```
α = UndeterminedLieSeries[{"1", "2"}, αs];
β = UndeterminedLieSeries[{"1", "2"}, βs];
γ = UndeterminedCWSeries[{"1", "2"}, γs];
V = Es[⟨1 → α, 2 → β⟩, γ];
κ = UndeterminedCWSeries[{"1"}, κs];
SeriesSolve[
 R+[2, 3] ** R+[1, 3] ** V ≡ V ** (R+[1, 3] // dΔ[1, 1, 2])
    && V ** (V // dA[1] // dA[2]) ≡ dε[1] ⋃ dε[2]
    && V ** (κ // dΔ[1, 1, 2]) // dc[1] // dc[2] ≡ κ ⋃ (κ // dσ[1, 2]),
 {α, β, γ, κ},
 Arbitrator → 0 (* should be 0 or a pure function that takes a list of
  unsettled variables and returns a list of their arbitrated values *)
]

SetOptions[SeriesSolve, Arbitrator → 0
  (* should be 0 or a pure function that takes a list of unsettled
   variables and returns a list of their arbitrated values *)];

SeriesSolve[{
  α = LS[{"1", "2"}, αs], β = LS[{"1", "2"}, βs],
  γ = CWS[{"1", "2"}, γs], κ = CWS[{"1"}, κs]
 },
 V = Es[⟨1 → α, 2 → β⟩, γ];
 ħ^-1 (ζs[R+[2, 3] ** R+[1, 3]] ** V ≡ V ** (ζs[R+[1, 3]] // dΔ[1, 1, 2]))
    && V ** (V // dA[1] // dA[2]) ≡ Es[⟨1 → 0, 2 → 0⟩, 0]
    && V ** (κ // dΔ[1, 1, 2]) // dc[1] // dc[2] ≡ κ ⋃ (κ // dσ[1, 2])]
```

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
$SeriesShowDegree = 3;
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords,
  AllLyndonWords, AllWords, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS,
  CC, Crop, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, EulerE,
  Exp, InvertLieMorphism, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW,
  LyndonFactorization, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
  Support, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ι, Λ, ħ, ⏤, ⏜}.

AwCalculus` implements / extends
 {*, **, E, ≡, dA, deg, dm, dS, dΔ, dη, dσ, E1, Es, hA, hm, hS, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.
```

```
R⁺[a_, b_] // ζ_{t:(1|s)} := E_t[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
R⁻[a_, b_] // ζ_{t:(1|s)} := E_t[⟨a → LS[0], b → -LS[LW@a]⟩, CWS[0]];
ζ_{t:(1|s)}[K1_ ** K2_] := ζ_t[K1] ** ζ_t[K2];
ζ_{t:(1|s)}[K1_ K2_] := ζ_t[K1] ζ_t[K2];
```

```
SeriesSolve[{
  α = LS[{"1", "2"}, αs], β = LS[{"1", "2"}, βs],
  γ = CWS[{"1", "2"}, γs], κ = CWS[{"1"}, κs]
 },
 V = E_s[⟨1 → α, 2 → β⟩, γ]; Cap = Es[⟨1 → LS[0]⟩, κ];
 ħ^-1 (ζ_s[R⁺[2, 3] ** R⁺[1, 3]] ** V ≡ V ** (ζ_s[R⁺[1, 3]] // dΔ[1, 1, 2]))
  && V ** (V // dA[1] // dA[2]) ≡ E_s[⟨1 → LS[0], 2 → LS[0]⟩, CWS[0]]
  && (V ** (Cap // dΔ[1, 1, 2]) // dc[1] // dc[2]) ≡
   (Cap (Cap // dσ[1, 2]) // dc[1] // dc[2])
]
```

