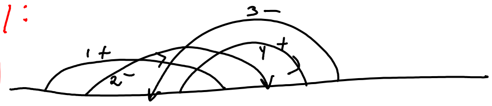
1. KAL of August 6 Summary and expansion   in 2008-08
2. Det and Tr   in 2008-07
3. wAlex   in 2008-07
4. AcademicPensieve/2008-07/nb/WFormulaForAlexander.pdf

Digest: [Everything here ignores $D_L =$  & $D_R =$ ]

From 1:
(Edits: $Z \to \zeta$)



$a_i :=$ arrow $\# i$
$d_i :=$ direction of a_i here: $(++-+)$
$s_i :=$ sign of a_i here: $(+--+)$

T - the trapping matrix:
$$T_{ij} = \begin{cases} 1 & a_j \text{ ends within the open span of } a_i \\ 0 & \text{otherwise} \end{cases}$$
$$S = \mathrm{diag}(s_i d_i)$$

Let $B = T(e^{xS} - 1)$ $C = (I+T)S$
$$\zeta = \mathrm{Tr}(I-B)^{-1}BC = \mathrm{Tr}\left[(I-B)^{-1}C - C\right]$$

Proposition $\zeta = \frac{d}{dx}\log(\det(I-B))$

Conjecture $\zeta(X) = -X\frac{A'(X)}{A(X)}$, with $A(X)$ being the Alexander polynomial.

Conjecture: $A(X) = \det(I-B)$, perhaps up to a power of X & a sign.

From 3:
(w/ $\log z \to EZ$)
$$\zeta = Z^{-1}EZ = \text{The "glow" of } Z$$
The Euler operator

Moral: Also assuming $W_k W_l = W_{k+l}$ we have
$$Z^{-1}xZ' = \zeta = -x\,A'/A \Rightarrow Z = -A$$

must be a sign error because of the unknot.

Without $W_k W_l = W_{k+l}$, we should have [yet still ignoring D_L & D_K]
$$Z = \exp_{AW}\left[W\left(\log_{\mathbb{Q}[[x]]} A(e^x)\right)\right]$$

Aside Wouldn't it be nice to have a direct construction of the map $\mathcal{A}^w \to \mathbb{Q}[D_L, D_R, w_i]$ in terms of some determinant or trace of some "trapping matrix"?

Proof diagram:

$$\begin{tikzcd}
\widetilde{EZ} \in IAM(K) \arrow[d] & \mathbb{Q}[[x]] \otimes (\text{Framing stuff}) \arrow[l] \arrow[dl] \\
EZ \in \mathcal{A}^w &
\end{tikzcd}$$

provisional name

Applying $\tilde{E}$:

$$A(K)(X) := \det\big(\overbrace{I + T(I - X^{-S})}^{B(X)}\big) \leftarrow \text{From WKO}$$

$$Z(K) = \underbrace{\exp_{\mathcal{A}^w}(sl_L(K)D_L)\cdot\exp_{\mathcal{A}^w}(sl_R(K)D_R)}_{\textit{self linking coded in arrows}}\cdot\underbrace{\exp_{\mathcal{A}^w}\left(w\left(\log_{\mathbb{Q}[x]}A(K)(e^x)\right)\right)}_{\textit{Alexander coded in wheels}},$$

$$\Downarrow \tilde{E}$$

$$\tilde{E}Z(K) = \underbrace{sl_L(K)D_L + sl_R(K)D_R}_{=:SL} + w\left(E(\log A(K)(e^x))\right)$$

$$= SL + w\left(x\frac{\partial}{\partial x}\log\det B(e^x)\right)$$

$$= SL + w\left(x e^x \frac{\partial}{\partial X}\log\det B(X)\Big|_{X=e^x}\right)$$

$$\frac{d}{d\epsilon}\det A_\epsilon = \frac{d}{d\epsilon}\det A_0 \det A_0^{-1}A_\epsilon$$
$$= \det A_0 \operatorname{tr} A_0^{-1}\frac{d}{d\epsilon}A_\epsilon$$

$$= SL + w\left(x e^x \operatorname{tr}\left(B(X)^{-1}\frac{\partial}{\partial X}B\right)\Big|_{X=e^x}\right)$$

Feb 11: $(-)$ missing?

