

Pensieve Header: Alexander blobs Results: R4 only, with f21==0.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2011-08\\w-Computations"];
<< "AlexanderBlobs-Program.m"

r[i_, j_] := Diag[1, ar[i, j]];
b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]]
-Diag[h[1], ar[2, 3]] + Diag[h[2], ar[1, 3]]
b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]] + b[r[1, 3], r[2, 3]]
0

R[i_, j_] := DExp[r[i, j]];
ModDegree[4, R[1, 2]]
Diag[1] + Diag[1, ar[1, 2]] +
   $\frac{1}{2}$  Diag[1, ar[1, 2], ar[1, 2]] +  $\frac{1}{6}$  Diag[1, ar[1, 2], ar[1, 2], ar[1, 2]]
ModDegree[7, R[1, 2] ** R[1, 3] ** R[2, 3] - R[2, 3] ** R[1, 3] ** R[1, 2]]
0

v[1] = 0;
d = 2;
ModDegree[d + 1,
  Print[
    v[d] = v[d - 1] -  $\frac{1}{2}$  Diag[1, ar[1, 2]]
  ];
  V = DExp[v[d]];
  V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]
 $-\frac{1}{2}$  Diag[1, ar[1, 2]]
0

d = 3;
ModDegree[d + 1,
  Print[
    v[d] = v[d - 1] + (1 / 24) Diag[h[1], ar[1, 2]] + (1 / 12) Diag[h[2], ar[1, 2]]
  ];
  V = DExp[v[d]];
  V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]
 $-\frac{1}{2}$  Diag[1, ar[1, 2]] +  $\frac{1}{24}$  Diag[h[1], ar[1, 2]] +  $\frac{1}{12}$  Diag[h[2], ar[1, 2]]
0
```

```

d = 4;
ModDegree[d + 1,
  Print[
    v[d] = v[d - 1]
  ];
  V = DExp[v[d]];
  V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]

$$-\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]]$$

0

d = 5;
ModDegree[d + 1,
  Print[
    c[d, 0] = -1 / 720; c[d, 1] = -1 / 480; c[d, 2] = -1 / 720; c[d, 3] = -1 / 2880;
    v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 2 - k), ar[1, 2]], {k, 0, d - 2}]
  ];
  V = DExp[v[d]];
  V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]

$$-\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} +$$


$$\frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] -$$


$$\frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]]$$

0

d = 6;
ModDegree[d + 1,
  Print[
    c[6, _] = 0;
    v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 2 - k), ar[1, 2]], {k, 0, d - 2}]
  ];
  V = DExp[v[d]];
  V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]

$$-\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} +$$


$$\frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] -$$


$$\frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]]$$

0

```

```

d = 7;
ModDegree[d + 1,
Print[
  c[7, 0] = 1 / 30 240; c[7, 1] = 1 / 12 096; c[7, 2] = 1 / 9072;
  c[7, 3] = 1 / 12 096; c[7, 4] = 1 / 30 240; c[7, 5] = 1 / 181 440;
  v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 2 - k), ar[1, 2]], {k, 0, d - 2}]
];
V = DExp[v[d]];
V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]

```

$$\begin{aligned}
& -\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} + \\
& \frac{\text{Diag}[h[1]^5, \text{ar}[1, 2]]}{181\,440} + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{30\,240} - \frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{12\,096} - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{9072} + \frac{\text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{12\,096} + \frac{\text{Diag}[h[2]^5, \text{ar}[1, 2]]}{30\,240}
\end{aligned}$$

0

```
ModDegree[8, V ** Adjoint[V]]
```

```
Diag[1]
```

```

d = 8;
ModDegree[d + 1,
Print[
  c[8, _] = 0;
  v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 2 - k), ar[1, 2]], {k, 0, d - 2}]
];
V = DExp[v[d]];
V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]

```

$$\begin{aligned}
& -\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} + \\
& \frac{\text{Diag}[h[1]^5, \text{ar}[1, 2]]}{181\,440} + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{30\,240} - \frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{12\,096} - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{9072} + \frac{\text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{12\,096} + \frac{\text{Diag}[h[2]^5, \text{ar}[1, 2]]}{30\,240}
\end{aligned}$$

0

