EXTRA FIGURES

DROR BAR-NATAN AND ZSUZSANNA DANCSO
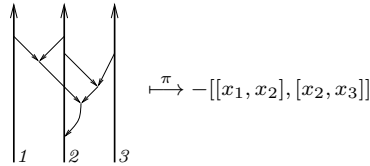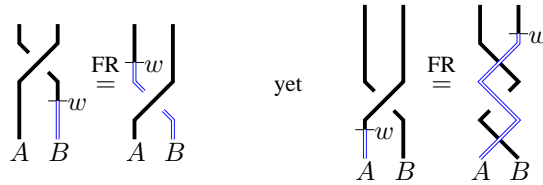
(0)

$$\overset{\pi}{\longmapsto} -[[x_1, x_2], [x_2, x_3]]$$

(1-1) figs/wTFgensWen@:

$$wTF = \mathrm{CA}\left\langle \left. \begin{matrix} \text{(diagrams)} \end{matrix} \right| \begin{matrix} \text{R1}^s,\ \text{R2, R3, R4, OC, CP,} \\ \text{FR, } W^2,\ \text{CW, TV} \end{matrix} \right| S_e, A_e, u_e, d_e \right\rangle.$$
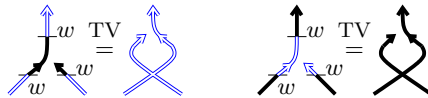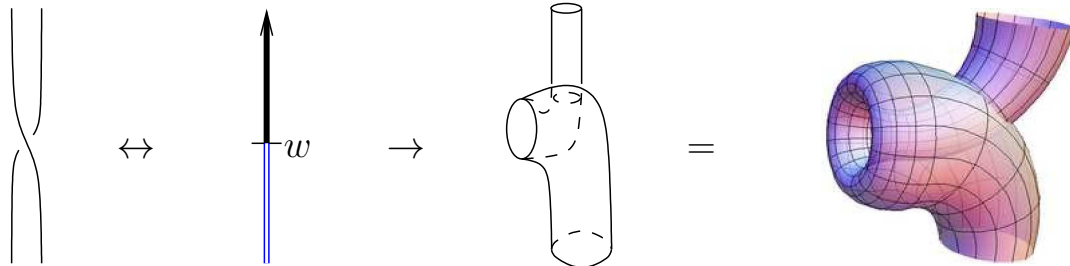
(1-2)

yet

(1-3) figs/CapWen@:

A cap can slide through a wen, hence a capped wen disappears, as shown on the right, to be denoted CW.
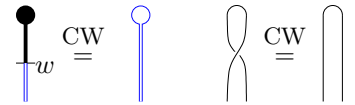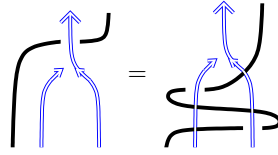

(1-4) figs/TVRel:

(1-5) figs/Wen2@:

$\leftrightarrow$ $\rightarrow$ $=$

(1-6) figs/SkelGenWen:

$$\mathcal{S} = \mathrm{CA}\left\langle \left. \begin{matrix} \text{(diagrams)} \end{matrix} \right| W^2,\ \text{CW, TV} \right\rangle.$$
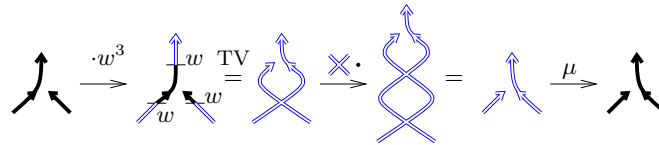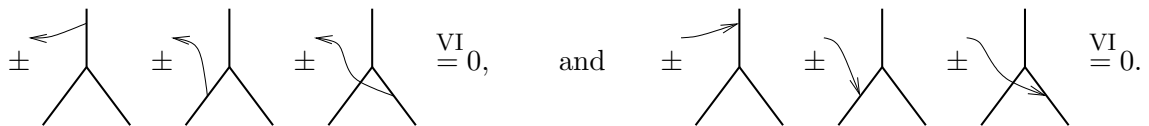
(1-7)

