

Pensieve header: Fuller output of for V and Cap. Even fuller: <VCapSolution-to-12.m>.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[<a → LS[0], b → LS[LW@a], CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[<x → α, y → β>, γ];
κ = CWS[{x}, κs]; Cap = Es[<x → LS[0], κ];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn = (V ** (V // dA[x] // dA[y]) ≡ Es[<x → LS[0], y → LS[0], CWS[0]]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
    (Cap * (Cap // dσ[x, y]) // dc[x] // dc[y]));
βs[x] = 1 / 2; βs[y] = 0;
SeriesSolve[{α, β, γ, κ}, (ħ-1 R4Eqn) && UnitarityEqn && CapEqn];
{V, κ}
```

FreeLie` implements / extends
 {*, +, **, \$SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↪}.

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, E1, Es, hA, hm, hS, hΔ, hη, hσ, RandomESeries, RandomESeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150909.

SeriesSolve: In degree 1 arbitrarily setting {κs[x] → 0}.

SeriesSolve: In degree 3 arbitrarily setting {αs[x, y] → 0}.

Out[]:=

$$\left\{ \text{Es} \left[\left\langle x \rightarrow \text{LS} \left[0, -\frac{\overline{xy}}{24}, 0, \dots \right], y \rightarrow \text{LS} \left[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \dots \right] \right\rangle, \text{CWS} \left[0, -\frac{\overline{xy}}{48}, 0, \dots \right] \right], \right. \\ \left. \text{CWS} \left[0, -\frac{\overline{xx}}{96}, 0, \dots \right] \right\}$$

In[]:= **V@{8} // Timing**

SeriesSolve: In degree 5 arbitrarily setting {αs[x, x, x, y] → 0}.

SeriesSolve: In degree 7 arbitrarily setting {αs[x, x, x, x, y] → 0}.

SeriesSolve: In degree 8 arbitrarily setting {αs[x, x, x, x, y, x, y] → 0}.

General: Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation.

Out[]=

$$\left\{ 9.625, \text{Es} \left[\left\langle x \rightarrow \text{LS} \left[0, -\frac{\overline{xy}}{24}, 0, \frac{7 \overline{xxxy}}{5760} - \frac{7 \overline{xyy}}{5760} + \frac{\overline{xyyy}}{1440}, 0, \right. \right. \right. \right.$$

$$-\frac{31 \overline{xxxxxy}}{967680} + \frac{31 \overline{xxxxyy}}{483840} - \frac{83 \overline{xxxyyy}}{967680} - \frac{31 \overline{xyxyy}}{725760} - \frac{31 \overline{xyxyy}}{645120} +$$

$$\frac{13 \overline{xyyy}}{241920} + \frac{101 \overline{xyxyy}}{1451520} + \frac{527 \overline{xyyyxy}}{5806080} - \frac{\overline{xyyyy}}{60480}, 0, \frac{127 \overline{xxxxxy}}{15482880} -$$

$$\frac{127 \overline{xxxxxyy}}{51609600} + \frac{2399 \overline{xxxxxyy}}{464486400} + \frac{127 \overline{xxxxyy}}{30965760} - \frac{2893 \overline{xxxxyy}}{464486400} -$$