```
V@{5}
```

```
Arbitrator called on {αs[2], κs[1]}...
Arbitrator called on {αs[122]}...
Arbitrator called on {αs[11122]}...
```

$$E_s\left[\left\langle 1 \to LS\left[0, -\frac{\overline{12}}{24}, 0, \frac{7\,\overline{1\,\overline{1\,\overline{12}}}}{5760} - \frac{7\,\overline{1\,\overline{12\,2}}}{5760} + \frac{\overline{\overline{12}\,\overline{2\,2}}}{1440}, 0, \ldots\right], 2 \to LS\left[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, 0, \right.\right.$$
$$\left.\left.\frac{\overline{1\,\overline{1\,\overline{12}}}}{5760} - \frac{1}{720}\,\overline{1\,\overline{\overline{12}\,2}} + \frac{1}{720}\,\overline{\overline{\overline{12}\,2}\,2}, -\frac{\overline{1\,\overline{1\,\overline{1\,\overline{12}}}}}{7680} + \frac{\overline{1\,\overline{1\,\overline{\overline{12}\,2}}}}{3840} - \frac{\overline{\overline{1\,\overline{12}}\,\overline{12}}}{6912}, \ldots\right]\right\rangle,$$
$$CWS\left[0, -\frac{\widehat{12}}{48}, 0, \frac{\widehat{1112}}{2880} + \frac{\widehat{1122}}{2880} + \frac{\widehat{1212}}{5760} + \frac{\widehat{1222}}{2880}, 0, \ldots\right]\right]$$

```
κ@{8} // Timing
```

```
Arbitrator called on {αs[2], κs[1]}...
Arbitrator called on {αs[122]}...
Arbitrator called on {αs[11122]}...
Arbitrator called on {αs[1111122]}...
Arbitrator called on {αs[11112122]}...
```

$$\left\{14.086890, CWS\left[0, -\frac{\widehat{11}}{96}, 0, \frac{\widehat{1111}}{11\,520}, 0, -\frac{\widehat{111111}}{725\,760}, 0, \frac{\widehat{11111111}}{38\,707\,200}, \ldots\right]\right\}$$

```
V // Λ
```

```
Arbitrator called on {αs[2], κs[1]}...

Arbitrator called on {αs[122]}...
```

$$E_1\Big[\Big\langle 1 \to \mathrm{LS}\Big[0, -\frac{\overline{12}}{24}, \frac{1}{96}\,\overline{1\,\overline{12}}, \ldots\Big],\ 2 \to \mathrm{LS}\Big[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, \frac{1}{96}\,\overline{1\,\overline{12}}, \ldots\Big]\Big\rangle,\ \mathrm{CWS}\Big[0, -\frac{\widehat{12}}{48}, 0, \ldots\Big]\Big]$$

(∇ // Λ)@{6}

```
Arbitrator called on {αs[2], κs[1]}...

Arbitrator called on {αs[122]}...