$$= SL + w\left(x e^x \cdot \operatorname{tr}\left((I + T(I - X^{-S}))^{-1}\cdot T\cdot S X^{-S-1}\right)\Big|_{X=e^x}\right)$$

$$= SL + w\left(x \operatorname{tr}\left((I + \underbrace{T(I - X^{-S})}_{-B})^{-1}\cdot\underbrace{T\cdot X^{-S}S}_{Q_1}\right)\Big|_{X=e^x}\right)$$

Q_2

$$= \Omega + W\Big(x\, tr\big((I-B)^{-1}\overbrace{B(I+T)S}^{Q_2}\big)\Big)_{x=e^x}$$

as $Q_1 - Q_2 = TX^{-s}S - B(I+T)S =$

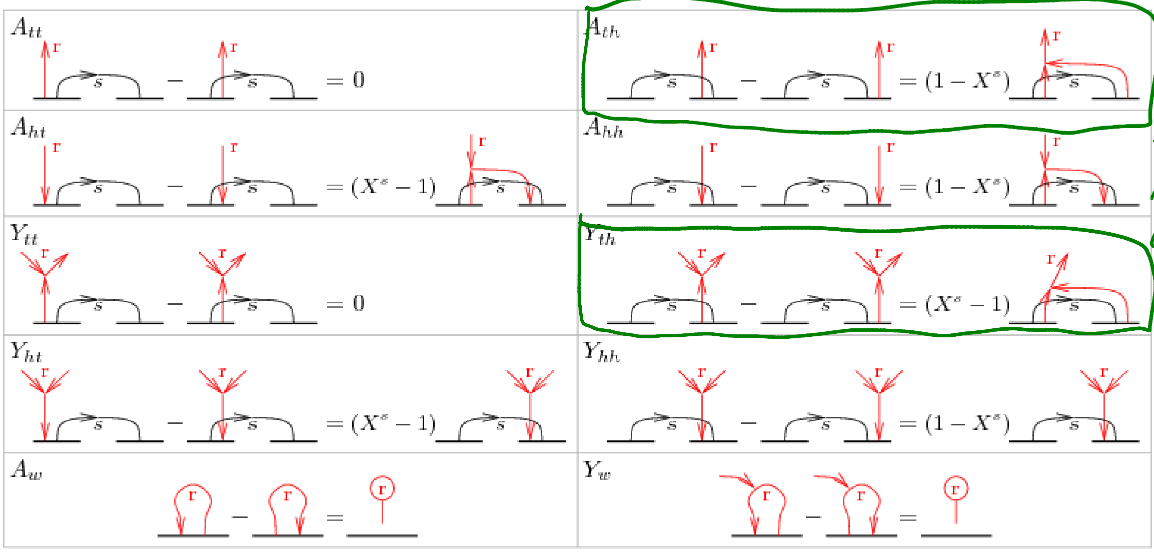
$$= [TX^{-s} - T(X^{-s}-I)(I+T)]S$$

$$= [(I - T(X^{-s}-I))T]S = (I-B)TS$$

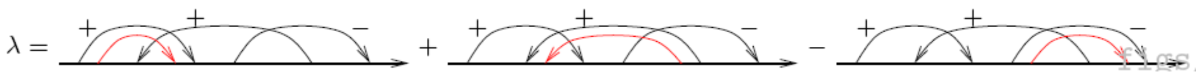
so $tr((I-B)^{-1}(Q_1-Q_2)) = tr(TS) = 0$.

And now the IAM part:

Some fixing of $X \leftrightarrow e^x$ is necessary.



MVRs



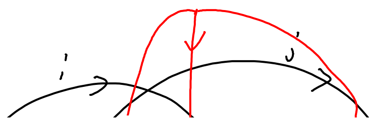
$$= \lambda_1 \quad + \quad \lambda_2 \quad - \quad \lambda_3$$

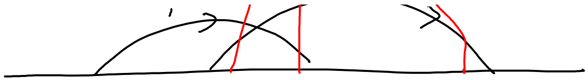
So −

Let a_i be the arrows of G; each carries a sign s_i and a direction d_i (+ for right, − for left) (consistent with Def 3.20). Let $\lambda_i \in IAM_k$ be "red excitation parallel to a_i"; so

$\lambda = \sum s_i \lambda_i$.

Let m_{ij} be:



Let η_{ij} be: 

formally, this is the Y diagram whose head is inside of the head of a_i, whose left tail is inside of the ....