$$\frac{727 \overline{xxxyxyy}}{87091200} - \frac{1397 \overline{xxxxyy}}{77414400} + \frac{271 \overline{xyyy}}{58060800} - \frac{15389 \overline{xyxyxy}}{1393459200} +$$

$$\frac{12809 \overline{xyxyyy}}{1393459200} - \frac{113 \overline{xxxyxy}}{24883200} - \frac{127 \overline{xxxxyxy}}{77414400} + \frac{13103 \overline{xxxyxy}}{1393459200} +$$

$$\frac{12409 \overline{xxxyyy}}{696729600} + \frac{1207 \overline{xyxyy}}{278691840} + \frac{11 \overline{xxxyxy}}{1382400} - \frac{19 \overline{xyyy}}{9676800} +$$

$$\frac{67 \overline{xyxyxyy}}{7257600} - \frac{1327 \overline{xyxyyy}}{348364800} + \frac{\overline{xyxyy}}{204800} + \frac{181 \overline{xxxyxy}}{38707200} +$$

$$\frac{2021 \overline{xxxyxy}}{696729600} - \frac{239 \overline{xxxyxy}}{199065600} - \frac{2171 \overline{xyyyxy}}{464486400} - \frac{631 \overline{xyyyxy}}{92897280} -$$

$$\frac{1751 \overline{xyxyxyy}}{696729600} - \frac{451 \overline{xyxyxyy}}{154828800} - \frac{997 \overline{xyxyxyxy}}{77414400} + \frac{\overline{xyyyyy}}{2419200}, \dots \Big],$$

$$y \rightarrow \text{LS} \left[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \frac{\overline{xxxy}}{5760} - \frac{1}{720} \overline{xyy} + \frac{1}{720} \overline{xyyy}, -\frac{\overline{xxxy}}{7680} + \frac{\overline{xxxy}}{3840} - \frac{\overline{xyxy}}{6912}, \right.$$

$$\left. -\frac{\overline{xxxxxy}}{645120} + \frac{23 \overline{xxxxyy}}{483840} - \frac{13 \overline{xxxyy}}{161280} - \frac{\overline{xyxyy}}{22680} \right]$$

$$\begin{aligned} & \frac{\overline{\text{xxxxxxy}}}{4838400} + \frac{\overline{\text{xxxxxyy}}}{4838400} + \frac{\overline{\text{xxxxxyxy}}}{4838400} + \frac{\overline{\text{xxxxxyyy}}}{4838400} + \frac{\overline{\text{xxxxyxy}}}{4838400} + \frac{\overline{\text{xxxxyxyy}}}{4838400} + \\ & \frac{\overline{\text{xxxxyxyy}}}{4838400} + \frac{\overline{\text{xxxxyyyy}}}{4838400} + \frac{\overline{\text{xxxyxxxxy}}}{9676800} + \frac{\overline{\text{xxxyxxyy}}}{4838400} + \frac{\overline{\text{xxxyxyxy}}}{4838400} + \frac{\overline{\text{xxxyxyyy}}}{4838400} + \\ & \frac{\overline{\text{xxxyyxy}}}{4838400} + \frac{\overline{\text{xxxyyxyy}}}{4838400} + \frac{\overline{\text{xxxyyyyxy}}}{4838400} + \frac{\overline{\text{xxxyyyyxyy}}}{4838400} + \frac{\overline{\text{xyyxyxy}}}{4838400} + \frac{\overline{\text{xyyxyxyy}}}{4838400} + \\ & \frac{\overline{\text{xyyxyxyy}}}{4838400} + \frac{\overline{\text{xyyxyyyy}}}{4838400} + \frac{\overline{\text{xyyxyyyy}}}{4838400} + \frac{\overline{\text{xyyxyxyy}}}{9676800} + \frac{\overline{\text{xyyxyxyy}}}{4838400} + \\ & \frac{\overline{\text{xyyxyxyy}}}{4838400} + \frac{\overline{\text{xyyxyyyy}}}{4838400} + \frac{\overline{\text{xyyxyyyy}}}{4838400} + \frac{\overline{\text{xyyxyxyy}}}{9676800} + \frac{\overline{\text{xyyxyxyy}}}{4838400}, \dots \end{aligned}$$