Arbitrator called on {αs[11122]}...
```

$$E_1\Big[\Big\langle 1 \to \mathrm{LS}\Big[0, -\frac{\overline{12}}{24}, \frac{1}{96}\,\overline{1\,\overline{12}},$$
$$\frac{\overline{1\,1\,\overline{12}}}{2880} - \frac{1}{480}\,\overline{1\,\overline{12}\,2} + \frac{\overline{12\,2\,2}}{1440}, -\frac{\overline{7\,1\,1\,\overline{1\,12}}}{23\,040} + \frac{\overline{31\,1\,1\,\overline{12\,2}}}{69\,120} - \frac{\overline{1\,\overline{12\,2\,2}}}{5760} + \frac{\overline{1\,\overline{12}\,\overline{12}}}{13\,824},$$
$$-\frac{\overline{1\,1\,1\,1\,\overline{12}}}{322\,560} + \frac{\overline{19\,1\,1\,1\,\overline{12\,2}}}{241\,920} - \frac{\overline{361\,1\,1\,\overline{12\,2\,2}}}{2\,903\,040} - \frac{\overline{41\,1\,\overline{12}\,\overline{12\,2}}}{322\,560} - \frac{\overline{13\,1\,1\,\overline{12}\,\overline{12}}}{161\,280} +$$
$$\frac{\overline{1\,\overline{12\,2\,2\,2}}}{12\,096} + \frac{\overline{41\,\overline{12}\,\overline{12\,2\,2}}}{362\,880} + \frac{\overline{41\,1\,\overline{12\,2\,12}}}{483\,840} - \frac{\overline{12\,2\,222}}{60\,480}, \ldots\Big],$$
$$2 \to \mathrm{LS}\Big[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, \frac{1}{96}\,\overline{1\,\overline{12}}, \frac{1}{960}\,\overline{1\,1\,\overline{12}} - \frac{1}{320}\,\overline{1\,\overline{12\,2}} + \frac{1}{720}\,\overline{\overline{12\,2}\,2},$$
$$-\frac{\overline{7\,1\,1\,\overline{1\,12}}}{23\,040} + \frac{\overline{31\,1\,1\,\overline{12\,2}}}{69\,120} - \frac{\overline{1\,\overline{12\,2}\,2}}{5760} + \frac{\overline{1\,\overline{12}\,\overline{12}}}{13\,824},$$
$$-\frac{\overline{19\,1\,1\,1\,\overline{12}}}{967\,680} + \frac{\overline{29\,1\,1\,\overline{12\,2}}}{241\,920} - \frac{\overline{521\,1\,1\,\overline{12\,2\,2}}}{2\,903\,040} - \frac{\overline{41\,1\,\overline{12}\,\overline{12\,2}}}{241\,920} - \frac{\overline{31\,1\,1\,\overline{12}\,\overline{12}}}{290\,304} +$$
$$\frac{\overline{1\,\overline{12\,2\,2}\,2}}{8064} + \frac{\overline{41\,\overline{12}\,\overline{12\,2}\,2}}{241\,920} + \frac{\overline{121\,1\,\overline{12\,2}\,\overline{12}}}{967\,680} - \frac{\overline{12\,2\,222}}{30\,240}, \ldots\Big]\Big\rangle,$$
$$\mathrm{CWS}\Big[0, -\frac{\widehat{12}}{48}, 0, \frac{\widehat{1112}}{2880} + \frac{\widehat{1122}}{2880} + \frac{\widehat{1212}}{5760} + \frac{\widehat{1222}}{2880}, 0,$$
$$-\frac{\widehat{111112}}{120\,960} - \frac{\widehat{111122}}{120\,960} - \frac{\widehat{111212}}{120\,960} - \frac{\widehat{111222}}{120\,960} - \frac{\widehat{112112}}{241\,920} - \frac{\widehat{112122}}{120\,960} -$$
$$\frac{\widehat{112212}}{120\,960} - \frac{\widehat{112222}}{120\,960} - \frac{\widehat{121212}}{362\,880} - \frac{\widehat{121222}}{120\,960} - \frac{\widehat{122122}}{241\,920} - \frac{\widehat{122222}}{120\,960}, \ldots\Big]\Big]$$

```
{α = LS[{"1", "2"}, αs], β = LS[{"1", "2"}, βs],
 γ = CWS[{"1", "2"}, γs], κ = CWS[{"1"}, κs]}
```

$$\Big\{\text{LS}\Big[\overline{1}\,\alpha s[1]+\overline{2}\,\alpha s[2],\ \overline{1\,2}\,\alpha s[12],\ \overline{1\,\overline{1\,2}}\,\alpha s[112]+\overline{\overline{1\,2}\,2}\,\alpha s[122],\ \ldots\Big],$$
$$\text{LS}\Big[\overline{1}\,\beta s[1]+\overline{2}\,\beta s[2],\ \overline{1\,2}\,\beta s[12],\ \overline{1\,\overline{1\,2}}\,\beta s[112]+\overline{\overline{1\,2}\,2}\,\beta s[122],\ \ldots\Big],$$
$$\text{CWS}\Big[\widehat{1}\,\gamma s[1]+\widehat{2}\,\gamma s[2],\ \widehat{11}\,\gamma s[11]+\widehat{12}\,\gamma s[12]+\widehat{22}\,\gamma s[22],$$
$$\widehat{111}\,\gamma s[111]+\widehat{112}\,\gamma s[112]+\widehat{122}\,\gamma s[122]+\widehat{222}\,\gamma s[222],\ \ldots\Big],$$
$$\text{CWS}\Big[\widehat{1}\,\kappa s[1],\ \widehat{11}\,\kappa s[11],\ \widehat{111}\,\kappa s[111],\ \ldots\Big]\Big\}$$

