Pensieve header: Testing if KV is equivalent to R4 + Unitarity of Φ_V.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
V12 = V // dσ[{x, y} → {1, 2}];
ΦV = (V12 // dA)^σ[12,3] ** (V12 // dA)^σ[1,2] ** V12^σ[2,3] ** V12^σ[1,23];
UnitarityOfPhi = (ΦV ** dA[ΦV]) ≡ Es[⟨1 → LS[0], 2 → LS[0], 3 → LS[0]⟩, CWS[0]];
βs[x] = 1 / 2; βs[y] = 0;
SeriesSolve[{α, β, γ}, (ħ^-1 R4Eqn) && UnitarityOfPhi];
V
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⟞, ⟟}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA,
  hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150909.

SeriesSolve: In degree 1 arbitrarily setting {αs[x] → 0}.

SeriesSolve: In degree 3 arbitrarily setting {αs[x, y, y] → 0}.
```

Out[•]=

$$\mathrm{Es}\left[\left\langle x \to \mathrm{LS}\left[0, -\frac{\overline{xy}}{24}, 0, \ldots\right], y \to \mathrm{LS}\left[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \ldots\right]\right\rangle, \mathrm{CWS}\left[0, -\frac{\widehat{xy}}{48}, 0, \ldots\right]\right]$$

In[•]:=
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V@{7} // Timing
```

SeriesSolve: In degree 5 arbitrarily setting {αs[x, x, x, y, y] → 0, αs[x, x, y, y, y] → 0, αs[x, y, y, y, y] → 0}.

SeriesSolve: In degree 7 arbitrarily setting {αs[x, x, x, x, x, y, y] → 0, αs[x, x, x, x, y, y, y] → 0, αs[x, x, x, y, y, y, y] → 0, αs[x, x, y, x, y, y, y] → 0, αs[x, x, y, y, y, y, y] → 0, αs[x, y, y, y, y, y, y] → 0}.

Out[•]=

$$\Big\{146.141,$$

$$\mathrm{Es}\Big[\Big\langle x \to \mathrm{LS}\Big[0, -\frac{\overline{xy}}{24}, 0, \frac{7\,x\,\overline{x\,\overline{xy}}}{5760} - \frac{7\,x\,\overline{\overline{xy}\,y}}{5760} + \frac{\overline{\overline{xy}\,y\,y}}{1440}, 0, -\frac{31\,x\,x\,x\,\overline{x\,\overline{xy}}}{967\,680} + \frac{31\,x\,x\,x\,\overline{\overline{xy}\,y}}{483\,840} -$$
$$\frac{83\,x\,x\,\overline{x\,\overline{\overline{xy}\,y}\,y}}{967\,680} - \frac{31\,x\,\overline{x\,\overline{xy}}\,\overline{\overline{xy}\,y}}{725\,760} - \frac{31\,x\,x\,\overline{\overline{xy}\,\overline{xy}}}{645\,120} + \frac{13\,x\,\overline{\overline{xy}\,y\,y}\,y}{241\,920} + \frac{101\,\overline{xy}\,\overline{\overline{xy}\,y\,y}}{1\,451\,520} +$$
$$\frac{527\,x\,\overline{x\,\overline{xy}\,y}\,\overline{xy}}{5\,806\,080} - \frac{\overline{\overline{xy}\,y\,y\,y}}{60\,480}, 0, \ldots\Big],$$

$$y \to \mathrm{LS}\Big[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \frac{x\,\overline{x\,\overline{xy}}}{5760} - \frac{1}{720}\,x\,\overline{\overline{xy}\,y} + \frac{1}{720}\,\overline{\overline{xy}\,y\,y}, -\frac{x\,x\,\overline{x\,\overline{xy}}}{7680} + \frac{x\,x\,\overline{\overline{xy}\,y}}{3840} - \frac{\overline{x\,\overline{xy}}\,\overline{xy}}{6912},$$
$$-\frac{x\,x\,x\,\overline{x\,\overline{xy}}}{645\,120} + \frac{23\,x\,x\,x\,\overline{\overline{xy}\,y}}{483\,840} - \frac{13\,x\,x\,\overline{\overline{xy}\,y\,y}}{161\,280} - \frac{x\,\overline{xy}\,\overline{\overline{xy}\,y}}{22\,680} -$$
$$\frac{41\,x\,\overline{x\,\overline{xy}}\,\overline{xy}}{580\,608} + \frac{x\,\overline{\overline{xy}\,y\,y}\,y}{15\,120} + \frac{\overline{xy}\,\overline{\overline{xy}\,y\,y}}{12\,096} + \frac{71\,x\,\overline{\overline{xy}\,y}\,\overline{xy}}{483\,840} - \frac{\overline{\overline{xy}\,y\,y\,y}}{30\,240},$$
$$\frac{x\,x\,x\,x\,\overline{x\,\overline{xy}}}{258\,048} - \frac{5\,x\,x\,x\,x\,\overline{\overline{xy}\,y}}{387\,072} + \frac{x\,x\,x\,\overline{x\,\overline{\overline{xy}\,y}\,y}}{64\,512} + \frac{x\,x\,\overline{x\,\overline{xy}}\,\overline{\overline{xy}\,y}}{96\,768} + \frac{5\,x\,x\,\overline{x\,\overline{xy}}\,\overline{xy}}{290\,304} - \frac{x\,x\,\overline{x\,\overline{\overline{xy}\,y}\,y}\,y}{96\,768} -$$
$$\frac{17\,x\,\overline{xy}\,\overline{x\,\overline{\overline{xy}\,y}\,y}\,y}{1\,451\,520} - \frac{x\,x\,\overline{\overline{xy}\,y\,y}\,\overline{xy}}{60\,480} - \frac{x\,\overline{xy}\,x\,\overline{\overline{xy}\,y}}{207\,360} - \frac{7\,x\,x\,\overline{xy}\,x\,\overline{xy}}{1\,658\,880} + \frac{x\,\overline{\overline{xy}\,y\,y}\,\overline{xy}}{207\,360}, \ldots\Big]\Big\rangle,$$
$$\mathrm{CWS}\Big[0, -\frac{\widehat{xy}}{48}, 0, \frac{\widehat{xxxy}}{2880} + \frac{\widehat{xxyy}}{2880} + \frac{\widehat{xyxy}}{5760} + \frac{\widehat{xyyy}}{2880}, 0, -\frac{\widehat{xxxxxy}}{120\,960} - \frac{\widehat{xxxxyy}}{120\,960} - \frac{\widehat{xxxyxy}}{120\,960} - \frac{\widehat{xxxyyy}}{120\,960} -$$
$$\frac{\widehat{xxyxxy}}{241\,920} - \frac{\widehat{xxyxyy}}{120\,960} - \frac{\widehat{xxyyxy}}{120\,960} - \frac{\widehat{xxyyyy}}{120\,960} - \frac{\widehat{xyxyxy}}{362\,880} - \frac{\widehat{xyxyyy}}{120\,960} - \frac{\widehat{xyyxyy}}{241\,920} - \frac{\widehat{xyyyyy}}{120\,960}, 0, \ldots\Big]\Big]\Big\}$$

In[•]:= Φ_V

Out[•]=

$$\text{Es}\left[\left\langle 1 \to \text{LS}\left[0, \frac{\overline{23}}{24}, 0, \ldots\right], 2 \to \text{LS}\left[0, -\frac{\overline{13}}{24}, 0, \ldots\right], 3 \to \text{LS}\left[0, \frac{\overline{12}}{24}, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, 0, \ldots]\right]$$