text{Diag}\left[h[2]^3, \text{ar}[2, 1]\right]}{46\,080}$$

```
{0, 0}
```

```
d = 6; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1]
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$$-\frac{1}{4}\,\text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4}\,\text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{32}\,\text{Diag}[h[1], \text{ar}[1, 2]] +$$

$$\frac{5}{96}\,\text{Diag}[h[1], \text{ar}[2, 1]] - \frac{19\,\text{Diag}\left[h[1]^3, \text{ar}[1, 2]\right]}{46\,080} - \frac{11\,\text{Diag}\left[h[1]^3, \text{ar}[2, 1]\right]}{15\,360} +$$

$$\frac{5}{96}\,\text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32}\,\text{Diag}[h[2], \text{ar}[2, 1]] - \frac{169\,\text{Diag}\left[h[1]^2\,h[2], \text{ar}[1, 2]\right]}{138\,240} -$$

$$\frac{187\,\text{Diag}\left[h[1]^2\,h[2], \text{ar}[2, 1]\right]}{138\,240} - \frac{187\,\text{Diag}\left[h[1]\,h[2]^2, \text{ar}[1, 2]\right]}{138\,240} -$$

$$\frac{169\,\text{Diag}\left[h[1]\,h[2]^2, \text{ar}[2, 1]\right]}{138\,240} - \frac{11\,\text{Diag}\left[h[2]^3, \text{ar}[1, 2]\right]}{15\,360} - \frac{19\,\text{Diag}\left[h[2]^3, \text{ar}[2, 1]\right]}{46\,080}$$

```
{0, 0}
```

```
d = 7; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1] + Sum[c[2 k] Diag[h[1]^k h[2]^(d - 2 - k), ar[1, 2]] +
       c[2 k + 1] Diag[h[1]^k h[2]^(d - 2 - k), ar[2, 1]], {k, 0, d - 2}] /.
```

$$\left\{c[0] \to \frac{13}{1\,161\,216}, c[1] \to \frac{37}{4\,644\,864}, c[2] \to \frac{2669}{69\,672\,960}, c[3] \to \frac{1279}{34\,836\,480}, \right.$$

$$c[4] \to \frac{2539}{34\,836\,480}, c[5] \to \frac{125}{1\,741\,824}, c[6] \to \frac{125}{1\,741\,824}, c[7] \to \frac{2539}{34\,836\,480},$$

$$\left. c[8] \to \frac{1279}{34\,836\,480}, c[9] \to \frac{2669}{69\,672\,960}, c[10] \to \frac{37}{4\,644\,864}, c[11] \to \frac{13}{1\,161\,216}\right\}$$

```
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$$-\frac{1}{4} \mathrm{Diag}[1, ar[1, 2]] + \frac{1}{4} \mathrm{Diag}[1, ar[2, 1]] + \frac{1}{32} \mathrm{Diag}[h[1], ar[1, 2]] +$$

$$\frac{5}{96} \mathrm{Diag}[h[1], ar[2, 1]] - \frac{19\,\mathrm{Diag}\left[h[1]^3, ar[1, 2]\right]}{46\,080} - \frac{11\,\mathrm{Diag}\left[h[1]^3, ar[2, 1]\right]}{15\,360} +$$

$$\frac{37\,\mathrm{Diag}\left[h[1]^5, ar[1, 2]\right]}{4\,644\,864} + \frac{13\,\mathrm{Diag}\left[h[1]^5, ar[2, 1]\right]}{1\,161\,216} + \frac{5}{96} \mathrm{Diag}[h[2], ar[1, 2]] +$$

$$\frac{1}{32} \mathrm{Diag}[h[2], ar[2, 1]] - \frac{169\,\mathrm{Diag}\left[h[1]^2 h[2], ar[1, 2]\right]}{138\,240} - \frac{187\,\mathrm{Diag}\left[h[1]^2 h[2], ar[2, 1]\right]}{138\,240} +$$