```
V = Es[⟨1 → α, 2 → β⟩, γ]
```

$$\text{E}_s\Big[\Big\langle 1\to\text{LS}\Big[\overline{1}\,\alpha s[1]+\overline{2}\,\alpha s[2],\ \overline{1\,2}\,\alpha s[12],\ \overline{1\,\overline{1\,2}}\,\alpha s[112]+\overline{\overline{1\,2}\,2}\,\alpha s[122],\ \ldots\Big],$$
$$2\to\text{LS}\Big[\overline{1}\,\beta s[1]+\overline{2}\,\beta s[2],\ \overline{1\,2}\,\beta s[12],\ \overline{1\,\overline{1\,2}}\,\beta s[112]+\overline{\overline{1\,2}\,2}\,\beta s[122],\ \ldots\Big]\Big\rangle,$$
$$\text{CWS}\Big[\widehat{1}\,\gamma s[1]+\widehat{2}\,\gamma s[2],\ \widehat{11}\,\gamma s[11]+\widehat{12}\,\gamma s[12]+\widehat{22}\,\gamma s[22],$$
$$\widehat{111}\,\gamma s[111]+\widehat{112}\,\gamma s[112]+\widehat{122}\,\gamma s[122]+\widehat{222}\,\gamma s[222],\ \ldots\Big]\Big]$$

```
V ** (Es[⟨1 → 0⟩, κ] // dΔ[1, 1, 2]) // dc[1] // dc[2]
```

$$\text{E}_s\Big[\langle\rangle,\ \text{CWS}\Big[-\widehat{1}\,\alpha s[1]-\widehat{2}\,\beta s[2]+\widehat{1}\,\gamma s[1]+\widehat{2}\,\gamma s[2]+\widehat{1}\,\kappa s[1]+\widehat{2}\,\kappa s[1],$$
$$\widehat{12}\,\alpha s[12]+\frac{1}{2}\,\widehat{12}\,\alpha s[1]\,\alpha s[2]-\widehat{12}\,\alpha s[2]\,\beta s[1]-\widehat{12}\,\beta s[12]+\frac{1}{2}\,\widehat{12}\,\beta s[1]\,\beta s[2]+$$
$$\widehat{11}\,\gamma s[11]+\widehat{12}\,\gamma s[12]+\widehat{22}\,\gamma s[22]+\widehat{11}\,\kappa s[11]+2\,\widehat{12}\,\kappa s[11]+\widehat{22}\,\kappa s[11],$$
$$\widehat{112}\,\alpha s[112]+\frac{1}{2}\,\widehat{112}\,\alpha s[1]\,\alpha s[12]-\widehat{122}\,\alpha s[122]+\frac{1}{6}\,\widehat{112}\,\alpha s[1]^2\,\alpha s[2]-$$
$$\frac{1}{2}\,\widehat{122}\,\alpha s[12]\,\alpha s[2]-\frac{1}{6}\,\widehat{122}\,\alpha s[1]\,\alpha s[2]^2-\widehat{112}\,\alpha s[12]\,\beta s[1]-\frac{1}{2}\,\widehat{112}\,\alpha s[1]\,\alpha s[2]\,\beta s[1]+$$
$$\frac{1}{2}\,\widehat{122}\,\alpha s[2]^2\,\beta s[1]+\frac{1}{2}\,\widehat{112}\,\alpha s[2]\,\beta s[1]^2-\widehat{112}\,\beta s[112]+\widehat{122}\,\alpha s[2]\,\beta s[12]+$$
$$\frac{1}{2}\,\widehat{112}\,\beta s[1]\,\beta s[12]+\widehat{122}\,\beta s[122]-\frac{1}{2}\,\widehat{122}\,\alpha s[2]\,\beta s[1]\,\beta s[2]-\frac{1}{6}\,\widehat{112}\,\beta s[1]^2\,\beta s[2]-$$
$$\frac{1}{2}\,\widehat{122}\,\beta s[12]\,\beta s[2]+\frac{1}{6}\,\widehat{122}\,\beta s[1]\,\beta s[2]^2+\widehat{111}\,\gamma s[111]+\widehat{112}\,\gamma s[112]+\widehat{122}\,\gamma s[122]+$$
$$\widehat{222}\,\gamma s[222]+\widehat{111}\,\kappa s[111]+3\,\widehat{112}\,\kappa s[111]+3\,\widehat{122}\,\kappa s[111]+\widehat{222}\,\kappa s[111],\ \ldots\Big]\Big]$$