$x@{8}$

$$\text{CWS}\left[0, -\frac{\overline{\text{xx}}}{96}, 0, \frac{\overline{\text{xxxx}}}{11520}, 0, -\frac{\overline{\text{xxxxxx}}}{725760}, 0, \frac{\overline{\text{xxxxxxxx}}}{38707200}, \dots\right]$$

The same thing, copy-paste ready and machine readable:

Sum[{**V**[1],**x**[**k**], **V**[1],**y**[**k**], **V**[2],**k**], {**k**, 8}] // **InputForm**

$$\begin{aligned} & -\text{LW}[x, y]/24 + (7*\text{LW}[x, x, x, y])/5760 - (7*\text{LW}[x, x, y, y])/5760 + \\ & \text{LW}[x, y, y, y]/1440 - (31*\text{LW}[x, x, x, x, y, y])/967680 + \\ & (31*\text{LW}[x, x, x, x, y, y])/483840 - (31*\text{LW}[x, x, x, y, x, y, y])/645120 - \\ & (83*\text{LW}[x, x, x, y, y, y, y])/967680 - (31*\text{LW}[x, x, y, x, y, y, y])/725760 + \\ & (527*\text{LW}[x, x, y, y, x, y, x, y, y])/5806080 + (13*\text{LW}[x, x, y, y, y, y, y, y])/241920 + \\ & (101*\text{LW}[x, y, x, y, y, y, y, y, y])/1451520 - \text{LW}[x, y, y, y, y, y, y, y, y, y]/60480 + \\ & (127*\text{LW}[x, x, x, x, x, x, x, y, y])/154828800 - (127*\text{LW}[x, x, x, x, x, x, x, x, y, y])/ \\ & 51609600 + (127*\text{LW}[x, x, x, x, x, x, x, y, x, y])/30965760 + \\ & (2399*\text{LW}[x, x, x, x, x, y, y, y, y])/464486400 - (127*\text{LW}[x, x, x, x, y, x, x, x, y, y])/ \\ & 77414400 - (1397*\text{LW}[x, x, x, x, y, y, x, y, y])/77414400 - \\ & (2893*\text{LW}[x, x, x, x, y, y, y, y, y])/464486400 - (113*\text{LW}[x, x, x, y, x, x, y, y, y])/ \\ & 24883200 + (11*\text{LW}[x, x, x, y, x, y, x, y, y])/1382400 - \\ & (727*\text{LW}[x, x, x, y, x, y, y, y, y])/87091200 + (2021*\text{LW}[x, x, x, y, y, x, x, y, y])/ \\ & 696729600 + (13103*\text{LW}[x, x, x, y, y, x, y, y, y])/1393459200 + \\ & (12409*\text{LW}[x, x, x, y, y, y, x, y, y])/696729600 + (271*\text{LW}[x, x, x, y, y, y, y, y, y])/ \\ & 58060800 + (181*\text{LW}[x, x, y, x, x, y, x, y, y])/38707200 + \\ & \text{LW}[x, x, y, x, x, y, y, y, y, y]/204800 - (15389*\text{LW}[x, x, y, x, y, x, y, y, y, y])/ \\ & 1393459200 - (239*\text{LW}[x, x, y, x, y, y, x, y, y, y])/199065600 + \\ & (12809*\text{LW}[x, x, y, x, y, y, y, y, y])/1393459200 - (997*\text{LW}[x, x, y, y, x, y, x, y, y, y])/ \\ & 77414400 + (1207*\text{LW}[x, x, y, y, x, y, y, y, y])/278691840 - \\ & (2171*\text{LW}[x, x, y, y, y, x, y, y, y])/464486400 - (631*\text{LW}[x, x, y, y, y, y, x, y, y, y])/ \\ & 92897280 - (19*\text{LW}[x, x, y, y, y, y, y, y, y])/9676800 + \\ & (67*\text{LW}[x, y, x, y, x, y, y, y, y])/7257600 - (451*\text{LW}[x, y, x, y, y, x, y, y, y])/ \\ & 154828800 - (1327*\text{LW}[x, y, x, y, y, y, y, y, y])/348364800 - \\ & (1751*\text{LW}[x, y, y, x, y, y, y, y, y, y])/696729600 + \text{LW}[x, y, y, y, y, y, y, y, y, y]/2419200, \\ & \text{LW}[x]/2 - \text{LW}[x, y]/12 + \text{LW}[x, x, x, y]/5760 - \text{LW}[x, x, y, y]/720 + \\ & \text{LW}[x, y, y, y]/720 - \text{LW}[x, x, x, x, y]/7680 + \text{LW}[x, x, x, y, y]/3840 - \\ & \text{LW}[x, x, y, x, y]/6912 - \text{LW}[x, x, x, x, x, y]/645120 + \\ & (23*\text{LW}[x, x, x, x, y, y, y])/483840 - (41*\text{LW}[x, x, x, y, x, y, y])/580608 - \\ & (13*\text{LW}[x, x, x, y, y, y, y])/161280 - \text{LW}[x, x, y, x, y, y, y, y, y, y]/22680 + \end{aligned}$$

$$\begin{aligned}
 & (71 * LW[x, x, y, y, x, y]) / 483840 + LW[x, x, y, y, y, y] / 15120 + \\
 & LW[x, y, x, y, y, y] / 12096 - LW[x, y, y, y, y, y] / 30240 + \\
 & LW[x, x, x, x, x, y] / 258048 - (5 * LW[x, x, x, x, x, y]) / 387072 + \\
 & (5 * LW[x, x, x, x, y, x, y]) / 290304 + LW[x, x, x, x, y, y, y] / 64512 - \\
 & (7 * LW[x, x, x, y, x, x, y]) / 1658880 + LW[x, x, x, y, x, y, y] / 96768 - \\
 & LW[x, x, x, y, y, x, y] / 60480 - LW[x, x, x, y, y, y, y] / 96768 - \\
 & LW[x, x, y, x, x, y, y] / 207360 - (17 * LW[x, x, y, x, y, y, y]) / 1451520 + \\
 & LW[x, x, y, y, y, x, y] / 207360 + LW[x, x, x, x, x, x, x, y] / 77414400 - \\
 & (587 * LW[x, x, x, x, x, x, y, y]) / 464486400 + (541 * LW[x, x, x, x, x, y, x, y]) / \\
 & 154828800 + (253 * LW[x, x, x, x, x, y, y, y]) / 66355200 - \\
 & (157 * LW[x, x, x, x, y, x, x, y]) / 49766400 + (59 * LW[x, x, x, x, y, x, y, y]) / \\
 & 1393459200 - (24457 * LW[x, x, x, x, y, y, x, y]) / 1393459200 - \\
 & (43 * LW[x, x, x, x, y, y, y, y]) / 7257600 - (199 * LW[x, x, x, y, x, x, y, y]) / \\
 & 29030400 + (271 * LW[x, x, x, y, x, y, x, y]) / 30965760 - \\
 & (12143 * LW[x, x, x, y, x, y, y, y]) / 1393459200 + (2281 * LW[x, x, x, y, y, x, x, y]) / \\
 & 464486400 + (7367 * LW[x, x, x, y, y, x, y, y]) / 696729600 + \\
 & (5939 * LW[x, x, x, y, y, y, x, y]) / 278691840 + (73 * LW[x, x, x, y, y, y, y, y]) / \\
 & 14515200 + (4057 * LW[x, x, y, x, x, y, x, y]) / 696729600 + \\
 & (47 * LW[x, x, y, x, x, y, y, y]) / 6451200 - (1217 * LW[x, x, y, x, y, x, y, y]) / \\
 & 99532800 + (3 * LW[x, x, y, x, y, y, x, y]) / 5734400 + \\
 & (1423 * LW[x, x, y, x, y, y, y, y]) / 116121600 - (17513 * LW[x, x, y, y, x, y, x, y]) / \\
 & 1393459200 + (3263 * LW[x, x, y, y, x, y, y, y]) / 696729600 - \\
