Pensieve header: Testing if KV is equivalent to R4 + Unitarity of Φ_V.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
V12 = V // dσ[{x, y} → {1, 2}];
ΦV = (V12 // dA)^σ[12,3] ** (V12 // dA)^σ[1,2] ** V12^σ[2,3] ** V12^σ[1,23];
UnitarityOfPhi = (ΦV ** dA[ΦV]) ≡ Es[⟨1 → LS[0], 2 → LS[0], 3 → LS[0]⟩, CWS[0]];
βs[x] = 1 / 2; βs[y] = 0;
SeriesSolve[{α, β, γ},
  (ħ^-1 R4Eqn) && UnitarityOfPhi && (ΦV // dc[1] // dc[2] // dc[3])[[2]] ≡ CWS[0]];
V
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⁀, ⁀}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA,
  hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150909.
```

SeriesSolve: In degree 3 arbitrarily setting {αs[x, y] → 0}.

Out[•]=

$$\text{Es}\left[\left\langle \text{x} \to \text{LS}\left[0, -\frac{\overline{\text{x y}}}{24}, 0, \ldots\right], \text{y} \to \text{LS}\left[\frac{\overline{\text{x}}}{2}, -\frac{\overline{\text{x y}}}{12}, 0, \ldots\right]\right\rangle, \text{CWS}\left[0, -\frac{\widehat{\text{xy}}}{48}, 0, \ldots\right]\right]$$

In[•]:=
```
V@{5} // Timing
```

SeriesSolve: In degree 5 arbitrarily setting {αs[x, x, x, y, y] → 0}.

Out[•]=

$$\left\{5.15625,\ \text{Es}\left[\left\langle x \to \text{LS}\left[0,\ -\frac{\overline{xy}}{24},\ 0,\ \frac{7\,\overline{x\,\overline{x\,\overline{xy}}}}{5760} - \frac{7\,\overline{x\,\overline{\overline{xy}\,y}}}{5760} + \frac{\overline{\overline{\overline{xy}\,y}\,y}}{1440},\ 0,\ \ldots\right],\ y \to \right.\right.$$

$$\left.\left.\text{LS}\left[\frac{\overline{x}}{2},\ -\frac{\overline{xy}}{12},\ 0,\ \frac{\overline{x\,\overline{x\,\overline{xy}}}}{5760} - \frac{1}{720}\,\overline{x\,\overline{\overline{xy}\,y}} + \frac{1}{720}\,\overline{\overline{\overline{xy}\,y}\,y},\ -\frac{\overline{xx\,\overline{x\,\overline{xy}}}}{7680} + \frac{\overline{xx\,\overline{\overline{xy}\,y}}}{3840} - \frac{\overline{\overline{x\,\overline{xy}}\,\overline{xy}}}{6912},\ \ldots\right]\right\rangle,\right.$$

$$\left.\text{CWS}\left[0,\ -\frac{\widehat{xy}}{48},\ 0,\ \frac{\widehat{xxxy}}{2880} + \frac{\widehat{xxyy}}{2880} + \frac{\widehat{xyxy}}{5760} + \frac{\widehat{xyyy}}{2880},\ 0,\ \ldots\right]\right]\right\}$$

In[•]:= **V@{7} // Timing**

SeriesSolve: In degree 7 arbitrarily setting {as[x, x, x, x, x, y, y] → 0}.