```

d = 9;
ModDegree[d + 1,
Print[
  c[9, 0] = -1 / 1 209 600; c[9, 1] = -1 / 345 600;
  c[9, 2] = -1 / 172 800; c[9, 3] = -1 / 138 240; c[9, 4] = -1 / 172 800;
  c[9, 5] = -1 / 345 600; c[9, 6] = -1 / 1 209 600; c[9, 7] = -1 / 9 676 800;
  v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 2 - k), ar[1, 2]], {k, 0, d - 2}]
];
V = DExp[v[d]];
V ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V
]

-  $\frac{1}{2}$  Diag[1, ar[1, 2]] +  $\frac{1}{24}$  Diag[h[1], ar[1, 2]] -  $\frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880}$  +
 $\frac{\text{Diag}[h[1]^5, \text{ar}[1, 2]]}{181\,440}$  -  $\frac{\text{Diag}[h[1]^7, \text{ar}[1, 2]]}{9\,676\,800}$  +  $\frac{1}{12}$  Diag[h[2], ar[1, 2]] -
 $\frac{1}{720}$  Diag[h[1]^2 h[2], ar[1, 2]] +  $\frac{\text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{30\,240}$  -  $\frac{\text{Diag}[h[1]^6 h[2], \text{ar}[1, 2]]}{1\,209\,600}$  -
 $\frac{1}{480}$  Diag[h[1] h[2]^2, ar[1, 2]] +  $\frac{\text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{12\,096}$  -  $\frac{\text{Diag}[h[1]^5 h[2]^2, \text{ar}[1, 2]]}{345\,600}$  -
 $\frac{1}{720}$  Diag[h[2]^3, ar[1, 2]] +  $\frac{\text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{9\,072}$  -  $\frac{\text{Diag}[h[1]^4 h[2]^3, \text{ar}[1, 2]]}{172\,800}$  +
 $\frac{\text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{12\,096}$  -  $\frac{\text{Diag}[h[1]^3 h[2]^4, \text{ar}[1, 2]]}{138\,240}$  +  $\frac{\text{Diag}[h[2]^5, \text{ar}[1, 2]]}{30\,240}$  -
 $\frac{\text{Diag}[h[1]^2 h[2]^5, \text{ar}[1, 2]]}{172\,800}$  -  $\frac{\text{Diag}[h[1] h[2]^6, \text{ar}[1, 2]]}{345\,600}$  -  $\frac{\text{Diag}[h[2]^7, \text{ar}[1, 2]]}{1\,209\,600}$ 
0
ModDegree[10, V ** Adjoint[V]]
Diag[1]
v[9] /. Diag[hs_, ar[1, 2]] -> hs /. {h[1] -> h1, h[2] -> h2}
-  $\frac{1}{2}$  +  $\frac{h1}{24}$  -  $\frac{h1^3}{2880}$  +  $\frac{h1^5}{181\,440}$  -  $\frac{h1^7}{9\,676\,800}$  +  $\frac{h2}{12}$  -  $\frac{h1^2 h2}{720}$  +  $\frac{h1^4 h2}{30\,240}$  -  $\frac{h1^6 h2}{1\,209\,600}$  -  $\frac{h1 h2^2}{480}$  +  $\frac{h1^3 h2^2}{12\,096}$  -
 $\frac{h1^5 h2^2}{345\,600}$  -  $\frac{h2^3}{720}$  +  $\frac{h1^2 h2^3}{9\,072}$  -  $\frac{h1^4 h2^3}{172\,800}$  +  $\frac{h1 h2^4}{12\,096}$  -  $\frac{h1^3 h2^4}{138\,240}$  +  $\frac{h2^5}{30\,240}$  -  $\frac{h1^2 h2^5}{172\,800}$  -  $\frac{h1 h2^6}{345\,600}$  -  $\frac{h2^7}{1\,209\,600}$ 
-1 209 600 * 8 * Table[c[9, k], {k, 0, 7}]
{8, 28, 56, 70, 56, 28, 8, 1}
Table[Binomial[8, k], {k, 1, 8}]
{8, 28, 56, 70, 56, 28, 8, 1}

```