

Pensieve header:  $\mu$ -calculus programs, continues pensieve://2013-11/DoubleTree/.

“d” is “ht”: along tube strands, heads appear before tails.

## Program

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Domain[f_List] := First /@ f;
f \ key_ := DeleteCases[f, key → _];
f \ keys_List := Fold[#1 \ #2 &, f, keys];
f1_List ≡ f2_List := Domain[f1] === Domain[f2] && (And @@ (
  ((# /. f1) ≡ (# /. f2)) & /@ Domain[f1]
));
LieMorphism[mor_] [f_List] := MapAt[LieMorphism[mor], f, {All, 2}];
Domain[M[f_List, _]] := Domain[f];
M[λ1_, ω1_] ≡ M[λ2_, ω2_] := (λ1 ≡ λ2) && (ω1 ≡ ω2);
M[_ , ω_] [W] := ω;
M[λ_, _] [x_] := (x /. λ);

CC[u_, γ_LieSeries] := LieMorphism[⟨u⟩ → Ad[γ][⟨u⟩]];
CC_u [γ_] := CC[u, γ];
RC[u_, γ_LieSeries, ub_] [ser_] :=
  StableApply[LieMorphism[⟨u⟩ → Ad[γ][⟨ub⟩]], ser];
RC[u_, γ_LieSeries] [ser_] := ser // RC[u, γ, ⟨u⟩] // LieMorphism[⟨u⟩ → ⟨u⟩];
RC_u [γ_] := RC[u, γ];

J[u_LW, γ_] := Module[{s}, ∫01 (γ // RC_u[s γ] // div_u // CC_u[-s γ]) ds];
J[u_, γ_] := J[⟨u⟩, γ];
J_u [γ_] := J[u, γ];

M /: M[λ1_, ω1_] ∪ M[λ2_, ω2_] := M[λ1 ∪ λ2, ω1 + ω2];
M /: M[λ1_, ω1_] M[λ2_, ω2_] := M[λ1 ∪ λ2, ω1 + ω2];

tσ[us_List → vs_List] [ser_LieSeries | ser_CWSeries | ser_List] :=
  ser // LieMorphism[Thread[(LW/@us) → (LW/@vs)]];
tσ[u_, v_] := tσ[{u} → {v}];
tσ[us_List → vs_List] [μ_M] := tσ[us → vs] /@ μ;
hσ[xs_List → ys_List] [λ_List] := Union[λ \ xs, Thread[ys → (xs /. λ)]];
hσ[x_, y_] := hσ[{x} → {y}];
hσ[xs_List → ys_List] [M[λ_, ω_]] := M[λ // hσ[xs → ys], ω];
dσ[as_List → bs_List] [μ_] := μ // tσ[as → bs] // hσ[as → bs];
dσ[a_, b_] [μ_] := μ // tσ[a, b] // hσ[a, b];

hη[xs___] [M[λ_, ω_]] := M[λ \ {xs}, ω];
tη[us___] [ser_LieSeries | ser_CWSeries | ser_List] :=
  ser // LieMorphism[(LW[#] → 0) & /@ {us}];
tη[us___] [μ_M] := tη[us] /@ μ;
dη[as___] [μ_M] := μ // hη[as] // tη[as];

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tm[u_, v_, w_][λ_List] := λ // LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩];
tm[u_, v_, w_][M[λ_, ω_]] := LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩] /@ M[λ, ω];
hm[x_, y_, z_][λ_List] := Union[λ \ {x, y}, {z → BCH[x /. λ, y /. λ]}];
hm[x_, y_, z_][M[λ_, ω_]] := M[λ // hm[x, y, z], ω];
tha[u_, x_][λ_List] := MapAt[RC[u, x /. λ], λ, {All, 2}];
tha[u_, x_][M[λ_, ω_]] :=
  M[λ // tha[u, x], (ω + J[u, x /. λ]) // RC[u, x /. λ]];
dm[a_, b_, c_][μ_] := μ // tha[⟨a⟩, b] // tm[⟨a⟩, ⟨b⟩, ⟨c⟩] // hm[a, b, c];
dm[a_, b_, rest_, c_][μ_] := μ // dm[b, rest, b] // dm[a, b, c];

