Pensieve header: Misguided triality, continuing TrialityComputations.nb at pensieve://2013-11/DoubleTree/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< WKO4.m
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords,
  AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence,
  BracketForm, BS, CC, Crop, CW, CWS, CWSeries, D, Deg, DegreeScale,
  DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA,
  LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism,
  New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t,
  tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⟞, ⟝}.

FreeLie` is in the public domain. Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm,
  dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.
```

SeriesSolve::ArbitrarilySetting : In degree 1 arbitrarily setting {κs[x] → 0}.

SeriesSolve::ArbitrarilySetting : In degree 3 arbitrarily setting {αs[x, y, y] → 0}.

SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {αs[x, x, x, y, y] → 0}.

General::stop : Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation. ≫

$$\text{Es}\left[\left\langle x \to \text{LS}[0,\,0,\,0,\,0,\,\ldots]\right\rangle,\ \text{CWS}\left[0,\ \frac{\overline{xx}}{24},\ 0,\ -\frac{\overline{xxxx}}{2880},\ \ldots\right]\right]$$

```
ρ3[ξ_Es] := ξ // dS[y] // dΔ[y, y, z] // dm[x, z, x] // dσ[{x, y} → {y, x}];
V0 // ρ3
```

$$\text{Es}\left[\left\langle x \to \text{LS}\left[-\frac{\overline{y}}{2},\ \frac{\overline{xy}}{12},\ 0,\ -\frac{1}{720}\,\overline{x\,\overline{x\,\overline{xy}}}+\frac{1}{720}\,\overline{x\,\overline{\overline{xy}\,y}}-\frac{\overline{\overline{\overline{xy}\,y}\,y}}{5760},\ \ldots\right],\right.$$
$$\left.y \to \text{LS}\left[-\frac{\overline{y}}{2},\ \frac{\overline{xy}}{24},\ -\frac{1}{96}\,\overline{\overline{xy}\,y},\ -\frac{\overline{x\,\overline{x\,\overline{xy}}}}{1440}+\frac{7\,\overline{x\,\overline{\overline{xy}\,y}}}{5760}-\frac{\overline{\overline{\overline{xy}\,y}\,y}}{2880},\ \ldots\right]\right\rangle,$$
$$\text{CWS}\left[-\frac{\widehat{y}}{2},\ \frac{\widehat{xy}}{48}+\frac{\widehat{yy}}{48},\ 0,\ -\frac{\widehat{xxxy}}{2880}-\frac{\widehat{xxyy}}{2880}-\frac{\widehat{xyxy}}{5760}-\frac{\widehat{xyyy}}{2880}-\frac{\widehat{yyyy}}{5760},\ \ldots\right]\right]$$

```
V0@{5}
```

SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {αs[x, x, x, y, y] → 0}.

$$\text{Es}\left[\left\langle x \to \text{LS}\left[0,\ -\frac{\overline{xy}}{24},\ 0,\ \frac{7\,\overline{x\,\overline{x\,\overline{xy}}}}{5760}-\frac{7\,\overline{x\,\overline{\overline{xy}\,y}}}{5760}+\frac{\overline{\overline{\overline{xy}\,y}\,y}}{1440},\ 0,\ \ldots\right],\ y \to \text{LS}\left[\frac{\overline{x}}{2},\ -\frac{\overline{xy}}{12},\ 0,\right.\right.$$
$$\left.\left.\frac{\overline{x\,\overline{x\,\overline{xy}}}}{5760}-\frac{1}{720}\,\overline{x\,\overline{\overline{xy}\,y}}+\frac{1}{720}\,\overline{\overline{\overline{xy}\,y}\,y},\ -\frac{\overline{x\,\overline{x\,\overline{x\,\overline{xy}}}}}{7680}+\frac{\overline{x\,\overline{x\,\overline{\overline{xy}\,y}}}}{3840}-\frac{\overline{\overline{x\,\overline{xy}}\,\overline{xy}}}{6912},\ \ldots\right]\right\rangle,$$
$$\text{CWS}\left[0,\ -\frac{\overline{xy}}{48},\ 0,\ \frac{\overline{xxxy}}{2880}+\frac{\overline{xxyy}}{2880}+\frac{\overline{xyxy}}{5760}+\frac{\overline{xyyy}}{2880},\ 0,\ \ldots\right]\right]$$