$$\frac{1279\,\mathrm{Diag}\left[h[1]^4 h[2], ar[1, 2]\right]}{34\,836\,480} + \frac{2669\,\mathrm{Diag}\left[h[1]^4 h[2], ar[2, 1]\right]}{69\,672\,960} -$$

$$\frac{187\,\mathrm{Diag}\left[h[1] h[2]^2, ar[1, 2]\right]}{138\,240} - \frac{169\,\mathrm{Diag}\left[h[1] h[2]^2, ar[2, 1]\right]}{138\,240} +$$

$$\frac{125\,\mathrm{Diag}\left[h[1]^3 h[2]^2, ar[1, 2]\right]}{1\,741\,824} + \frac{2539\,\mathrm{Diag}\left[h[1]^3 h[2]^2, ar[2, 1]\right]}{34\,836\,480} -$$

$$\frac{11\,\mathrm{Diag}\left[h[2]^3, ar[1, 2]\right]}{15\,360} - \frac{19\,\mathrm{Diag}\left[h[2]^3, ar[2, 1]\right]}{46\,080} + \frac{2539\,\mathrm{Diag}\left[h[1]^2 h[2]^3, ar[1, 2]\right]}{34\,836\,480} +$$

$$\frac{125\,\mathrm{Diag}\left[h[1]^2 h[2]^3, ar[2, 1]\right]}{1\,741\,824} + \frac{2669\,\mathrm{Diag}\left[h[1] h[2]^4, ar[1, 2]\right]}{69\,672\,960} +$$

$$\frac{1279\,\mathrm{Diag}\left[h[1] h[2]^4, ar[2, 1]\right]}{34\,836\,480} + \frac{13\,\mathrm{Diag}\left[h[2]^5, ar[1, 2]\right]}{1\,161\,216} + \frac{37\,\mathrm{Diag}\left[h[2]^5, ar[2, 1]\right]}{4\,644\,864}$$

{0, 0}

```
Solve[(# == 0) & /@ {-c[1]/4 + c[10]/4, -c[3]/4 + c[8]/4, -3 c[0]/4 + 3 c[11]/4,
    -3 c[2]/4 + 3 c[9]/4, -c[5]/4 + c[6]/4, -3 c[4]/4 + 3 c[7]/4, 59/18 579 456 - 3 c[10]/4 + c[11]/4,
    143/7 962 624 + c[2]/4 - 3 c[3]/4, 4961/139 345 920 + c[4]/4 - 3 c[5]/4, 2539/52 254 720 - 2 c[4]/3,
    2669/104 509 440 - 2 c[2]/3, -13/1 741 824 + 2 c[0]/3}, c /@ Range[0, 11]]
```

$$\left\{\left\{c[0] \to \frac{13}{1\,161\,216}, c[1] \to \frac{37}{4\,644\,864}, c[2] \to \frac{2669}{69\,672\,960}, c[3] \to \frac{1279}{34\,836\,480},\right.\right.$$
$$c[4] \to \frac{2539}{34\,836\,480}, c[5] \to \frac{125}{1\,741\,824}, c[6] \to \frac{125}{1\,741\,824}, c[7] \to \frac{2539}{34\,836\,480},$$
$$\left.\left.c[8] \to \frac{1279}{34\,836\,480}, c[9] \to \frac{2669}{69\,672\,960}, c[10] \to \frac{37}{4\,644\,864}, c[11] \to \frac{13}{1\,161\,216}\right\}\right\}$$

```
d = 8; ModDegree[d + 1,
  Print[
   v[d] = v[d - 1]
  ];
  V[d] = DExp[v[d]];
  {R4Eqn[V[d]], TwistEqn[V[d]]}
 ] // PullDiags
```