(1-8)
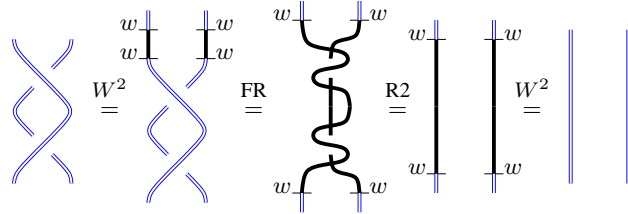
(1-9) figs/AdjointV:

(1-10) figs/wTFprojgens@ (appears in two places!):

$$\mathcal{A}^{(s)w} = \text{CA} \left\langle \; \begin{array}{c} \text{(generators)} \end{array} \; \middle| \; \begin{array}{c} \text{relations as in} \\ \text{Section ??} \end{array} \; \middle| \; \begin{array}{c} \text{operations as in} \\ \text{Section ??} \end{array} \; \right\rangle .$$

$$\mathcal{A}^{(s)w} = \text{CA} \left\langle \; \begin{array}{c} \text{(generators)} \end{array} \; \middle| \; \begin{array}{c} \overrightarrow{4T}, \text{TC}, \text{VI}, \text{CP}, \\ W^2, \text{TW}, \text{CW}, \text{FR}, \\ (\text{RI for } \mathcal{A}^{sw}) \end{array} \; \middle| \; S_e, A_e, u_e, d_e \; \right\rangle .$$

(1-11) figs/VI@:

(2-1) figs/ZofWen1:

2

(2-2) figs/ZofWen2:

$$1 \quad = \quad \text{(diagram)} \quad = \quad \text{(diagram)}$$
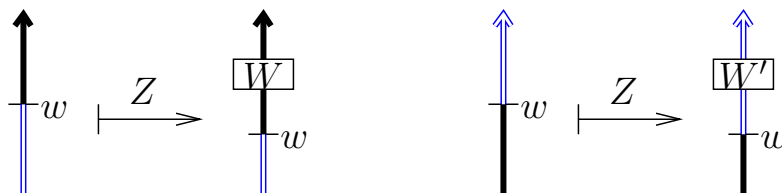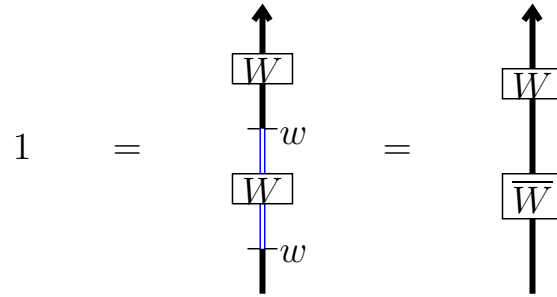
(2-3) figs/ZofWen3: (Cancelled)

$$\xmapsto{Z}$$

$$\xmapsto{Z}$$
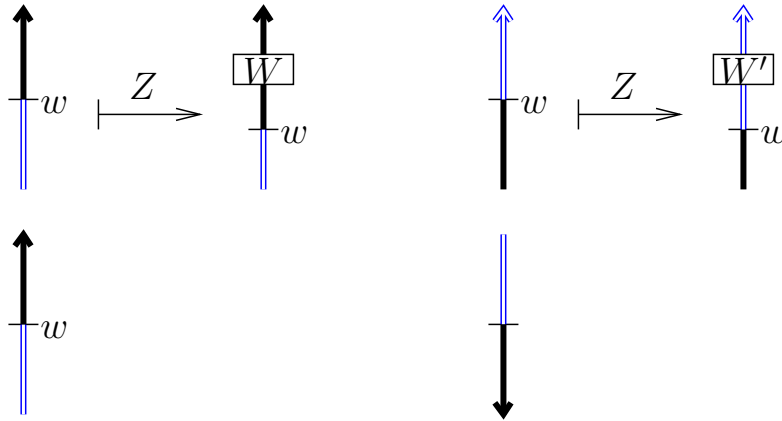
(2-4) figs/ZofWen4: (Postponed)

(2-5) figs/ColouredCrossings:

(3-1) figs/FRCalc:

$$= \quad \xleftarrow{Z} \quad \overset{FR}{=} \quad \xmapsto{Z} \quad =$$
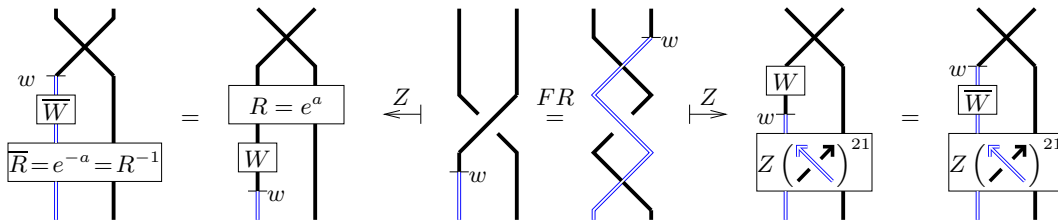
The proof of $(R^{-1})^{21} = Z\left(\text{(diagram)}\right)$

(3-2) figs/TVUnitarity:

$$= \quad \xleftarrow{Z} \quad \overset{TV}{=} \quad \xmapsto{Z} \quad S_1 S_2(V^{-1})$$

3
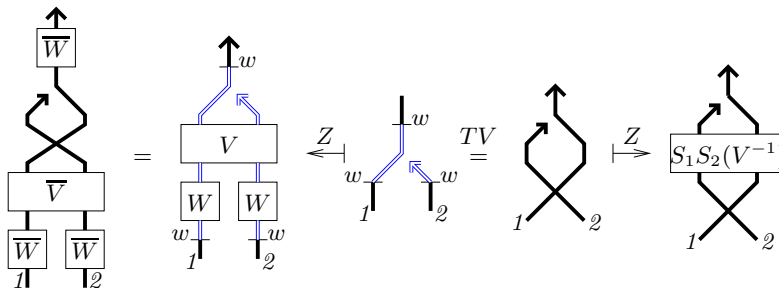
(3-3) figs/SkelGen:

$$\mathcal{S}^0 = \mathrm{CA}\left\langle \text{↑}, \text{⋀}, \text{⋁} \right\rangle$$

(4-1) figs/wTFgensWen:

$$wTF = \mathrm{CA}\left\langle \text{⤢}, \text{⤡}, \text{↑}, \dagger_w, \text{⋀}, \text{⋁} \,\middle|\, \begin{matrix} \text{R1}^s, \text{R2, R3, R4, OC, CP,} \\ \text{FR, } W^2, \text{ CW, TV} \end{matrix} \,\middle|\, S_e, u_e, d_e \right\rangle.$$

(4-2) figs/SkelGenWen@:

$$\mathcal{S} = \mathrm{CA}\left\langle \text{↑}, \dagger_w, \text{⋀}, \text{⋁} \,\middle|\, W^2, \text{ CW, TV} \right\rangle.$$

(4-3) figs/TheTwistedVertex:
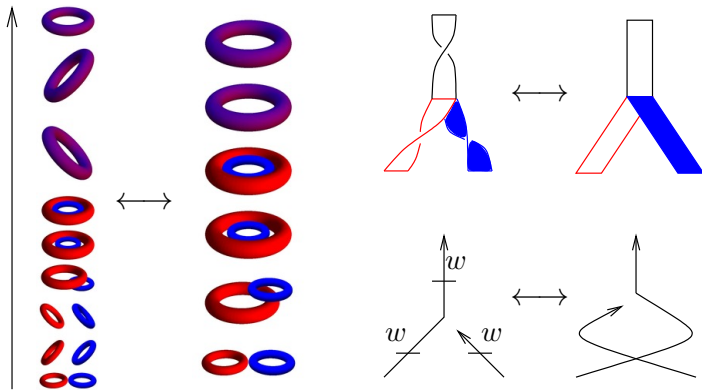
The last wen relation describes the interaction of wens and vertices, as illustrated on the left. In the band notation the non-filled band represents the larger circle, and the band the inner/smaller circle, as usual. Conjugating a vertex by three wens switches the top and bottom bands, as shown in the figure on the left. Alternatively in the flying circle interpretation, if both rings flip, then merge, and then the merged ring flips again, this is homotopic to no flips, except the fly-in direction (from below or from above) has changed. We denote this relation by TV, for *Twisted Vertex*. Diagrammatically, we have obtained the relation shown below: ???
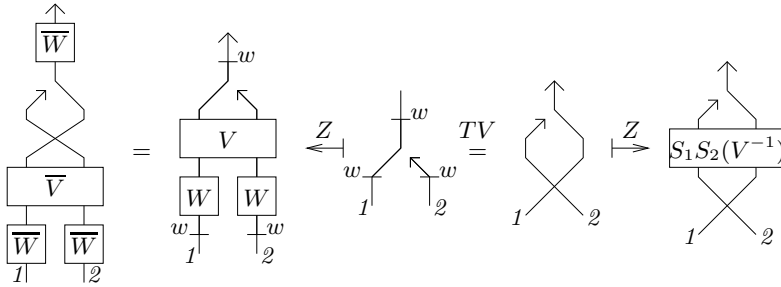
(4-4) figs/wTFprojgensWen and figs/wTFprojgens:

$$\mathcal{A}^{(s)w} = \mathrm{CA}\left\langle \text{⊢�ᵀ}, \text{↑}, \dagger_w, \text{⋀}, \text{⋁} \,\middle|\, \begin{matrix} \overrightarrow{4T}, \text{TC, VI, CP, } W^2, \text{ TW,} \\ \text{CW, FR, (RI for } \mathcal{A}^{sw}) \end{matrix} \,\middle|\, S_e, u_e, d_e \right\rangle.$$

$$\mathcal{A}^{sw} = \mathrm{CA}\left\langle \text{⊢⤤}, \text{↑}, \text{⋀}, \text{⋁} \,\middle|\, \overrightarrow{4T}, \text{TC, VI, CP, RI} \,\middle|\, S_e, u_e, d_e, - \right\rangle.$$
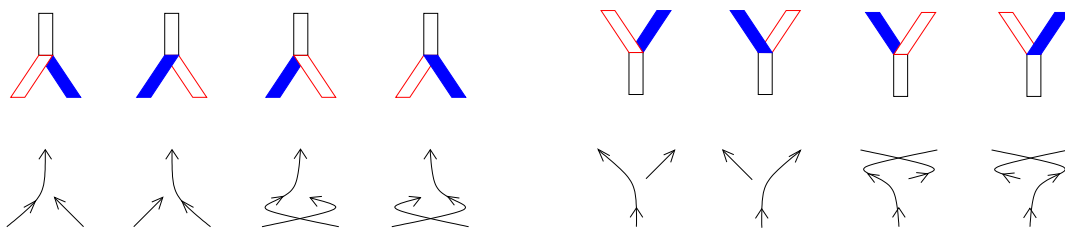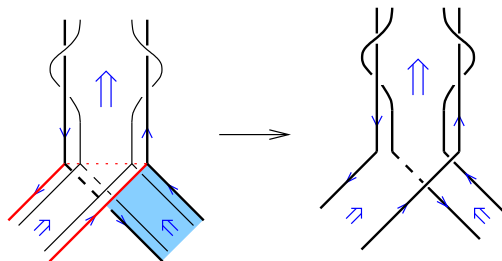
(4-5) figs/ZofWen2@:

(4-6) figs/TVUnitarity@:

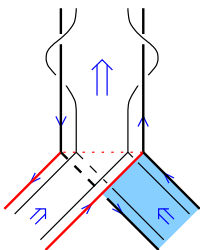(4-7) figs/NooseEquations, figs/NooseCappedProof, figs/NooseCapped:
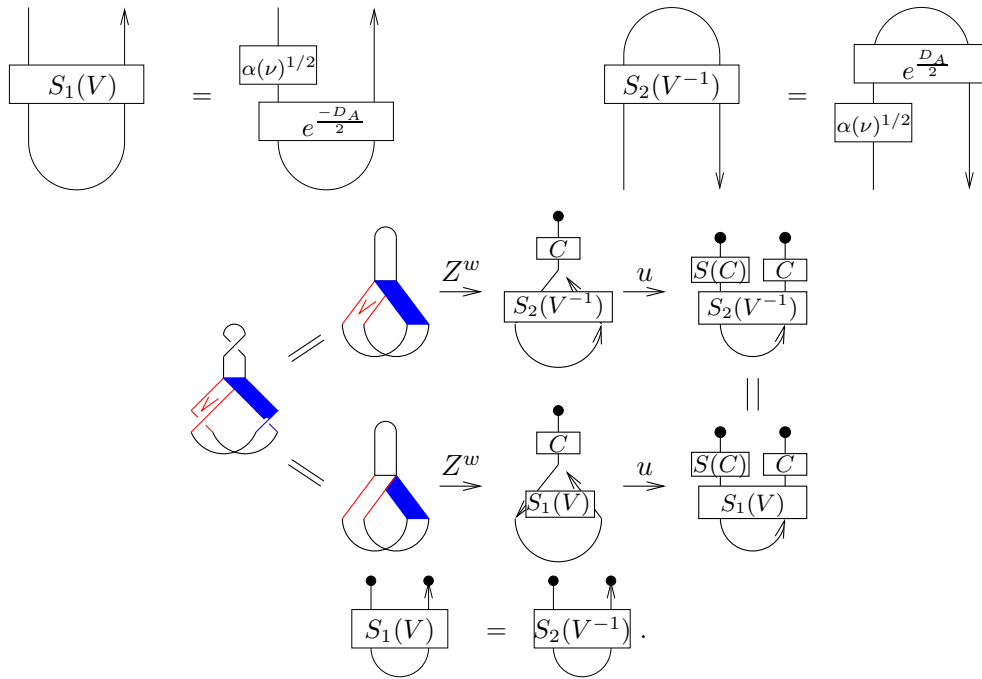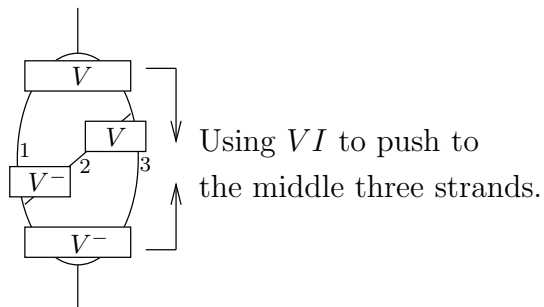

(5-1) figs/VertexTypes@:

(5-2) figs/BandUnzip@:

5

(5-3) figs/DetailedVertex@:
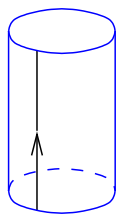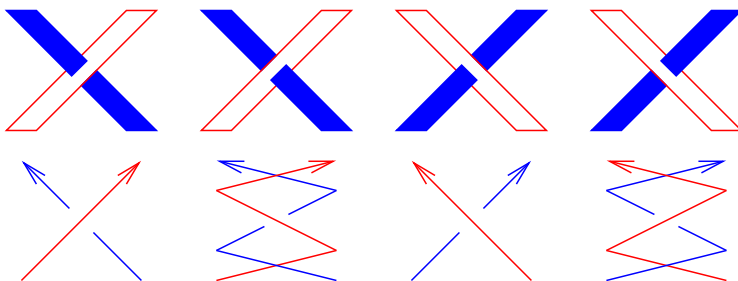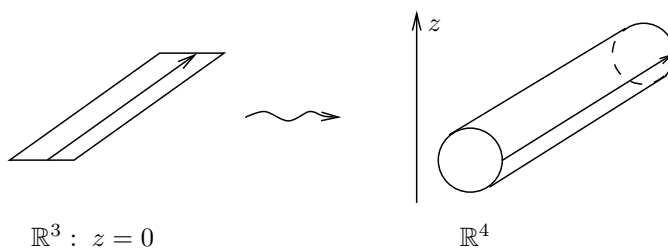

(5-4) figs/Associator:

Using VI to push to the middle three strands.

(5-5) figs/TubeOrientation@:

(5-6) figs/BandCrossings@:

(5-7) figs/PushMembranes@:

$\mathbb{R}^3 : z = 0$ $\mathbb{R}^4$

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