Out[•]=

$$\Big\{151.453,$$

$$\text{Es}\Big[\Big\langle x\to \text{LS}\Big[0,-\frac{\overline{xy}}{24},0,\frac{7\,x\,\overline{x\,\overline{xy}}}{5760}-\frac{7\,x\,\overline{\overline{xy}\,y}}{5760}+\frac{\overline{\overline{xy}\,y\,y}}{1440},0,-\frac{31\,x\,x\,x\,\overline{x\,\overline{xy}}}{967\,680}+\frac{31\,x\,x\,x\,\overline{\overline{xy}\,y}}{483\,840}-$$

$$\frac{83\,x\,x\,\overline{\overline{xy}\,y\,y}}{967\,680}-\frac{31\,x\,\overline{xy}\,\overline{\overline{xy}\,y}}{725\,760}-\frac{31\,x\,\overline{x\,\overline{xy}}\,\overline{xy}}{645\,120}+\frac{13\,x\,\overline{\overline{xy}\,y\,y\,y}}{241\,920}+\frac{101\,\overline{xy}\,\overline{\overline{xy}\,y\,y}}{1\,451\,520}+$$

$$\frac{527\,x\,\overline{\overline{xy}\,y}\,\overline{xy}}{5\,806\,080}-\frac{\overline{\overline{xy}\,y\,y\,y}}{60\,480},0,\ldots\Big],$$

$$y\to \text{LS}\Big[\frac{\overline{x}}{2},-\frac{\overline{xy}}{12},0,\frac{x\,\overline{x\,\overline{xy}}}{5760}-\frac{1}{720}\,x\,\overline{\overline{xy}\,y}+\frac{1}{720}\,\overline{\overline{xy}\,y\,y},-\frac{x\,x\,\overline{x\,\overline{xy}}}{7680}+\frac{x\,x\,\overline{\overline{xy}\,y}}{3840}-\frac{\overline{x\,\overline{xy}}\,\overline{xy}}{6912},$$

$$-\frac{x\,x\,x\,\overline{x\,\overline{xy}}}{645\,120}+\frac{23\,x\,x\,x\,\overline{\overline{xy}\,y}}{483\,840}-\frac{13\,x\,x\,\overline{\overline{xy}\,y\,y}}{161\,280}-\frac{x\,\overline{xy}\,\overline{\overline{xy}\,y}}{22\,680}-$$

$$\frac{41\,x\,\overline{x\,\overline{xy}}\,\overline{xy}}{580\,608}+\frac{x\,\overline{\overline{xy}\,y\,y\,y}}{15\,120}+\frac{\overline{xy}\,\overline{\overline{xy}\,y\,y}}{12\,096}+\frac{71\,x\,\overline{\overline{xy}\,y}\,\overline{xy}}{483\,840}-\frac{\overline{\overline{xy}\,y\,y\,y}}{30\,240},$$

$$\frac{x\,x\,x\,x\,\overline{x\,\overline{xy}}}{258\,048}-\frac{5\,x\,x\,x\,x\,\overline{\overline{xy}\,y}}{387\,072}+\frac{x\,x\,x\,\overline{\overline{xy}\,y\,y}}{64\,512}+\frac{x\,x\,\overline{x\,\overline{xy}}\,\overline{\overline{xy}\,y}}{96\,768}+\frac{5\,x\,x\,\overline{x\,\overline{xy}}\,\overline{xy}}{290\,304}-\frac{x\,x\,\overline{\overline{xy}\,y\,y\,y}}{96\,768}-$$

$$\frac{17\,x\,\overline{xy}\,\overline{\overline{xy}\,y\,y}}{1\,451\,520}-\frac{x\,x\,\overline{\overline{xy}\,y}\,\overline{xy}}{60\,480}-\frac{x\,\overline{xy}\,\overline{x\,\overline{xy}\,y}}{207\,360}-\frac{7\,x\,\overline{x\,\overline{xy}}\,\overline{x\,\overline{xy}}}{1\,658\,880}+\frac{x\,\overline{\overline{xy}\,y\,y}\,\overline{xy}}{207\,360},\ldots\Big]\Big\rangle,$$