```
(V ** (Es[⟨1 → 0⟩, κ] // dΔ[1, 1, 2]) // dc[1] // dc[2]) ≡
 (Es[⟨1 → 0⟩, κ] (Es[⟨1 → 0⟩, κ] // dσ[1, 2]) // dc[1] // dc[2])
```

$$
\begin{aligned}
&\text{BS}\Big[\text{True},\\
&-\text{CW}[1]\,\alpha s[1]-\text{CW}[2]\,\beta s[2]+\text{CW}[1]\,\gamma s[1]+\text{CW}[2]\,\gamma s[2]+\text{CW}[1]\,\kappa s[1]+\text{CW}[2]\,\kappa s[1]==\\
&\quad \text{CW}[1]\,\kappa s[1]+\text{CW}[2]\,\kappa s[1],\\
&-\text{CW}[1]\,\alpha s[1]-\text{CW}[2]\,\beta s[2]+\text{CW}[1]\,\gamma s[1]+\text{CW}[2]\,\gamma s[2]+\text{CW}[1]\,\kappa s[1]+\text{CW}[2]\,\kappa s[1]==\\
&\quad \text{CW}[1]\,\kappa s[1]+\text{CW}[2]\,\kappa s[1]\ \&\&\ \text{CW}[12]\,\alpha s[12]+\frac{1}{2}\text{CW}[12]\,\alpha s[1]\,\alpha s[2]-\text{CW}[12]\,\alpha s[2]\,\beta s[1]-\\
&\qquad \text{CW}[12]\,\beta s[12]+\frac{1}{2}\text{CW}[12]\,\beta s[1]\,\beta s[2]+\text{CW}[11]\,\gamma s[11]+\text{CW}[12]\,\gamma s[12]+\text{CW}[22]\,\gamma s[22]+\\
&\qquad \text{CW}[11]\,\kappa s[11]+2\,\text{CW}[12]\,\kappa s[11]+\text{CW}[22]\,\kappa s[11]==\text{CW}[11]\,\kappa s[11]+\text{CW}[22]\,\kappa s[11],\\
&-\text{CW}[1]\,\alpha s[1]-\text{CW}[2]\,\beta s[2]+\text{CW}[1]\,\gamma s[1]+\text{CW}[2]\,\gamma s[2]+\text{CW}[1]\,\kappa s[1]+\text{CW}[2]\,\kappa s[1]==\\
&\quad \text{CW}[1]\,\kappa s[1]+\text{CW}[2]\,\kappa s[1]\ \&\&\ \text{CW}[12]\,\alpha s[12]+\frac{1}{2}\text{CW}[12]\,\alpha s[1]\,\alpha s[2]-\text{CW}[12]\,\alpha s[2]\,\beta s[1]-\\
&\qquad \text{CW}[12]\,\beta s[12]+\frac{1}{2}\text{CW}[12]\,\beta s[1]\,\beta s[2]+\text{CW}[11]\,\gamma s[11]+\text{CW}[12]\,\gamma s[12]+\text{CW}[22]\,\gamma s[22]+\\
&\qquad \text{CW}[11]\,\kappa s[11]+2\,\text{CW}[12]\,\kappa s[11]+\text{CW}[22]\,\kappa s[11]==\text{CW}[11]\,\kappa s[11]+\text{CW}[22]\,\kappa s[11]\ \&\&\\
&\quad \text{CW}[112]\,\alpha s[112]+\frac{1}{2}\text{CW}[112]\,\alpha s[1]\,\alpha s[12]-\text{CW}[122]\,\alpha s[122]+\frac{1}{6}\text{CW}[112]\,\alpha s[1]^2\,\alpha s[2]-\\
&\qquad \frac{1}{2}\text{CW}[122]\,\alpha s[12]\,\alpha s[2]-\frac{1}{6}\text{CW}[122]\,\alpha s[1]\,\alpha s[2]^2-\text{CW}[112]\,\alpha s[12]\,\beta s[1]-\\
&\qquad \frac{1}{2}\text{CW}[112]\,\alpha s[1]\,\alpha s[2]\,\beta s[1]+\frac{1}{2}\text{CW}[122]\,\alpha s[2]^2\,\beta s[1]+\frac{1}{2}\text{CW}[112]\,\alpha s[2]\,\beta s[1]^2-\\
&\qquad \text{CW}[112]\,\beta s[112]+\text{CW}[122]\,\alpha s[2]\,\beta s[12]+\frac{1}{2}\text{CW}[112]\,\beta s[1]\,\beta s[12]+\\
&\qquad \text{CW}[122]\,\beta s[122]-\frac{1}{2}\text{CW}[122]\,\alpha s[2]\,\beta s[1]\,\beta s[2]-\frac{1}{6}\text{CW}[112]\,\beta s[1]^2\,\beta s[2]-\\
&\qquad \frac{1}{2}\text{CW}[122]\,\beta s[12]\,\beta s[2]+\frac{1}{6}\text{CW}[122]\,\beta s[1]\,\beta s[2]^2+\text{CW}[111]\,\gamma s[111]+\text{CW}[112]\,\gamma s[112]+\\
&\qquad \text{CW}[122]\,\gamma s[122]+\text{CW}[222]\,\gamma s[222]+\text{CW}[111]\,\kappa s[111]+3\,\text{CW}[112]\,\kappa s[111]+\\
&\qquad 3\,\text{CW}[122]\,\kappa s[111]+\text{CW}[222]\,\kappa s[111]==\text{CW}[111]\,\kappa s[111]+\text{CW}[222]\,\kappa s[111],\ \ldots\Big]
\end{aligned}
$$