tΔ[u_, v_, w_][μ_M] := LieMorphism[⟨u⟩ → ⟨v⟩ + ⟨w⟩] /@ μ;
hΔ[x_, y_, z_][M[λ_, ω_]] := M[Union[λ \ x, {y → (x /. λ), z → (x /. λ)}], ω];
dΔ[a_, b_, c_][μ_M] := μ // tΔ[a, b, c] // hΔ[a, b, c];
dΔ[a_, {b_}] := dσ[a, b];
dΔ[a_, {b_, c_, rest_}][μ_M] := μ // dΔ[a, b, c] // dΔ[c, {c, rest}];

dP[pl_List][μ_] := Module[{σ, len, μ1},
  len = Length[pl];
  μ1 = μ // dσ[Range[len] → First /@ pl];
  Do[μ1 = μ1 // dΔ[pl[[i, 1]], pl[[i]]], {i, len}];
  μ1
];
dP[pl_Integer] := dP[IntegerDigits /@ {pl}];

tS[u_][μ_M] := LieMorphism[⟨u⟩ → -⟨u⟩] /@ μ;
hS[x_][M[λ_, ω_]] := M[Union[λ \ x, {x → -(x /. λ)}], ω];
dS[a_][μ_] := μ // tS[a] // hS[a] // tha[a, a];
dS[a_, rest_][μ_] := μ // dS[a] // dS[rest];

tA[u_][expr_] := expr;
hA[x_][M[λ_, ω_]] := M[Union[λ \ x, {x → -(x /. λ)}], ω];
dA[a_][μ_] := μ // hA[a] // tha[a, a];
dA[a_, rest_][μ_] := μ // dA[a] // dA[rest];

dc[a_][μ_] := μ // hS[a] // tha[a, a] // hS[a] // hη[a];

te[u_] := M[{}, MakeCWSeries[0]];
he[x_] := M[{x → MakeLieSeries[0]}, MakeCWSeries[0]];
de[a_] := te[a] ∪ he[a];
de[a_, rest_] := de[a] ∪ de[rest];
ρ+[u_, x_] := M[{x → MakeLieSeries[⟨u⟩]}, MakeCWSeries[0]];
ρ-[u_, x_] := M[{x → MakeLieSeries[-⟨u⟩]}, MakeCWSeries[0]];
R+[a_, b_] := ρ+[a, b] ∪ M[{a → MakeLieSeries[0]}, MakeCWSeries[0]];
R-[a_, b_] := ρ-[a, b] ∪ M[{a → MakeLieSeries[0]}, MakeCWSeries[0]];
R[a_, b_, p_] /; a ≠ b :=
  M[{a → MakeLieSeries[0], b → MakeLieSeries[p ⟨a⟩]}, MakeCWSeries[0]];
R[a_, a_, p_] := M[{a → MakeLieSeries[p ⟨a⟩]}, MakeCWSeries[0]];
Θ[a_, b_, p_] := (R[a, a, p / 2] // dΔ[a, a, b]) ** R[a, a, -p / 2] ** R[b, b, -p / 2];
Θ[a_, b_] := Θ[a, b, 1];

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M /:  $\mu_{1\_M} ** \mu_{2\_M} :=$  Module[{d1, d2, dom,  $\mu_{1p} = \mu_1$ ,  $\mu_{2p} = \mu_2$ , s},
  {d1 = Domain[ $\mu_1$ ], d2 = Domain[ $\mu_2$ ], dom = Union[d1, d2]};
  Do[ $\mu_{1p} = \mu_{1p} \cup de[a]$ , {a, Complement[d2, d1]}];
  Do[ $\mu_{2p} = \mu_{2p} \cup de[a]$ , {a, Complement[d1, d2]}];
  s = Max[dom] - Min[dom] + 1;
  Do[ $\mu_{2p} = \mu_{2p} // d\sigma[a, a + s]$ , {a, dom}];
  Fold[({#1 //  $dm[{\#2, \#2 + s, \#2}]$ ) &,  $\mu_{1p} \cup \mu_{2p}$ , dom]
];

Rot120[ $\mu_{\_}$ ] :=  $\mu // dS[2] // d\Delta[2, 2, 3] // dm[1, 3, 1] // dP[2, 1]$ 

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