

Pensieve header: Fuller output of for the Borromean tangle. Even fuller: <Borromean-to-7.m>

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
```

```
<< FreeLie.m;
```

```
<< AwCalculus.m;
```

```
Rl[a_, b_] := El[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
```

```
iRl[a_, b_] := El[⟨a → LS[0], b → -LS[LW@a]⟩, CWS[0]];
```

```
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
```

```
iRs[a_, b_] := Es[⟨a → LS[0], b → -LS[LW@a]⟩, CWS[0]];
```

```
ξ = iRs[r, 6] Rs[2, 4] iRs[g, 9] Rs[5, 7] iRs[b, 3] Rs[8, 1];
```

```
Do[ξ = ξ // dm[r, k, r], {k, 1, 3}];
```

```
Do[ξ = ξ // dm[g, k, g], {k, 4, 6}];
```

```
Do[ξ = ξ // dm[b, k, b], {k, 7, 9}];
```

```
ξ
```

FreeLie` implements / extends

```
{*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↪}.
```

FreeLie` is in the public domain. Dror Bar-Natan is committed to

support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm,

hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed

to support it within reason until July 15, 2022. This is version 150909.

$$\text{Es}\left[\left\langle\begin{array}{l} b \rightarrow \text{LS}[0, \overline{gr}, \frac{1}{2} \overline{ggr} + \overline{brg} + \frac{1}{2} \overline{grr}, \dots], \\ g \rightarrow \text{LS}[0, -\overline{br}, \frac{1}{2} \overline{bbr} - \overline{bgr} - \overline{brg} + \frac{1}{2} \overline{brr}, \dots], \\ r \rightarrow \text{LS}[0, \overline{bg}, \frac{1}{2} \overline{bbg} + \overline{bgr} + \frac{1}{2} \overline{brr}, \dots] \end{array}\right\rangle, \text{CWS}[0, 0, 2 \overline{bgr}, \dots]\right]$$

ξ@{5} // Timing

{0.34375,

$$\begin{aligned}
 & \text{Es} \left[\left(b \rightarrow \text{LS} \left[0, \overline{gr}, \frac{1}{2} \overline{ggr} + \overline{brg} + \frac{1}{2} \overline{grr}, -\frac{1}{2} \overline{bbrg} + \frac{1}{6} \overline{gggr} + \frac{1}{4} \overline{grrr} - \frac{1}{2} \overline{bgbr} - \right. \right. \\
 & \quad \frac{1}{2} \overline{brgg} - \frac{1}{2} \overline{brrg} + \frac{1}{6} \overline{grrr}, \frac{1}{6} \overline{bbbrg} + \frac{1}{6} \overline{bbgbr} + \frac{1}{4} \overline{bbrgg} + \\
 & \quad \frac{1}{4} \overline{bbrrg} + \frac{1}{24} \overline{gggrr} + \frac{1}{12} \overline{ggrrr} + \frac{1}{12} \overline{grrrr} + \frac{1}{2} \overline{bgbrg} + \frac{1}{4} \overline{bgbrr} - \\
 & \quad \frac{3}{2} \overline{brggr} - \frac{1}{2} \overline{brbrg} - \frac{1}{6} \overline{bbrbg} - \overline{bgrgr} - 2 \overline{bgrrg} - \frac{1}{12} \overline{ggrgr} + \\
 & \quad \left. \frac{1}{4} \overline{bggbr} - \frac{9}{2} \overline{brgrr} + \frac{1}{6} \overline{brggg} - \frac{3}{4} \overline{brrgg} + \frac{1}{6} \overline{brrrg} + \frac{1}{24} \overline{grrrr}, \dots \right], \\
 & g \rightarrow \text{LS} \left[0, -\overline{br}, \frac{1}{2} \overline{bbr} - \overline{bgr} - \overline{brg} + \frac{1}{2} \overline{brr}, -\frac{1}{6} \overline{bbbr} - \frac{1}{2} \overline{bbgr} - \frac{1}{2} \overline{bggr} - \right. \\