```
unknowns = {
  α = LS[{"1", "2"}, αs], β = LS[{"1", "2"}, βs],
  γ = CWS[{"1", "2"}, γs], κ = CWS[{"1"}, κs]
 }
```

$$
\begin{aligned}
&\Big\{\text{LS}\Big[\overline{1}\,\alpha s[1]+\overline{2}\,\alpha s[2],\ \overline{1\,2}\,\alpha s[12],\ \overline{1\,\overline{1\,2}}\,\alpha s[112]+\overline{\overline{1\,2}\,2}\,\alpha s[122],\ \ldots\Big],\\
&\ \text{LS}\Big[\overline{1}\,\beta s[1]+\overline{2}\,\beta s[2],\ \overline{1\,2}\,\beta s[12],\ \overline{1\,\overline{1\,2}}\,\beta s[112]+\overline{\overline{1\,2}\,2}\,\beta s[122],\ \ldots\Big],\\
&\ \text{CWS}\Big[\widehat{1}\,\gamma s[1]+\widehat{2}\,\gamma s[2],\ \widehat{11}\,\gamma s[11]+\widehat{12}\,\gamma s[12]+\widehat{22}\,\gamma s[22],\\
&\quad \widehat{111}\,\gamma s[111]+\widehat{112}\,\gamma s[112]+\widehat{122}\,\gamma s[122]+\widehat{222}\,\gamma s[222],\ \ldots\Big],\\
&\ \text{CWS}\Big[\widehat{1}\,\kappa s[1],\ \widehat{11}\,\kappa s[11],\ \widehat{111}\,\kappa s[111],\ \ldots\Big]\Big\}
\end{aligned}
$$

```
V = Es[⟨1 → α, 2 → β⟩, γ]
```

$$\mathrm{E_s}\Big[\Big\langle 1 \to \mathrm{LS}\Big[\overline{1}\,\alpha s[1] + \overline{2}\,\alpha s[2],\ \overline{1\,2}\,\alpha s[12],\ \overline{1\,\overline{1\,2}}\,\alpha s[112] + \overline{\overline{1\,2}\,2}\,\alpha s[122],\ \ldots\Big],$$
$$2 \to \mathrm{LS}\Big[\overline{1}\,\beta s[1] + \overline{2}\,\beta s[2],\ \overline{1\,2}\,\beta s[12],\ \overline{1\,\overline{1\,2}}\,\beta s[112] + \overline{\overline{1\,2}\,2}\,\beta s[122],\ \ldots\Big]\Big\rangle,$$
$$\mathrm{CWS}\Big[\widehat{1}\,\gamma s[1] + \widehat{2}\,\gamma s[2],\ \widehat{11}\,\gamma s[11] + \widehat{12}\,\gamma s[12] + \widehat{22}\,\gamma s[22],$$
$$\widehat{111}\,\gamma s[111] + \widehat{112}\,\gamma s[112] + \widehat{122}\,\gamma s[122] + \widehat{222}\,\gamma s[222],\ \ldots\Big]\Big]$$

