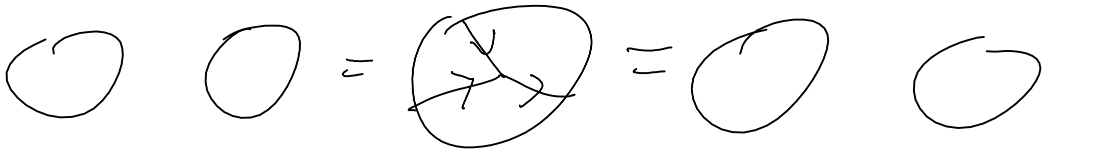
<u>Thm</u> The obvious inclusion $\iota:\mathcal{D}^w \to \mathcal{D}^{tw}$ descends to an isomorphism $\mathcal{A}^w \to \mathcal{A}^{tw}$, and in $\mathcal{A}^{tw}$, the $\overrightarrow{IHX}$ relation is satisfied.

(The proof, joint with D Thurston, is modeled after ...)

<u>Proof</u> ~~Let $\mathcal{D}^{tw}_K$ be the subset of $\mathcal{D}^{tw}_K$ consisting of diagrams with at most K internal vertices, and let R_K be the collection of $\overrightarrow{STU}$ relations that can be written within $\mathcal{D}^w_K$. We will prove by induction on K that the obvious inclusion $\iota_K:\mathcal{D}^w \to \mathcal{D}^{tw}$ descends to an iso. As everything is graded and the number of internal vertices is always bounded by twice the degree, the inductive statement at large enough K proves that ι is an isomorphism.~~

<u>2nd attempt</u>

To show that ι induces a well defined map $\bar\iota:\mathcal{A}^w \to \mathcal{A}^{tw}$ we just need to show that it carries relations to relations; that is, that the $4T$ follows from the $\overrightarrow{STU}$ relations. Indeed,

 (*)

It is clear that $\bar\iota$ is onto; indeed, it is by definition onto the diagrams that have no internal vertices, and given the connectedness of w-Jac diagrams, it is clear that internal vertices in a w-Jac diagram can be eliminated one by one using the $\overrightarrow{STU}$ relations. To complete the proof that $\bar{\iota}$ is an isomorphism it is enough to show that this "elimination of the internals" procedure is well defined – that it's result is independent of the order in which $\overrightarrow{STU}$ relations are applied to eliminate internal vertices. Indeed, this done, This done, the elimination map would by definition satisfy the $\overrightarrow{STU}$ relations and thus descend to a well defined inverse of $\bar{\iota}$.

On diagrams with one internal vertex, equation (*) shows that all ways of eliminating that vertex are equivalent mod $\overrightarrow{4T}$, and hence the elimination map is well defined on such diagrams. Now assume we have shown that the elimination map is well defined on all diagrams with at most 7 internal vertices, and let D be a diagram with 8. Let e and e' be edges in D that connect the skeleton to an internal vertex. We need to show that any elimination procedure that begins with e yields the same answer, mod 4T, as any elimination procedure that begins with e′.

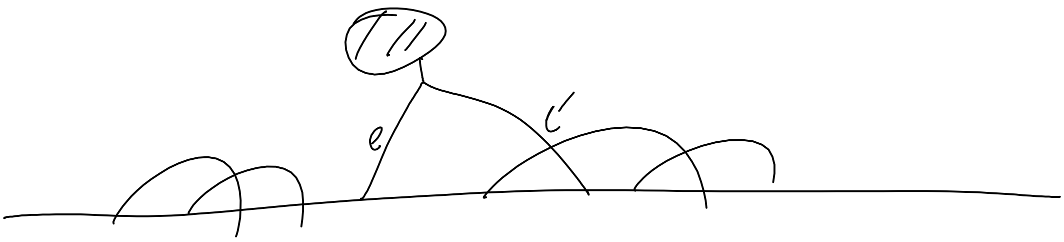
Case I e & e′ connect the skeleton to different internal vertices.

In this case, . . . Y = Y

Case II e & e′ are connected to the same internal vertex, yet some other edge e″ exists . . .

We use the transitivity of equality.

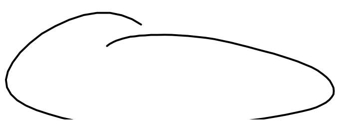
Case III Case III is what remains if neither Case I nor case II hold; in that case, D must have the following schematic form:



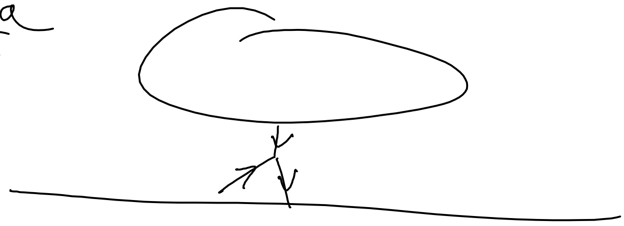
(The "blob" is not connected to the skeleton other than via e or e′, further arrows may exist outside the blob, but no further vertices.)

Case III splits into two subcases, depending upon the orientations of e & e′:

Case IIIa

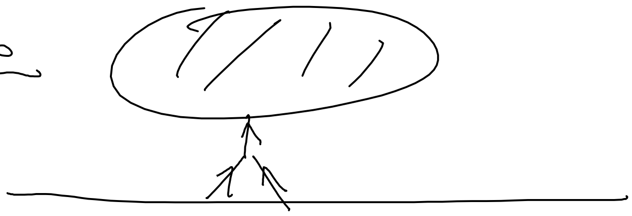


Case III a

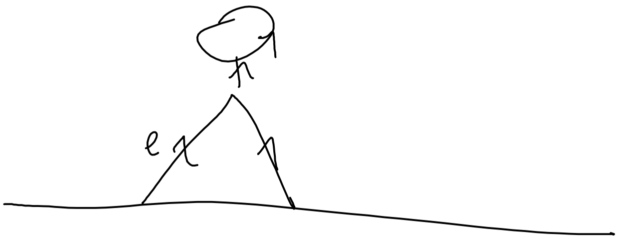


case IIIa cannot exist for arrow-orientation reasons.

Case III b



In this case the "blobs" must be minimal



using STU along e we get

which clearly holds.