```
V0 // ρ3 // ρ3 // ρ3
```

$$\text{Es}\Big[\Big\langle x\to \text{LS}\Big[0,-\frac{\overline{xy}}{24},0,\frac{7\,\overline{x\,\overline{x\overline{xy}}}}{5760}-\frac{7\,\overline{x\,\overline{\overline{xy}y}}}{5760}+\frac{\overline{\overline{\overline{xy}y}\,y}}{1440},\ldots\Big],$$
$$y\to \text{LS}\Big[\frac{\overline{x}}{2},-\frac{\overline{xy}}{12},0,\frac{\overline{x\,\overline{x\overline{xy}}}}{5760}-\frac{1}{720}\,\overline{x\,\overline{\overline{xy}y}}+\frac{1}{720}\,\overline{\overline{\overline{xy}y}\,y},\ldots\Big]\Big\rangle,$$
$$\text{CWS}\Big[0,-\frac{\widehat{xy}}{48},0,\frac{\widehat{xxxy}}{2880}+\frac{\widehat{xxyy}}{2880}+\frac{\widehat{xyxy}}{5760}+\frac{\widehat{xyyy}}{2880},\ldots\Big]\Big]$$

```
θ[x_, s_] := Module[{y}, θs[x, y, s] // dm[x, y, x]];
θ[1, 1]@{12}
```

$$\text{Es}\Big[\big\langle 1\to \text{LS}\big[2\,\overline{1},0,0,0,0,0,0,0,0,0,\ldots\big]\big\rangle,$$
$$\text{CWS}\Big[\widehat{1},\frac{\widehat{11}}{2},0,-\frac{\widehat{1111}}{12},0,\frac{\widehat{111111}}{45},0,-\frac{17\,\widehat{11111111}}{2520},0,\frac{31\,\widehat{1111111111}}{14\,175},\ldots\Big]\Big]$$

```
FindSequenceFunction[{1/2, 1/12, 1/45, 17/2520, 31/14175}]
```

$$\text{FindSequenceFunction}\Big[\Big\{\frac{1}{2},\frac{1}{12},\frac{1}{45},\frac{17}{2520},\frac{31}{14\,175}\Big\}\Big]$$

```
Vpp = (V0 ** θs[x, y, -1/2]) θ[z, -1/4] // dm[y, z, y] // ρ3 // ρ3
```

$$\text{Es}\Big[\Big\langle x\to \text{LS}\Big[0,-\frac{\overline{xy}}{24},0,\frac{7\,\overline{x\,\overline{x\overline{xy}}}}{5760}-\frac{7\,\overline{x\,\overline{\overline{xy}y}}}{5760}+\frac{\overline{\overline{\overline{xy}y}\,y}}{1440},\ldots\Big],$$
$$y\to \text{LS}\Big[\frac{\overline{x}}{2},-\frac{\overline{xy}}{12},0,\frac{\overline{x\,\overline{x\overline{xy}}}}{5760}-\frac{1}{720}\,\overline{x\,\overline{\overline{xy}y}}+\frac{1}{720}\,\overline{\overline{\overline{xy}y}\,y},\ldots\Big]\Big\rangle,$$
$$\text{CWS}\Big[\frac{\widehat{x}}{4},\frac{5\,\widehat{xx}}{96}+\frac{\widehat{xy}}{48},0,-\frac{23\,\widehat{xxxx}}{46\,080}-\frac{\widehat{xxxy}}{2880}-\frac{\widehat{xxyy}}{2880}-\frac{\widehat{xyxy}}{5760}-\frac{\widehat{xyyy}}{2880},\ldots\Big]\Big]$$

```
(V0 // dA[x, y]) ** Vpp
```

$$\text{Es}\Big[\langle x\to \text{LS}[0,0,0,0,\ldots], y\to \text{LS}[0,0,0,0,\ldots]\rangle,$$
$$\text{CWS}\Big[\frac{\widehat{x}}{4},\frac{5\,\widehat{xx}}{96}+\frac{\widehat{xy}}{24},0,-\frac{23\,\widehat{xxxx}}{46\,080}-\frac{\widehat{xxxy}}{1440}-\frac{\widehat{xxyy}}{1440}-\frac{\widehat{xyxy}}{2880}-\frac{\widehat{xyyy}}{1440},\ldots\Big]\Big]$$

```
((V0 // dA[x, y]) ** Vpp ** (Cap ** Cap // dΔ[x, x, y]))@{6}
```

$$\text{Es}\Big[\langle x\to \text{LS}[0,0,0,0,0,0,\ldots], y\to \text{LS}[0,0,0,0,0,0,\ldots]\rangle,$$
$$\text{CWS}\Big[\frac{\overline{x}}{4},\frac{\overline{xx}}{32}-\frac{\overline{yy}}{48},0,-\frac{\overline{xxxx}}{3072}+\frac{\overline{yyyy}}{5760},0,\frac{\overline{xxxxxx}}{184\,320}-\frac{\overline{yyyyyy}}{362\,880},\ldots\Big]\Big]$$