 & \quad \frac{1}{2} \overline{bbrg} - \frac{1}{4} \overline{brrr} + \frac{1}{2} \overline{bgrr} + \frac{1}{2} \overline{bgbr} + \overline{brgr} - \overline{bgrg} - \frac{1}{2} \overline{brgg} + \frac{1}{2} \overline{brrg} - \\
 & \quad \frac{1}{6} \overline{brrr}, \frac{1}{24} \overline{bbbb} - \frac{1}{6} \overline{bbbrg} - \frac{1}{4} \overline{bbggr} - \frac{1}{6} \overline{bbbrg} + \frac{1}{12} \overline{bbbrr} - \\
 & \quad \frac{3}{4} \overline{bbgrr} - \frac{1}{6} \overline{bgggr} + \frac{1}{4} \overline{bgrrr} + \frac{1}{3} \overline{bbgbr} + \frac{1}{2} \overline{bbrgr} - \frac{1}{2} \overline{bbgrg} - \\
 & \quad \frac{1}{4} \overline{bbrgg} + \frac{5}{4} \overline{bbrrg} + \frac{1}{12} \overline{brrrr} - \frac{1}{6} \overline{bgrrr} - \frac{1}{2} \overline{bgbgr} - \frac{1}{2} \overline{bgbrg} - \\
 & \quad \frac{1}{4} \overline{bgbrr} + \frac{1}{2} \overline{brggr} + 2 \overline{brbrg} - \frac{1}{2} \overline{brgrr} + \frac{1}{6} \overline{bbrbg} - \frac{1}{12} \overline{bbrbr} - \\
 & \quad \overline{bgrbr} + \frac{1}{2} \overline{bgrgr} - \frac{1}{2} \overline{bggrg} + \frac{1}{2} \overline{bgrrg} + \frac{1}{4} \overline{bggbr} - \frac{1}{2} \overline{brrgr} + \\
 & \quad \left. \overline{brgrr} - \frac{1}{2} \overline{brggg} - \frac{1}{6} \overline{brggg} + \frac{1}{4} \overline{brrgg} - \frac{1}{6} \overline{brrrg} + \frac{1}{24} \overline{brrrr}, \dots \right], \\
 & r \rightarrow \text{LS} \left[0, \overline{bg}, \frac{1}{2} \overline{bbg} + \overline{bgr} + \frac{1}{2} \overline{bgg}, \frac{1}{6} \overline{bbbg} + \frac{1}{2} \overline{bbgr} + \frac{1}{2} \overline{bggr} + \frac{1}{4} \overline{bbgg} + \right. \\
 & \quad \frac{1}{2} \overline{bgrr} + \frac{1}{6} \overline{bggg}, \frac{1}{24} \overline{bbbb} + \frac{1}{6} \overline{bbbrg} + \frac{1}{4} \overline{bbggr} + \frac{1}{12} \overline{bbbrr} + \\
 & \quad \frac{1}{4} \overline{bbgrr} + \frac{1}{6} \overline{bgggr} + \frac{1}{4} \overline{bgrrr} - \overline{bbgrg} + \frac{1}{12} \overline{bbggg} - 2 \overline{bbrgg} + \\
 & \quad \left. \frac{1}{6} \overline{bgrrr} + \frac{1}{2} \overline{bgbgr} - \overline{bgbrg} - \frac{1}{12} \overline{bbgbr} - \frac{1}{2} \overline{bgrgr} + \frac{1}{24} \overline{bgggg}, \dots \right], \\
 & \text{CWS} \left[0, 0, 2 \overline{bgr}, \overline{bbgr} - \overline{bgr} + \overline{bggr} - \overline{bgrg} + \overline{bgrr} - \overline{brgr}, \right. \\
 & \quad \frac{\overline{bbgr} - \overline{bbgr}}{3} + \frac{\overline{bbgr}}{2} + \frac{\overline{bbgr}}{2} + \frac{\overline{bbgr}}{2} + \frac{\overline{bbgr}}{2} + \frac{\overline{bbgr}}{2} - \\
 & \quad \frac{3 \overline{bbrgr}}{2} + \frac{\overline{bgrr}}{2} - \frac{3 \overline{bggr}}{2} + \frac{\overline{bggr}}{3} - \frac{\overline{bggr}}{2} + \frac{\overline{bgrr}}{2} + \\
 & \quad \left. \frac{\overline{bgrgg}}{2} - \frac{3 \overline{bgrrg}}{2} + \frac{\overline{bgrrr}}{3} + \frac{\overline{brgrr}}{2} - \frac{\overline{brgrr}}{2} + \frac{\overline{brrgr}}{2}, \dots \right]]]
 \end{aligned}$$

The same thing, copy-paste ready and machine readable:

```
Sum[{S[1]_r[k], S[1]_g[k], S[1]_b[k], S[2][k]}, {k, 5}] // InputForm
{LW[b, g] + LW[b, b, g]/2 + LW[b, g, g]/2 + LW[b, g, r] + LW[b, b, b, g]/6 + LW[b, b,
  LW[b, g, g, g]/6 + LW[b, g, g, r]/2 + LW[b, g, r, r]/2 + LW[b, b, b, b, g]/24 + LW
  LW[b, b, b, g, r]/6 - LW[b, b, g, b, g]/12 + LW[b, b, g, g, g]/12 + LW[b, b, g, g,
  LW[b, b, g, r, r]/4 - 2*LW[b, b, r, g, g] + LW[b, g, b, g, r]/2 - LW[b, g, b, r, g]
  LW[b, g, g, g, r]/6 + LW[b, g, g, r, r]/4 - LW[b, g, r, g, r]/2 + LW[b, g, r, r, r]
-LW[b, r] + LW[b, b, r]/2 - LW[b, g, r] - LW[b, r, g] + LW[b, r, r]/2 - LW[b, b, b,
  LW[b, b, r, g]/2 - LW[b, b, r, r]/4 + LW[b, g, b, r]/2 - LW[b, g, g, r]/2 - LW[b,
  LW[b, r, g, g]/2 + LW[b, r, g, r] + LW[b, r, r, g]/2 - LW[b, r, r, r]/6 + LW[b, b,
  LW[b, b, b, g, r]/6 - LW[b, b, b, r, g]/6 + LW[b, b, b, r, r]/12 + LW[b, b, g, b,
  LW[b, b, g, r, g]/2 - (3*LW[b, b, g, r, r])/4 + LW[b, b, r, b, g]/6 - LW[b, b, r,
  LW[b, b, r, g, g]/4 + LW[b, b, r, g, r]/2 + (5*LW[b, b, r, r, g])/4 + LW[b, b, r,
  LW[b, g, b, g, r]/2 - LW[b, g, b, r, g]/2 - LW[b, g, b, r, r]/4 + LW[b, g, g, b, r]
  LW[b, g, g, r, g]/2 + LW[b, g, g, r, r]/4 - LW[b, g, r, b, r] - LW[b, g, r, g, g]/
  LW[b, g, r, r, g]/2 - LW[b, g, r, r, r]/6 + 2*LW[b, r, b, r, g] - LW[b, r, g, g, g]
  LW[b, r, g, r, g] - LW[b, r, g, r, r]/2 + LW[b, r, r, g, g]/4 - LW[b, r, r, g, r]/
  LW[b, r, r, r, r]/24, LW[g, r] + LW[b, r, g] + LW[g, g, r]/2 + LW[g, r, r]/2 - LW
  LW[b, g, b, r]/2 - LW[b, r, g, g]/2 - LW[b, r, r, g]/2 + LW[g, g, g, r]/6 + LW[g,
  LW[b, b, b, r, g]/6 + LW[b, b, g, b, r]/6 - LW[b, b, r, b, g]/6 + LW[b, b, r, g, g]
  LW[b, g, b, r, g]/2 + LW[b, g, b, r, r]/4 + LW[b, g, g, b, r]/4 - LW[b, g, r, g, r]
  LW[b, r, b, r, g]/2 + LW[b, r, g, g, g]/6 - (3*LW[b, r, g, g, r])/2 - (9*LW[b, r,
  (3*LW[b, r, r, g, g])/4 + LW[b, r, r, r, g]/6 + LW[g, g, g, g, r]/24 + LW[g, g, g,
  LW[g, g, r, g, r]/12 + LW[g, g, r, r, r]/12 + LW[g, r, r, r, r]/24,
  2*CW[b, g, r] + CW[b, b, g, r] - CW[b, g, b, r] + CW[b, g, g, r] - CW[b, g, r, g] +
  CW[b, r, g, r] + CW[b, b, b, g, r]/3 - CW[b, b, g, b, r]/2 + CW[b, b, g, g, r]/2 +
  CW[b, b, g, r, r]/2 + CW[b, b, r, b, g]/2 - (3*CW[b, b, r, g, r])/2 + CW[b, g, b,
  (3*CW[b, g, g, b, r])/2 + CW[b, g, g, g, r]/3 - CW[b, g, g, r, g]/2 + CW[b, g, g,
  (3*CW[b, g, r, r, g])/2 + CW[b, g, r, r, r]/3 + CW[b, r, g, g, r]/2 - CW[b, r, g,
```

The same thing, machine readable and to degree 7, is in the file quoted below, in the same folder:

```
Put[Sum[{S[1]_r[k], S[1]_g[k], S[1]_b[k], S[2][k]}, {k, 7}], "Borromean-to-7.m"]
TimeUsed[]
6.814
```

The same thing, machine readable and to degree 8, is in the file quoted below, in the same folder:

```
Put[Sum[{S[1]_r[k], S[1]_g[k], S[1]_b[k], S[2][k]}, {k, 8}], "Borromean-to-8.m"]
$Aborted
TimeUsed[]
```