```
arbitrator = Replace[#, _ → 0, 1] &
```

```
Replace[#1, _ → 0, 1] &
```

```
d = 1
```

```
1
```

```
eqns = ħ^-1 (ζs[R+[2, 3] ** R+[1, 3]] ** V ≡ V ** (ζs[R+[1, 3]] // dΔ[1, 1, 2]));
lineqs = eqns[d]
```

$$-\frac{\mathrm{LW}[12]}{2} - \mathrm{LW}[12]\,\alpha s[2] + \mathrm{LW}[12]\,\beta s[1] == 0$$

```
lineqs = If[Head[lineqs] === And, List @@ lineqs, List@lineqs]
```

$$\Big\{-\frac{\mathrm{LW}[12]}{2} - \mathrm{LW}[12]\,\alpha s[2] + \mathrm{LW}[12]\,\beta s[1] == 0\Big\}$$

```
gens = Union[Cases[lineqs, _LW | _CW, ∞]]
```

```
{LW[12]}
```

```
lineqs = Flatten[Replace[lineqs ,
    lhs_ == rhs_ :↦ ((Coefficient[lhs, #] == Coefficient[rhs, #]) & /@ gens),
    {1}]]
```

$$\Big\{-\frac{1}{2} - \alpha s[2] + \beta s[1] == 0\Big\}$$

```
vars = Union @@ ((#[d, UndeterminedCoefficients]) & /@ unknowns)
```

```
{αs[1], αs[2], βs[1], βs[2], γs[1], γs[2], κs[1]}
```

```
{sol} = Solve[lineqs, vars]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\Big\{\Big\{\beta s[1] \to \frac{1}{2} + \alpha s[2]\Big\}\Big\}$$

```
fvars = Complement[vars, First /@ sol]
```

```
{αs[1], αs[2], βs[2], γs[1], γs[2], κs[1]}
```

```
MapThread[(#1 = #2) &, {fvars, arbitrator[fvars]}]
```

```
{0, 0, 0, 0, 0, 0}
```

```
sol /. (Rule → Set)
```

$$\left\{\frac{1}{2}\right\}$$

```
(#[d] = #[d]) & /@ (First /@ unknowns)
```

$$\left\{0, \frac{\text{LW}[1]}{2}, 0, 0\right\}$$

```
d = 2
```

$$2$$

```
eqns = ħ^-1 (ζs[R⁺[2, 3] ** R⁺[1, 3]] ** V ≡ V ** (ζs[R⁺[1, 3]] // dΔ[1, 1, 2]));
lineqs = eqns[d]
```

$$-\frac{\text{LW}[112]}{24}+\frac{\text{LW}[122]}{12}-\text{LW}[112]\,\alpha\text{s}[12]+\text{LW}[122]\,\beta\text{s}[12]==0$$

```
lineqs = If[Head[lineqs] === And, List @@ lineqs, List@lineqs]
```

$$\left\{-\frac{\text{LW}[112]}{24}+\frac{\text{LW}[122]}{12}-\text{LW}[112]\,\alpha\text{s}[12]+\text{LW}[122]\,\beta\text{s}[12]==0\right\}$$

```
gens = Union[Cases[lineqs, _LW | _CW, ∞]]
```

$$\{\text{LW}[112], \text{LW}[122]\}$$

```
lineqs = Flatten[Replace[lineqs ,
    lhs_ == rhs_ :↦ ((Coefficient[lhs, #] == Coefficient[rhs, #]) & /@ gens),
    {1}]]
```

$$\left\{-\frac{1}{24}-\alpha\text{s}[12]==0, \frac{1}{12}+\beta\text{s}[12]==0\right\}$$

```
vars = Union @@ ((#[d, UndeterminedCoefficients]) & /@ unknowns)
```

$$\{\alpha\text{s}[12], \beta\text{s}[12], \gamma\text{s}[11], \gamma\text{s}[12], \gamma\text{s}[22], \kappa\text{s}[11]\}$$

```
{sol} = Quiet[Solve[lineqs, vars], {Solve::svars}]
```

$$\left\{\left\{\alpha\text{s}[12]\to-\frac{1}{24}, \beta\text{s}[12]\to-\frac{1}{12}\right\}\right\}$$