```
(V0[[2]] + ((2 Cp // tσ[x, y]) - 4 Cp + DegreeScale[1/2][4 Cp]))@{8}
```

$$\mathrm{CWS}\Big[0,\ \frac{\overline{xx}}{32}-\frac{\overline{xy}}{48}-\frac{\overline{yy}}{48},\ 0,\ -\frac{\overline{xxxx}}{3072}+\frac{\overline{xxxy}}{2880}+\frac{\overline{xxyy}}{2880}+\frac{\overline{xyxy}}{5760}+\frac{\overline{xyyy}}{2880}+\frac{\overline{yyyy}}{5760},$$
$$0,\ \frac{\overline{xxxxxx}}{184\,320}-\frac{\overline{xxxxxy}}{120\,960}-\frac{\overline{xxxxyy}}{120\,960}-\frac{\overline{xxxyxy}}{120\,960}-\frac{\overline{xxxyyy}}{120\,960}-\frac{\overline{xxyxxy}}{241\,920}-\frac{\overline{xxyxyy}}{120\,960}-$$
$$\frac{\overline{xxyyxy}}{120\,960}-\frac{\overline{xxyyyy}}{120\,960}-\frac{\overline{xyxyxy}}{362\,880}-\frac{\overline{xyxyyy}}{120\,960}-\frac{\overline{xyyxyy}}{241\,920}-\frac{\overline{xyyyyy}}{120\,960}-\frac{\overline{yyyyyy}}{362\,880},\ 0,$$
$$-\frac{17\,\overline{xxxxxxxx}}{165\,150\,720}+\frac{\overline{xxxxxxxy}}{4\,838\,400}+\frac{\overline{xxxxxxyy}}{4\,838\,400}+\frac{\overline{xxxxxyxy}}{4\,838\,400}+\frac{\overline{xxxxxyyy}}{4\,838\,400}+\frac{\overline{xxxxyxxy}}{4\,838\,400}+$$
$$\frac{\overline{xxxxyxyy}}{4\,838\,400}+\frac{\overline{xxxxyyxy}}{4\,838\,400}+\frac{\overline{xxxxyyyy}}{4\,838\,400}+\frac{\overline{xxxyxxxy}}{9\,676\,800}+\frac{\overline{xxxyxxyy}}{4\,838\,400}+\frac{\overline{xxxyxyxy}}{4\,838\,400}+$$
$$\frac{\overline{xxxyxyyy}}{4\,838\,400}+\frac{\overline{xxxyyxxy}}{4\,838\,400}+\frac{\overline{xxxyyxyy}}{4\,838\,400}+\frac{\overline{xxxyyyxy}}{4\,838\,400}+\frac{\overline{xxxyyyyy}}{4\,838\,400}+\frac{\overline{xxyxxyxy}}{4\,838\,400}+$$
$$\frac{\overline{xxyxxyyy}}{4\,838\,400}+\frac{\overline{xxyxyxyy}}{4\,838\,400}+\frac{\overline{xxyxyyxy}}{4\,838\,400}+\frac{\overline{xxyxyyyy}}{4\,838\,400}+\frac{\overline{xxyyxxyy}}{9\,676\,800}+\frac{\overline{xxyyxyxy}}{4\,838\,400}+$$
$$\frac{\overline{xxyyxyyy}}{4\,838\,400}+\frac{\overline{xxyyyxyy}}{4\,838\,400}+\frac{\overline{xxyyyyxy}}{4\,838\,400}+\frac{\overline{xxyyyyyy}}{4\,838\,400}+\frac{\overline{xyxyxyxy}}{19\,353\,600}+\frac{\overline{xyxyxyyy}}{4\,838\,400}+$$
$$\frac{\overline{xyxyyxyy}}{4\,838\,400}+\frac{\overline{xyxyyyyy}}{4\,838\,400}+\frac{\overline{xyyyxyyy}}{4\,838\,400}+\frac{\overline{xyyyyxyy}}{9\,676\,800}+\frac{\overline{xyyyyyyy}}{4\,838\,400}+\frac{\overline{yyyyyyyy}}{19\,353\,600},\ \ldots\Big]$$

```
(Vpp ** (Cap ** Cap // dΔ[x, x, y]))[[2]]@{8}
```

```
$Aborted
```

```
Cp = Cap[[2]]
```

$$\mathrm{CWS}\Big[0,\ -\frac{\overline{xx}}{96},\ 0,\ \frac{\overline{xxxx}}{11\,520},\ \ldots\Big]$$

```
((2 Cp // tσ[x, y]) - 4 Cp + DegreeScale[1/2][4 Cp])@{8}
```

$$\mathrm{CWS}\Big[0,\ \frac{\overline{xx}}{32}-\frac{\overline{yy}}{48},\ 0,\ -\frac{\overline{xxxx}}{3072}+\frac{\overline{yyyy}}{5760},\ 0,\ \frac{\overline{xxxxxx}}{184\,320}-\frac{\overline{yyyyyy}}{362\,880},\ \ldots\Big]$$

```
(V0 // dA[x, y]) ** Vpp ** (Cap ** Cap // dΔ[x, x, y])
```