Pensieve header: Calculations following Albert-Harinck-Torossian's arXiv:0802.2049.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
$SeriesShowDegree = 4;
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, EulerE, Exp,
  Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
  Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ι, Λ, σ, ħ, ⌐, ⌢}.
FreeLie` is in the public domain; yet Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.
AwCalculus` implements / extends {*, **, ≡, dA, dc, deg,
  dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.
AwCalculus` is in the public domain; yet Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.
```

```
{f = LS[{x, y}, fs], g = f // LieMorphism[LW@x → -LW@y, LW@y → -LW@x]};
msgs = SeriesSolve[{f},
  ħ^-1 (LS[LW@x + LW@y] - BCH[LW@y, LW@x] ≡ f - g - Ad[-LW@x][f] + Ad[LW@y][g])]
```

```
FreeLie`Private`MessageStream$340
```

```
Read[msgs]
```

```
{}
```

```
f@{6}
```

SeriesSolve::ArbitrarilySetting : In degree 1 arbitrarily setting {fs[x] → 0}.

SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {fs[x, x, x, y, y] → 0}.

$$\mathrm{LS}\Big[\frac{\overline{y}}{4},\ \frac{\overline{xy}}{24},\ -\frac{1}{48}\overline{x\overline{xy}}+\frac{1}{48}\overline{\overline{xy}y},\ -\frac{1}{180}\overline{x\overline{x\overline{xy}}}+\frac{1}{480}\overline{x\overline{\overline{xy}y}}+\frac{1}{360}\overline{\overline{\overline{xy}y}y},$$
$$\frac{\overline{x\overline{x\overline{xy}}}}{2880}+\frac{1}{360}\overline{x\overline{\overline{xy}y}y}+\frac{1}{480}\overline{\overline{xy}\,\overline{xy}y}-\frac{7\overline{x\overline{xy}\,\overline{xy}}}{1440}-\frac{1}{320}\overline{\overline{\overline{xy}y}yy},$$
$$\frac{\overline{x\overline{x\overline{x\overline{xy}}}}}{5040}+\frac{11\overline{x\overline{x\overline{x\overline{xy}y}}}}{13\,440}+\frac{\overline{x\overline{x\overline{xy}y}y}}{6048}+\frac{\overline{x\overline{xy}\,\overline{xy}y}}{6720}-\frac{19\overline{x\overline{x\overline{xy}}\,\overline{xy}}}{6720}-$$
$$\frac{43\overline{x\overline{\overline{xy}y}yy}}{40\,320}+\frac{\overline{xy}\,\overline{\overline{xy}y}y}{1260}+\frac{\overline{x\overline{xy}y\overline{xy}}}{1680}-\frac{\overline{\overline{xy}yyy}}{10\,080},\ \ldots\Big]$$

```
Read[msgs]
```

```
{{ArbitrarilySetting, 1, {Hold[fs[x]] → 0}}, {ArbitrarilySetting, 2, {}},
 {ArbitrarilySetting, 3, {}}, {ArbitrarilySetting, 4, {}},
 {ArbitrarilySetting, 5, {Hold[fs[x, x, x, y, y]] → 0}}, {ArbitrarilySetting, 6, {}}}
```

```
treqn = (
  divx[f] + divy[g] ≡ 1/2 tru[adSeries[ad/(e^ad - 1), LW@x][LW@u] +
      adSeries[ad/(e^ad - 1), LW@y][LW@u] - adSeries[ad/(e^ad - 1), BCH[LW@x, LW@y]][LW@u]])
BS[5 True, ...]
```

```
treqn@{6}
```

$$\mathrm{BS}\Big[5\ \mathrm{True},\ \frac{\overline{xxxxy}}{360}+\frac{\overline{xxxyy}}{120}-\frac{\overline{xxyxy}}{180}-\frac{\overline{xxyyy}}{120}+\frac{\overline{xyxyy}}{180}-\frac{\overline{xyyyy}}{360}==0,\ \frac{\overline{xxxxy}}{360}+\frac{\overline{xxxyy}}{120}-\frac{\overline{xxyxy}}{180}-\frac{\overline{xxyyy}}{120}+\frac{\overline{xyxyy}}{180}-\frac{\overline{xyyyy}}{360}==0,\ \ldots\Big]$$

