LIST OF EDITS FOR "FINITE TYPE INVARIANTS OF W-KNOTTED OBJECTS II: TANGLES, FOAMS AND THE KASHIWARA-VERGNE PROBLEM"

DROR BAR-NATAN AND ZSUZSANNA DANCSO

Below is a response to the referee's remaining questions and a list of corresponding edits in the paper.

(3): We believe there are two small misunderstandings here, as explained below. In the paper we have slightly extended the definition of v- and w-tangles as circuit algebras, as well as the discussion of their skeleta. The main reason for this is that the referee understands the paper very well and has read it in much detail. Therefore, if we have failed to communicate a point to the referee, we have probably failed to commuicate that to most readers. The changes are all on pages 12 and 13.

First, *on the subject of the "virtual crossings"* on the LHS and RHS of what is now Figure 2 (formerly Figure 3). These are in fact the same type of feature on both sides. On the RHS (skeleton), they are, as you say, simply part of the circuit algebra structure and there is nothing "virtual" about them, and the same statement is true on the LHS.

To clarify this, we first of all split the definition of v- and w-tangles into two parts: v-tangle diagrams in Definition 3.1, and the tangles themselves in Definition 3.4. Definition 3.1 is followed by an Example which explains Figure 2, and a Warning on "virtual crossings". We have also slightly extended what is now Remark 3.5, on the planar algebra presentation of v-tangles which involves virtual crossings and Reidemeister moves.

As for your question on numbering: we believe the misunderstanding is that the numbering in circuit algebras refers to boundary points, not arcs. To clarify this, we included in Figure 2 the wiring diagram D which produces the v-tangle diagram V. This diagram indeed takes three imputs from $vT_{2,2}$, but each of the inputs is simply a crossing (two are negative and one is positive). The ends of the crossings are numbered 1 through 4, say counterclockwise from the bottom left (this was left implicit in the definition). Correspondingly, the boundary points of the "input holes" in the wiring diagram D are also numbered 1 through 4, which determines how the crossings must be attached.

Now to obtain the skeleton, simply replace each crossing by what looks like a "virtual crossing" but is in fact only a wiring diagram with zero inputs of type (2,2): that is, an element of $\mathcal{S}_{2,2}$. The result is to be viewed as an element of $\mathcal{S}$, that is, a wiring diagram with zero inputs, which is an oriented 1-manifold with numbered boundary points. As such, the result is the same as the skeleton shown on the right of Figure 2. This is explained in Example 3.2 and in the caption of Figure 2.

(5): That is fair; we have ommitted these definitions.

Other: We have noticed that the contact information for one of the authors was out of date, we have corrected it.