Pensieve header: Solving R4, Cap, and Unitarity without assuming $W=1$. With Dancso, October 2023.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
ω = CWS[{x}, ωs]; Wen = Es[⟨x → LS[0]⟩, ω];

R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
CapWen = Cap ≡ ((Wen ** Cap) // dA[x] // dS[x]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
    (Cap * (Cap // dσ[x, y]) // dc[x] // dc[y]));
UnitarityEqn =
  ((V // dA[x] // dA[y]) ** (Wen * (Wen // dσ[x, y])) ** V) ≡ (Wen // dΔ[x, x, y]);

βs[x] = 1 / 2; βs[y] = 0;
ωs[x, x, x] = 1;
SeriesSolve[{α, β, γ, κ, ω}, (ħ^-1 R4Eqn) ∧ CapWen ∧ CapEqn ∧ UnitarityEqn];
Column@{V@{5}, κ@{5}, ω@{5}}
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⁀, ⁀}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA,
  hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150909.

SeriesSolve: In degree 1 arbitrarily setting {ωs[x] → 0}.

SeriesSolve: In degree 5 arbitrarily setting {γs[x, x, x, x, y] → 0}.
```

Out[•]=

$$\mathrm{Es}\Big[\Big\langle x \to \mathrm{LS}\Big[0, -\frac{\overline{xy}}{24}, 0, \frac{\overline{7x\,\overline{x\,\overline{xy}}}}{5760} - \frac{\overline{7x\,\overline{xy\,y}}}{5760} + \frac{\overline{\overline{xy}\,y\,y}}{1440}, 0, \ldots\Big],$$

$$y \to \mathrm{LS}\Big[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \frac{\overline{x\,\overline{x\,\overline{xy}}}}{5760} - \frac{1}{720}\overline{x\,\overline{\overline{xy}\,y}} + \frac{1}{720}\overline{\overline{\overline{xy}\,y}\,y}, -\frac{\overline{\overline{xx}\,\overline{x\,\overline{xy}}}}{7680} + \frac{\overline{\overline{xx}\,\overline{xy\,y}}}{3840} - \frac{\overline{\overline{x\,\overline{xy}}\,\overline{xy}}}{6912}, \ldots\Big]\Big\rangle,$$

$$\mathrm{CWS}\Big[0, -\frac{\widehat{xy}}{48}, \frac{3\widehat{xxy}}{2} + \frac{3\widehat{xyy}}{2}, \frac{\widehat{xxxy}}{2880} + \frac{\widehat{xxyy}}{2880} + \frac{\widehat{xyxy}}{5760} + \frac{\widehat{xyyy}}{2880}, 0, \ldots\Big]\Big]$$

$$\mathrm{CWS}\Big[0, -\frac{\widehat{xx}}{96}, -\frac{\widehat{xxx}}{2}, \frac{\widehat{xxxx}}{11\,520}, 0, \ldots\Big]$$

$$\mathrm{CWS}\Big[0, 0, \widehat{xxx}, 0, 0, \ldots\Big]$$