$$-\frac{1}{4} \operatorname{Diag}[1, ar[1, 2]] + \frac{1}{4} \operatorname{Diag}[1, ar[2, 1]] + \frac{1}{32} \operatorname{Diag}[h[1], ar[1, 2]] +$$
$$\frac{5}{96} \operatorname{Diag}[h[1], ar[2, 1]] - \frac{19 \operatorname{Diag}\left[h[1]^3, ar[1, 2]\right]}{46\,080} - \frac{11 \operatorname{Diag}\left[h[1]^3, ar[2, 1]\right]}{15\,360} +$$
$$\frac{37 \operatorname{Diag}\left[h[1]^5, ar[1, 2]\right]}{4\,644\,864} + \frac{13 \operatorname{Diag}\left[h[1]^5, ar[2, 1]\right]}{1\,161\,216} + \frac{5}{96} \operatorname{Diag}[h[2], ar[1, 2]] +$$
$$\frac{1}{32} \operatorname{Diag}[h[2], ar[2, 1]] - \frac{169 \operatorname{Diag}\left[h[1]^2 h[2], ar[1, 2]\right]}{138\,240} - \frac{187 \operatorname{Diag}\left[h[1]^2 h[2], ar[2, 1]\right]}{138\,240} +$$
$$\frac{1279 \operatorname{Diag}\left[h[1]^4 h[2], ar[1, 2]\right]}{34\,836\,480} + \frac{2669 \operatorname{Diag}\left[h[1]^4 h[2], ar[2, 1]\right]}{69\,672\,960} -$$
$$\frac{187 \operatorname{Diag}\left[h[1] h[2]^2, ar[1, 2]\right]}{138\,240} - \frac{169 \operatorname{Diag}\left[h[1] h[2]^2, ar[2, 1]\right]}{138\,240} +$$
$$\frac{125 \operatorname{Diag}\left[h[1]^3 h[2]^2, ar[1, 2]\right]}{1\,741\,824} + \frac{2539 \operatorname{Diag}\left[h[1]^3 h[2]^2, ar[2, 1]\right]}{34\,836\,480} -$$
$$\frac{11 \operatorname{Diag}\left[h[2]^3, ar[1, 2]\right]}{15\,360} - \frac{19 \operatorname{Diag}\left[h[2]^3, ar[2, 1]\right]}{46\,080} + \frac{2539 \operatorname{Diag}\left[h[1]^2 h[2]^3, ar[1, 2]\right]}{34\,836\,480} +$$
$$\frac{125 \operatorname{Diag}\left[h[1]^2 h[2]^3, ar[2, 1]\right]}{1\,741\,824} + \frac{2669 \operatorname{Diag}\left[h[1] h[2]^4, ar[1, 2]\right]}{69\,672\,960} +$$
$$\frac{1279 \operatorname{Diag}\left[h[1] h[2]^4, ar[2, 1]\right]}{34\,836\,480} + \frac{13 \operatorname{Diag}\left[h[2]^5, ar[1, 2]\right]}{1\,161\,216} + \frac{37 \operatorname{Diag}\left[h[2]^5, ar[2, 1]\right]}{4\,644\,864}$$

```
{0, 0}
```

```
Return[]
```

```
Clear[Phi];
Phi[d_] := ModDegree[d + 1,
  V[d] = DExp[v[d]];
  Phi[d] = PutOn[{1, 2}, 3, Adjoint[V[d]]] **
    Adjoint[V[d]] ** PutOn[2, 3, V[d]] ** PutOn[1, {2, 3}, V[d]]
 ]

Phi[2]
```

$$\text{Diag}[1] + \frac{1}{16} \text{Diag}[h[1], ar[2, 3]] - \frac{1}{16} \text{Diag}[h[2], ar[1, 3]] +$$

$$\frac{1}{16} \text{Diag}[h[2], ar[3, 1]] - \frac{1}{16} \text{Diag}[h[3], ar[2, 1]]$$

```
Pentagon[Phi_] := Phi ** PutOn[1, {2, 3}, 4, Phi] ** PutOn[2, 3, 4, Phi] -
    PutOn[{1, 2}, 3, 4, Phi] ** PutOn[1, 2, {3, 4}, Phi];
RR[d_] := ModDegree[d + 1, DExp[1 / 2 (r[1, 2] + r[2, 1])]];
Hexagon[Phi_, RR_] := PutOn[{1, 2}, 3, RR] - Phi ** PutOn[2, 3, RR] **
    PutOn[1, 3, 2, DInvert[Phi]] ** PutOn[1, 3, RR] ** PutOn[3, 1, 2, Phi];

ModDegree[3, Pentagon[Phi[2]]]
```