```
{F = LS[{x, y}, Fs], G = F // LieMorphism[LW@x → -LW@y, LW@y → -LW@x]};
SeriesSolve[{F},
 ħ^-1 (LS[LW@x + LW@y] - BCH[LW@y, LW@x] ≡ F - G - Ad[-LW@x][F] + Ad[LW@y][G]) &&
  divx[F] + divy[G] ≡ 1/2 tru[adSeries[ad/(e^ad - 1), LW@x][LW@u] +
      adSeries[ad/(e^ad - 1), LW@y][LW@u] - adSeries[ad/(e^ad - 1), BCH[LW@x, LW@y]][LW@u]]]
```

$$\Big\{\mathrm{LS}\Big[\frac{\overline{y}}{4},\ \frac{\overline{xy}}{24},\ -\frac{1}{48}\overline{x\overline{xy}}+\frac{1}{48}\overline{\overline{xy}y},\ -\frac{1}{180}\overline{x\overline{x\overline{xy}}}+\frac{1}{480}\overline{x\overline{\overline{xy}y}}+\frac{1}{360}\overline{\overline{\overline{xy}y}y},\ \ldots\Big]\Big\}$$

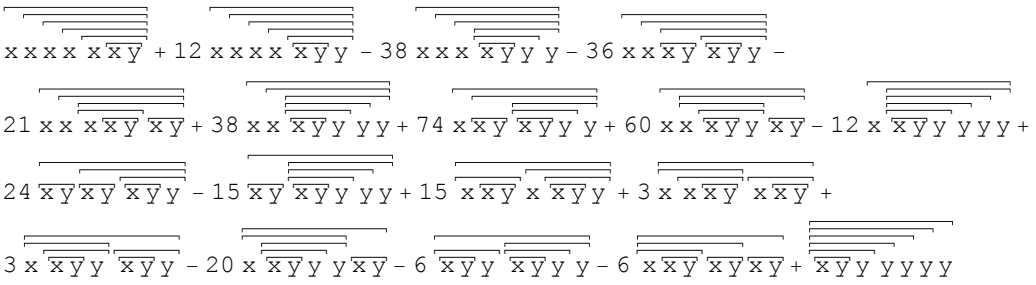
```
F@{6}
```

$$\mathrm{LS}\Big[\frac{\overline{y}}{4},\ \frac{\overline{xy}}{24},\ -\frac{1}{48}\overline{x\overline{xy}}+\frac{1}{48}\overline{\overline{xy}y},\ -\frac{1}{180}\overline{x\overline{x\overline{xy}}}+\frac{1}{480}\overline{x\overline{\overline{xy}y}}+\frac{1}{360}\overline{\overline{\overline{xy}y}y},$$
$$\frac{\overline{x\overline{x\overline{x\overline{xy}}}}}{2880}-\frac{1}{360}\overline{x\overline{x\overline{\overline{xy}y}}}+\frac{1}{360}\overline{x\overline{\overline{\overline{xy}y}y}}+\frac{1}{480}\overline{\overline{xy}\,\overline{\overline{xy}y}}+\frac{\overline{\overline{x\overline{xy}}\,\overline{xy}}}{1440}-\frac{\overline{\overline{\overline{\overline{xy}y}y}y}}{2880},$$
$$\frac{\overline{x\overline{x\overline{x\overline{\overline{xy}}}}}}{5040}-\frac{23\,\overline{x\overline{x\overline{x\overline{\overline{xy}}}}}}{40\,320}+\frac{\overline{x\overline{x\overline{\overline{\overline{xy}y}y}}}}{6048}+\frac{\overline{x\overline{\overline{xy}\,\overline{\overline{xy}y}}}}{6720}-$$
$$\frac{\overline{x\overline{x\overline{\overline{xy}}}\,\overline{\overline{xy}}}}{20\,160}+\frac{13\,\overline{x\overline{\overline{\overline{\overline{xy}y}y}y}}}{40\,320}+\frac{\overline{\overline{xy}\,\overline{\overline{\overline{xy}y}y}}}{1260}+\frac{\overline{x\overline{\overline{xy}y}\,\overline{xy}}}{1680}-\frac{\overline{\overline{\overline{\overline{xy}y}y}yy}}{10\,080},\ \ldots\Big]$$

```
2880 F@5 // Expand // TopBracketForm
```

$$\overline{x\overline{x\overline{x\overline{xy}}}}-8\,\overline{x\overline{x\overline{\overline{xy}y}}}+8\,\overline{x\overline{\overline{\overline{xy}y}y}}+6\,\overline{\overline{xy}\,\overline{\overline{xy}y}}+2\,\overline{\overline{x\overline{xy}}\,\overline{xy}}-\overline{\overline{\overline{\overline{xy}y}y}y}$$

```
120 960 F@7 // Expand // TopBracketForm
```



```
{A = LS[{x, y}, As], B = LS[{x, y}, Bs]};
SeriesSolve[{A, B},
 ħ^-1 (b[LW@x, A] + b[LW@y, B] ≡ LS[0]) && (div_x[A] + div_y[B] ≡ CWS[0])]
```

```
Arbitrarily setting {As[y] → 0}.
{LS[0, 0, 0, 0, ...], LS[0, 0, 0, 0, ...]}
```

```
A@{10}
```

```
Arbitrarily setting {As[x, x, x, x, y, x, y, y] → 0}.
Arbitrarily setting {As[x, x, x, x, x, x, y, x, y, y] → 0}.
LS[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...]
```

```
A@{16}
```

```
Arbitrarily setting {As[x, x, x, x, x, x, y, x, y, y, y] → 0}.
Arbitrarily setting
 {As[x, x, x, x, x, x, x, x, y, x, y, y] → 0, As[x, x, x, x, x, x, y, x, y, y, y, y] → 0}.
Arbitrarily setting
 {As[x, x, x, x, x, x, x, x, y, x, y, y, y] → 0, As[x, x, x, x, x, x, x, y, x, x, y, y] → 0}.
Arbitrarily setting
 {As[x, x, x, x, x, x, x, x, x, x, y, x, y, y] → 0, As[x, x, x, x, x, x, x, x, y, x, x, x, y, y] → 0,
  As[x, x, x, x, x, x, x, x, y, x, y, y, y, y] → 0}.
Arbitrarily setting {As[x, x, x, x, x, x, x, x, x, y, x, y, y, y] → 0,
  As[x, x, x, x, x, x, x, x, x, y, x, x, y, y, y] → 0,
  As[x, x, x, x, x, x, x, x, x, y, x, y, y, y, y, y] → 0}.
Arbitrarily setting {As[x, x, x, x, x, x, x, x, x, x, x, x, y, x, y, y] → 0,
  As[x, x, x, x, x, x, x, x, x, x, y, x, x, x, y, y] → 0,
  As[x, x, x, x, x, x, x, x, x, x, y, x, y, y, y, y] → 0,
  As[x, x, x, x, x, x, x, x, x, y, x, x, x, y, y, y] → 0,
  As[x, x, x, x, x, x, x, x, x, y, x, x, x, y, y, y] → 0}.
LS[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...]
```

```
{TimeUsed[], MaxMemoryUsed[]}
{25 781.6, 20 589 734 056}
```