$$\text{CWS}\Big[0,-\frac{\widehat{xy}}{48},0,\frac{\widehat{xxxy}}{2880}+\frac{\widehat{xxyy}}{2880}+\frac{\widehat{xyxy}}{5760}+\frac{\widehat{xyyy}}{2880},0,-\frac{\widehat{xxxxxy}}{120\,960}-\frac{\widehat{xxxxyy}}{120\,960}-\frac{\widehat{xxxyxy}}{120\,960}-\frac{\widehat{xxxyyy}}{120\,960}-$$

$$\frac{\widehat{xxyxxy}}{241\,920}-\frac{\widehat{xxyxyy}}{120\,960}-\frac{\widehat{xxyyxy}}{120\,960}-\frac{\widehat{xxyyyy}}{120\,960}-\frac{\widehat{xyxyxy}}{362\,880}-\frac{\widehat{xyxyyy}}{120\,960}-\frac{\widehat{xyyxyy}}{241\,920}-\frac{\widehat{xyyyyy}}{120\,960},0,\ldots\Big]\Big]\Big\}$$

In[•]:= **V // Λ**

Out[•]=

$$\text{El}\Big[\Big\langle x\to \text{LS}\Big[0,-\frac{\overline{xy}}{24},\frac{1}{96}\,\overline{x\,\overline{xy}},\ldots\Big],y\to \text{LS}\Big[\frac{\overline{x}}{2},-\frac{\overline{xy}}{12},\frac{1}{96}\,\overline{x\,\overline{xy}},\ldots\Big]\Big\rangle,\text{CWS}\Big[0,-\frac{\widehat{xy}}{48},0,\ldots\Big]\Big]$$

In[•]:= **Φ_V[[2]]@{5}**

Out[•]=

$$\text{CWS}[0,0,0,0,0,\ldots]$$

In[•]:=
```
(ΦV // Λ)[[2]]@{7}
```
Out[•]=
```
CWS[0, 0, 0, 0, 0, 0, 0, ...]
```

In[•]:=
```
(ΦV[[1]] // div)@{5}
```
Out[•]=
$$\text{CWS}\left[0,\,0,\,0,\,\frac{\widehat{1213}}{576}+\frac{\widehat{1232}}{576}+\frac{\widehat{1323}}{576},\,0,\,\ldots\right]$$

In[•]:=
```
(ΦV[[1]] // j)@{5}
```
Out[•]=
$$\text{CWS}\left[0,\,0,\,0,\,\frac{\widehat{1213}}{576}+\frac{\widehat{1232}}{576}+\frac{\widehat{1323}}{576},\,0,\,\ldots\right]$$

In[•]:=
```
dA[ΦV][[2]]@{7}
```
SeriesSolve: In degree 7 arbitrarily setting

{αs[x, x, x, x, y, y] → 0, αs[x, x, x, x, y, y, y] → 0, αs[x, x, x, y, y, y, y] → 0, αs[x, x, y, x, y, y, y] → 0, αs[x, x, y, y, y, y, y] → 0, αs[x, y, y, y, y, y, y] → 0}.

Out[•]=
```
CWS[0, 0, 0, 0, 0, 0, 0, ...]
```

In[•]:=
```
R4Eqn@{6}
```
Out[•]=
```
BS[7 True, ...]
```

In[•]:=
```
V
```
Out[•]=
$$\text{Es}\left[\left\langle \text{x}\to\text{LS}\left[0,\,-\frac{\overline{\text{x y}}}{24},\,0,\,\ldots\right],\,\text{y}\to\text{LS}\left[\frac{\overline{\text{x}}}{2},\,-\frac{\overline{\text{x y}}}{12},\,0,\,\ldots\right]\right\rangle,\,\text{CWS}\left[0,\,-\frac{\widehat{\text{xy}}}{48},\,0,\,\ldots\right]\right]$$

In[•]:=
```
UnitarityEqn = V ** (V // dA) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]
```
Out[•]=
```
BS[4 True, ...]
```

In[•]:=
```
UnitarityEqn@{6}
```
Out[•]=
```
BS[7 True, ...]
```

In[•]:=
```
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
CapEqn =
  (V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡ (Cap * (Cap // dσ[x, y]) // dc[x] // dc[y]);
SeriesSolve[{κ}, CapEqn];
Cap@{7}
```
SeriesSolve: In degree 1 arbitrarily setting {κs[x] → 0}.

SeriesSolve: No solution in degree 7.

Out[•]=
```
$Aborted
```

In[•]:=
```
Φ_V // dc[1] // dc[2] // dc[3]
```

Out[•]=
```
Es[⟨⟩, CWS[0, 0, 0, ...]]
```