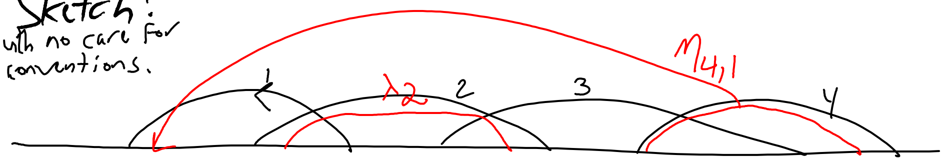
Try something more uniform .... "red tails to the right, red heads to the left"

The goal: $\lambda = \boxed{sl_L\delta_L + sl_R\delta_R} + \operatorname{tr}\left(B(I-B)^{-1}(I+T)S\right)\omega_1$

move to other side.

with $\qquad B := T(\exp(-xS) - I)$.

Sketch:
with no care for conventions.



$$\eta_i := \begin{pmatrix} \eta_{1i} \\ \eta_{2i} \\ \vdots \\ \eta_{4i} \end{pmatrix}$$

$$\lambda := \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{pmatrix} \qquad \delta := \begin{pmatrix} \delta_{d_1} \\ \vdots \\ \delta_{d_n} \end{pmatrix}$$

Lemma (First approx)
1. $\lambda_i - \delta_{d_i} = (B\eta_i)_i$
2. $\eta_i = B\eta_i + w_1(I+T)e_i$

Lemma (2nd iteration)
1. $\lambda - \delta = B\eta_i$
2. $\eta_i = B\eta_i$

Wrong approach!

λ should really be a matrix (perhaps a diagonal matrix?) (perhaps better, a matrix for $\lambda_i - \lambda_j$?) maybe $\lambda_i^i - \lambda_j^i$

New sketch:

Lemma: (more or less....) $\eta = B\eta + Tw_1$

$\Rightarrow \eta = (I-B)^{-1}Tw_1$

$\Rightarrow \operatorname{tr}\eta = (\operatorname{tr}(I-B)T)w_1$

Too clever; should not aim to merge Λ & Y.

Lemma
1. $\Lambda = B\eta + (\text{a traceless } w_1 \text{ factor})$
2. $\eta = B\eta + (I+T)S\cdot w_1$

$\Rightarrow \Lambda = (\text{traceless}) + B\eta = (\text{traceless}) + B(I-B)^{-1}(I+T)S\cdot w_1$

QED

~~First working version: With Λ & Y as in the paper (Feb 10) & with $S = \mathrm{diag}(s_i)$ & $D = \mathrm{diag}(d_i)$ (so S is not as in the Feb 10 paper),~~

~~**Lemma** 1. $\lambda - SL = \mathrm{Tr}(\Lambda \cdot S)$~~ Fix in nb: $S\Lambda \to \Lambda S$

~~2. $\Lambda = D \cdot B \cdot D \cdot Y - D \cdot T\, X^{-S \cdot D} \cdot w_1$~~

~~3. $Y = DBDY - DTX^{-SD} w_1$~~

$$\cancel{\Rightarrow \lambda - SL = \mathrm{Tr}(\Lambda S) = \mathrm{Tr}\, D \cdot B \cdot D\, Y S}$$

$$\cancel{= -\mathrm{Tr}\, DBD\,(I - DBD)^{-1} DTX^{-SD} \cdot S}$$

$$\cancel{= -\mathrm{Tr}\, B(I-B)^{-1} \underbrace{TX^{-SD} \cdot SD}_{\text{Traceless}}}$$

$$\cancel{= -\mathrm{Tr}\,(I-B)^{-1} TX^{-SD} \cdot SD}$$

~~Aside: $I + B(I-B)^{-1} = [(I-B)+B](I-B)^{-1} = (I-B)^{-1}$~~

Moral - Better change basis along D.

Final Version (optimism never killed anyone)

Lemma with Λ & Y as in the Feb 11 version of the paper, and with $S = \mathrm{diag}(s_i)$ & $D = \mathrm{diag}(d_i)$ (modify paper!), we have:

note: SD here is S in the paper

Lemma 1. $\lambda - SL = \mathrm{Tr}(D \cdot S \cdot \Lambda)$

2. $\Lambda = -BY - TX^{-SD} w_1$

3. $Y = BY + TX^{-SD} w_1$

$$\Rightarrow \lambda - SL = \mathrm{Tr}(DS\Lambda) = -\mathrm{Tr}(DSBY + DSTX^{-SD}) w_1$$

$$= -\mathrm{Tr}(DSB(I-B)^{-1} TX^{-SD} + DSTX^{-SD}) w_1$$

$$= -\mathrm{Tr}(DS(I-B)^{-1} TX^{-SD}) w_1$$