

Pensieve header: Knot Signatures as in KnotTheory` and as in Kashaev, with verifications up to 13 crossings. Continues pensieve://Talks/CMS-2112/.

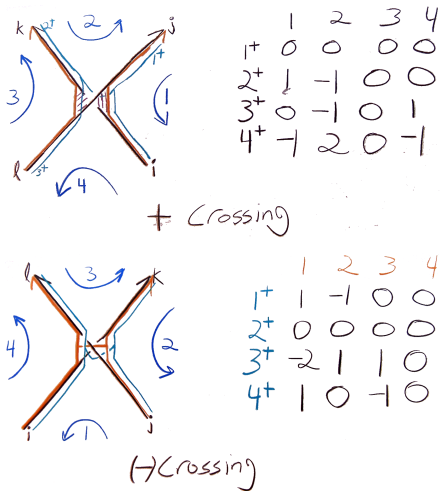
(Alt) In[]:=

Once [<< KnotTheory`]

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

(Alt) In[]:=

MatrixSignature[A_] := Total[Sign[Select[Eigenvalues[A], Abs[#] > 10⁻¹² &]]];
Writhe[K_] := Sum[If[PositiveQ[x], 1, -1], {x, List@@PD@K}];



tex

`\begin{multicols}{2}`

exec

`nb2tex$PDFWidth = 1.23 N[284/72];`

tex

`\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}`

pdf

The Bedlewo program

tex

`\[\resizebox{0.8\linewidth}{!}{\import{../Talks/CMS-2112/}{BedXings.pdf_t}} \]`

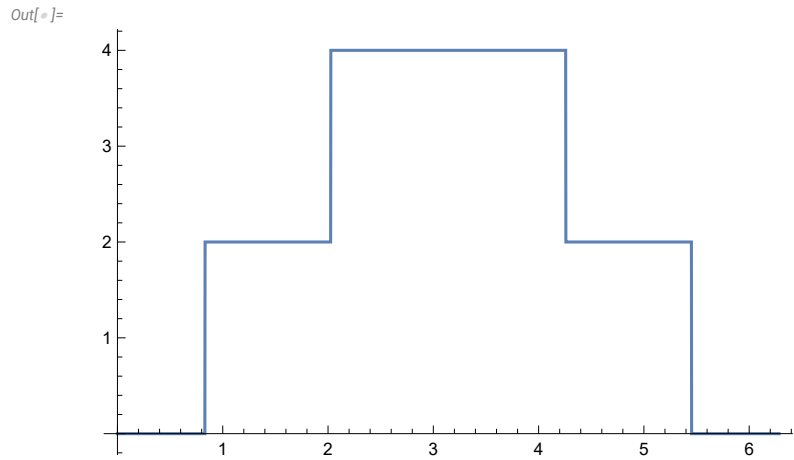
(Alt) In[]:=
pdf

```

Bed[K_, ω_] := Module[{t, r, XingsByArmpits, bends, faces, p, A, is},
  t = 1 - ω; r = t + t*;
  XingsByArmpits = List@@PD[K] /.
    x : X[i_, j_, k_, L_] => If[PositiveQ[x], X+[-i, j, k, -L], X_-[-j, k, L, -i]];
  bends = Times@@XingsByArmpits /. _[X][a_, b_, c_, d_] => pa,-d pb,-a pc,-b pd,-c;
  faces = bends //. px_,y_ py_,z_ => px,y,z;
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List@@x;
    A[[is, is]] += If[Head[x] === X+,
      
$$\begin{pmatrix} -r & -t & 2t & t^* \\ -t^* & 0 & t^* & 0 \\ 2t^* & t & -r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}, \begin{pmatrix} r & -t & -2t^* & t^* \\ -t^* & 0 & t^* & 0 \\ -2t & t & r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}],
    {x, XingsByArmpits}];
  MatrixSignature[A];$$

```

In[]:= Plot[Bed[K = Knot[8, 2], e^{i t}], {t, 0, 2 π}]
PositiveQ /@ (List@@PD[K])