 & LW[x, x, y, y, y, x, y, y] / 92160 - (151 * LW[x, x, y, y, y, y, x, y]) / 14515200 - \\
 & LW[x, x, y, y, y, y, y, y] / 403200 + (583 * LW[x, y, x, y, x, y, y, y]) / 38707200 - \\
 & (4589 * LW[x, y, x, y, y, x, y, y]) / 696729600 - LW[x, y, x, y, y, y, y, y] / 172800 - \\
 & LW[x, y, y, x, y, y, y, y] / 172800 + LW[x, y, y, y, y, y, y, y] / 1209600, \\
 & -CW[x, y] / 48 + CW[x, x, x, y] / 2880 + CW[x, x, y] / 2880 + CW[x, y, x, y] / 5760 + \\
 & CW[x, y, y] / 2880 - CW[x, x, x, x, y] / 120960 - CW[x, x, x, x, y, y] / 120960 - \\
 & CW[x, x, x, x, y, x, y] / 120960 - CW[x, x, x, x, y, y, y] / 120960 - \\
 & CW[x, x, x, y, x, x, y] / 120960 - CW[x, x, y, y, y, y] / 120960 - \\
 & CW[x, y, x, y, x, y] / 362880 - CW[x, y, x, y, y, y] / 120960 - \\
 & CW[x, y, y, x, y, y] / 241920 - CW[x, y, y, y, y, y] / 120960 + \\
 & CW[x, x, x, x, x, x, y] / 4838400 + CW[x, x, x, x, x, x, y, y] / 4838400 + \\
 & CW[x, x, x, x, x, y, x, y] / 4838400 + CW[x, x, x, x, x, y, y, y] / 4838400 + \\
 & CW[x, x, x, x, y, x, x, y] / 4838400 + CW[x, x, x, x, y, x, y, y] / 4838400 + \\
 & CW[x, x, x, x, y, y, x, y] / 4838400 + CW[x, x, x, x, y, y, y, y] / 4838400 + \\
 & CW[x, x, x, y, x, x, x, y] / 9676800 + CW[x, x, x, y, x, x, y, y] / 4838400 + \\
 & CW[x, x, x, y, x, y, x, y] / 4838400 + CW[x, x, x, y, x, y, y, y] / 4838400 + \\
 & CW[x, x, x, y, y, x, x, y] / 4838400 + CW[x, x, x, y, y, x, y, y] / 4838400 + \\
 & CW[x, x, x, y, y, y, x, y] / 4838400 + CW[x, x, x, y, y, y, y, y] / 4838400 + \\
 & CW[x, x, y, x, x, y, x, y] / 4838400 + CW[x, x, y, x, x, y, y, y] / 4838400 + \\
 & CW[x, x, y, x, y, x, x, y] / 4838400 + CW[x, x, y, x, y, x, y, y] / 4838400 + \\
 & CW[x, x, y, x, y, y, y, y] / 4838400 + CW[x, x, y, y, x, x, y, y] / 9676800 + \\
 & CW[x, x, y, y, x, y, x, y] / 4838400 + CW[x, x, y, y, x, y, y, y] / 4838400 + \\
 & CW[x, x, y, y, y, x, y, y] / 4838400 + CW[x, x, y, y, y, y, x, y] / 4838400 + \\
 & CW[x, x, y, y, y, y, y, y] / 4838400 + CW[x, y, x, y, x, y, x, y] / 19353600 + \\
 & CW[x, y, x, y, x, y, y, y] / 4838400 + CW[x, y, x, y, y, x, y, y] / 4838400 + \\
 & CW[x, y, x, y, y, y, y, y] / 4838400 + CW[x, y, y, x, y, y, y, y] / 4838400 + \\
 & CW[x, y, y, y, x, y, y, y] / 9676800 + CW[x, y, y, y, y, y, y, y] / 4838400
 \end{aligned}$$

Sum[x[k], {k, 8}] // InputForm

$$\begin{aligned}
 & -CW[x, x] / 96 + CW[x, x, x, x] / 11520 - CW[x, x, x, x, x, x] / 725760 + \\
 & CW[x, x, x, x, x, x, x, x] / 38707200
 \end{aligned}$$

The same thing, machine readable and to degree 11, is in the file quoted below, in the same folder:

