

Pensieve header: Solving R4, Unitarity, Cap, and Vertical Flip. Data file: <R4UniCapVFlipSolution-to-12.m>.

```

SetDirectory["~drorbn/AcademicPensieve/Projects/WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn = (V ** (V // dA[x] // dA[y]) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
  (Cap (Cap // dσ[x, y]) // dc[x] // dc[y]));
FlipEqn = V ** (V // dS) ≡ Rs[x, y];
βs[x] = 1/2; βs[y] = 0;
SeriesSolve[{α, β, γ, κ}, (ħ-1 R4Eqn) && UnitarityEqn && CapEqn && FlipEqn];
{V, κ}

FreeLie` implements / extends
{*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, EulerE, Exp,
  Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
  Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ℓ, Λ, σ, ħ, ←, ↪}.

FreeLie` is in the public domain; yet Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg,
  dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.

AwCalculus` is in the public domain; yet Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.

Arbitrarily setting {κs[x] → 0}.
{Es[⟨x → LS[0, - $\frac{\overline{xy}}{24}$ , 0, ...], y → LS[ $\frac{\overline{x}}{2}$ , - $\frac{\overline{xy}}{12}$ , 0, ...]⟩, CWS[0, - $\frac{\overline{xy}}{48}$ , 0, ...]],
  CWS[0, - $\frac{\overline{xx}}{96}$ , 0, ...]}

```

