Report on the article
"An Unexpected Cyclic Symmetry of $I\mathfrak{u}_n$"
by DROR BAR-NATAN AND ROLAND VAN DER VEEN
submitted to *Algebras and Representation Theory*.

1. Summary

An explicit order n automorphism Ψ of the Lie algebra $I\mathfrak{u}_n$ is described. Here $\mathfrak{u}_n$ denotes the Lie algebra of upper triangular complex matrices of size n. And $I\mathfrak{u}_n$ is the Drinfeld double of $\mathfrak{u}_n$ when the cobraket is 0. Namely, $I\mathfrak{u}_n$ is the semidirect product $\mathfrak{u}_n \ltimes \mathfrak{u}_n^*$, where $\mathfrak{u}_n^*$ is considered as an abelian ideal with the coadjoint action.

The automorphism is described explicitly: its matrix in the canonical basis is given. By construction $\Psi^{\circ n}$ is the identity and Ψ is bijective. To prove that it is a Lie algebra homomorphism, the authors report on explicit computations.

Moreover, these results are extended to $\mathfrak{gl}_{n+}^{\epsilon}$: a solvable approximation of $\mathfrak{gl}_n$ that degenerates on $I\mathfrak{u}_n$.

The motivations of the authors come from knot theory. Indeed, Lie algebras are used in knot theory to produce invariants by categorification. Then automorphisms of the Lie algebras are expected to become symmetries of knot invariants.

2. Comments for the authors

a.) Abstract. What is an order n cyclic automorphism? Is it different from an order n automorphism?

b.) p4. when ϵ is invertible should be replaced when ϵ is specialized to non-zero complex number.

c.) p5 : Knutsen $->$ Knutson

3. Opinion

The paper describes an exciting object. Indeed, most of time the automorphisms of a mathematical object translate important properties of it. The fact that the constructed automorphism appears by explicit computations could be interpreted as a lack of understanding of the Lie algebra $I\mathfrak{u}_n$. It raises a very intersting question: how to describe the automorphism naturally?

As mentioned by A. Knutson, a partial answer to this question is given in a paper by Knutson and Zinn-Justin. This work was mentioned to the authors after the first version of the refereed paper appear on ArXiv.

The paper is short, well-written and motivated by knot theory.
For these reasons I am convinced it should be published in Algebras and Representation Theory.