```
Put[Sum[{V[[1]]x[k], V[[1]]y[k], V[[2]] [k]}, {k, 11}], "VCapSolution-to-11.m"];
PutAppend[Sum[x[k], {k, 11}], "VCapSolution-to-11.m"]
Arbitrarily setting {αs[x, x, x, x, x, x, x, y, y] → 0}.
Arbitrarily setting {αs[x, x, x, x, x, x, y, x, y, y] → 0}.
Arbitrarily setting {αs[x, x, x, x, x, x, x, x, x, y, y] → 0, αs[x, x, x, x, x, x, y, x, y, y, y] → 0}.

{TimeUsed [], MaxMemoryUsed []}
{17 123.2, 10 071 646 624}

TwistEqn = (V ≡ Rs[x, y] ** (V // dσ[{x, y} → {y, x}]) **
  (E1[⟨x → LS[-LW@y / 2], y → LS[-LW@x / 2]⟩, CWS[0]] // T));
TwistEqn@{6}
BS[7 True, ...]

TrueQ[TwistEqn]@{6}
BS[7 True, ...]

TrueQ[TwistEqn]@{11}
BS[12 True, ...]

{TimeUsed [], MaxMemoryUsed []}
{35 991., 15 840 067 368}
```

The same thing, machine readable and to degree 12, is in the file quoted below, in the same folder:

```
Put[Sum[{V[[1]]x[k], V[[1]]y[k], V[[2]] [k]}, {k, 12}], "VCapSolution-to-12.m"];
PutAppend[Sum[x[k], {k, 12}], "VCapSolution-to-12.m"]
Arbitrarily setting
  {αs[x, x, x, x, x, x, x, x, y, x, y, y] → 0, αs[x, x, x, x, x, x, y, x, y, y, y, y] → 0}.

{TimeUsed [], MaxMemoryUsed []}
{140 028., 48 977 267 312}

TrueQ[TwistEqn]@{12}
BS[13 True, ...]
```

Added July 2023: Just for fun, testing the unitarity of Φ_V :

In[*]:= **V**
 Out[*]=

$$\text{Es} \left[\left\langle x \rightarrow \text{LS} \left[0, -\frac{\overline{xy}}{24}, 0, \dots \right], y \rightarrow \text{LS} \left[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \dots \right] \right\rangle, \text{CWS} \left[0, -\frac{\overline{xy}}{48}, 0, \dots \right] \right]$$

```
In[*]:= V12 = V // dσ[{x, y} → {1, 2}];
ϕV = (V12 // dA)σ[12,3] ** (V12 // dA)σ[1,2] ** V12σ[2,3] ** V12σ[1,23]
```

Out[*]=

$$\text{Es} \left[\left\langle 1 \rightarrow \text{LS} \left[0, \frac{\sqrt{23}}{24}, 0, \dots \right], 2 \rightarrow \text{LS} \left[0, -\frac{\sqrt{13}}{24}, 0, \dots \right], 3 \rightarrow \text{LS} \left[0, \frac{\sqrt{12}}{24}, 0, \dots \right] \right\rangle, \right. \\ \left. \text{CWS} [0, 0, 0, \dots] \right]$$

```
In[*]:= (ϕV ** dA[ϕV]) @ {4}
```

Out[*]=

$$\text{Es} \left[\left\langle 1 \rightarrow \text{LS} [0, 0, 0, 0, \dots], 2 \rightarrow \text{LS} [0, 0, 0, 0, \dots], 3 \rightarrow \text{LS} [0, 0, 0, 0, \dots] \right\rangle, \right. \\ \left. \text{CWS} [0, 0, 0, 0, \dots] \right]$$

```
In[*]:= Es[⟨1 → LS[0], 2 → LS[0], 3 → LS[0]⟩, CWS[0]]
```

Out[*]=

$$\text{Es} \left[\left\langle 1 \rightarrow \text{LS} [0, 0, 0, \dots], 2 \rightarrow \text{LS} [0, 0, 0, \dots], 3 \rightarrow \text{LS} [0, 0, 0, \dots] \right\rangle, \text{CWS} [0, 0, 0, \dots] \right]$$

```
In[*]:= UnitarityOfPhiV = (ϕV ** dA[ϕV]) ≡ Es[⟨1 → LS[0], 2 → LS[0], 3 → LS[0]⟩, CWS[0]]
```

Out[*]=

$$\text{BS} [4 \text{ True}, \dots]$$

```
In[*]:= UnitarityOfPhiV @ {7}
```

SeriesSolve: In degree 5 arbitrarily setting {as[x, x, x, y] → 0}.

SeriesSolve: In degree 7 arbitrarily setting {as[x, x, x, x, y] → 0}.

Out[*]=

$$\text{BS} [8 \text{ True}, \dots]$$