0

```
ModDegree[3, Hexagon[Phi[2], RR[2]]]
```

0

```
Phi[3]
```

$$\text{Diag}[1] + \frac{1}{32} \text{Diag}[h[1], ar[2, 3]] - \frac{5}{96} \text{Diag}[h[1], ar[3, 2]] - \frac{1}{24} \text{Diag}[h[2], ar[1, 3]] +$$

$$\frac{1}{24} \text{Diag}[h[2], ar[3, 1]] + \frac{5}{96} \text{Diag}[h[3], ar[1, 2]] - \frac{1}{32} \text{Diag}[h[3], ar[2, 1]]$$

```
ModDegree[4, Pentagon[Phi[3]]]
```

0

```
v[3]
```

$$-\frac{1}{4} \text{Diag}[1, ar[1, 2]] + \frac{1}{4} \text{Diag}[1, ar[2, 1]] + \frac{1}{32} \text{Diag}[h[1], ar[1, 2]] +$$

$$\frac{5}{96} \text{Diag}[h[1], ar[2, 1]] + \frac{5}{96} \text{Diag}[h[2], ar[1, 2]] + \frac{1}{32} \text{Diag}[h[2], ar[2, 1]]$$

```
V[3]
```

$$\text{Diag}[1] - \frac{1}{4} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{4} \text{Diag}[1, \text{ar}[2, 1]] +$$

$$\frac{1}{32} \text{Diag}[h[1], \text{ar}[1, 2]] + \frac{1}{48} \text{Diag}[h[1], \text{ar}[2, 1]] - \frac{1}{32} \text{Diag}[h[1], \text{ar}[2, 2]] +$$

$$\frac{1}{32} \text{Diag}[h[2], \text{ar}[1, 1]] + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] + \frac{1}{32} \text{Diag}[h[2], \text{ar}[2, 1]] +$$

$$\frac{1}{192} \text{Diag}[h[1]\, h[2], \text{ar}[2, 1]] + \frac{1}{192} \text{Diag}[h[1]\, h[2], \text{ar}[2, 2]] -$$

$$\frac{1}{192} \text{Diag}\left[h[2]^2, \text{ar}[1, 1]\right] - \frac{1}{192} \text{Diag}\left[h[2]^2, \text{ar}[1, 2]\right] + \frac{1}{32} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] -$$

$$\frac{1}{16} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{1}{32} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1]] -$$

$$\frac{1}{128} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[1, 2]] + \frac{1}{384} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 1]] +$$

$$\frac{1}{128} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 2]] + \frac{1}{192} \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 1]] -$$

$$\frac{1}{128} \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 2]] - \frac{1}{128} \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[1, 2]] +$$

$$\frac{1}{128} \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[2, 1]] - \frac{1}{48} \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[1, 2]] +$$

$$\frac{5}{384} \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{1}{128} \text{Diag}[h[2], \text{ar}[2, 1], \text{ar}[2, 1]] -$$

$$\frac{1}{384} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] + \frac{1}{128} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[2, 1]] -$$

$$\frac{1}{128} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1], \text{ar}[2, 1]] + \frac{1}{384} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1], \text{ar}[2, 1]]$$

```
ModDegree[4, V[3] ** Adjoint[V[3]]]
```

```
Diag[1]
```

```
ModDegree[4, Hexagon[Phi[3], RR[3]]]
```

```
0
```

```
Phi[4]
```

```
ModDegree[5, Pentagon[Phi[4]]]
```

```
ModDegree[5, Hexagon[Phi[4], RR[4]]]
```