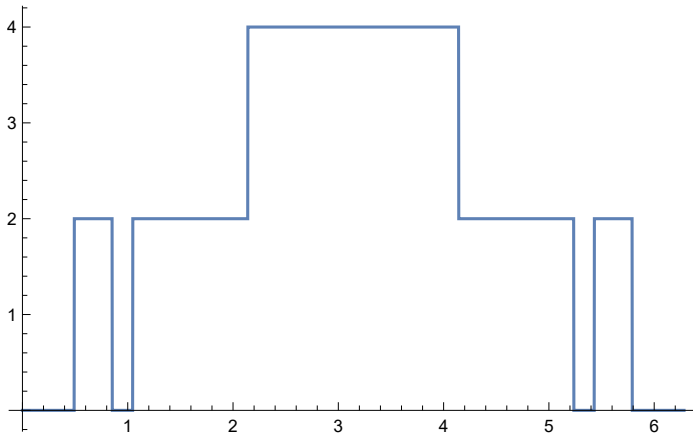


Out[]:= {False, False, True, True, False, False, False, False}

In[]:= Draw[Knot[8, 2]]

Out[]:= Draw[Knot[8, 2]]

```
In[ ]:= Plot [Bed [Knot@"K12a422", ei t], {t, 0, 2 π}]
Out[ ]=
```



tex

\columnbreak

pdf

The Kashaev Program

tex

\[\resizebox{0.8\linewidth}{!}{\import{../Talks/CMS-2112/}{KasXings.pdf_t}} \]

(Alt) In[]:=
pdf

```
Kas [K_, ω_] := Module [ {u, v, XingsByArmpits, bends, faces, p, A, is},
  u = Re [ω1/2]; v = Re [ω]; (* so v=2u2-1 *)
  XingsByArmpits = List @@ PD [K] /.
    x : X [i_, j_, k_, L_] => If [PositiveQ [x], X+ [-i, j, k, -L], X- [-j, k, L, -i]];
  bends = Times @@ XingsByArmpits /. _ [X] [a_, b_, c_, d_] => pa,-d pb,-a pc,-b pd,-c;
  faces = bends /. px-,y- py-,z- => px,y,z;
  A = Table [0, Length@faces, Length@faces];
  Do [is = Position [faces, #] [[1, 1]] & /@ List @@ x;
    A [[is, is]] += If [Head [x] === X+,
      
$$\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}, \begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}],
    {x, XingsByArmpits} ];
  (MatrixSignature [A] - Writhe [K]) / 2 ];$$

```

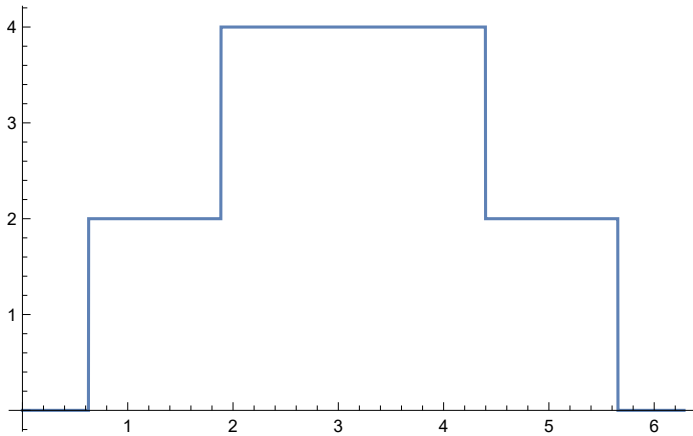
exec

nb2tex\$PDFWidth = 1.1 N[284/72];

tex

\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}

```
In[ ]:= Plot[Kas[Knot[5, 1], ei t], {t, 0, 2 π}]
Out[ ]=
```



tex

```
\end{multicols}
\vskip -3mm \hrule \vskip -7mm
\begin{multicols}{2}
```

pdf

Comparisons

pdf

```
In[ ]:= Sum[ω = ei RandomReal[{0, 2 π}]; Bed[K, ω] == Kas[K, ω], {10}, {K, AllKnots[{3, 10}]}]
Out[ ]=
pdf
2490 True
```

```
In[ ]:= Table[Bed[K, i], {K, AllKnots[{3, 5}]}]
Out[ ]=
{2, 0, 2, 2}
```

```
In[ ]:= Table[KnotSignature[K], {K, AllKnots[{3, 5}]}]
Out[ ]=
{-2, 0, -4, -2}
```

```
In[ ]:= Total@Table[Bed[K, -1] == -KnotSignature[K], {K, AllKnots[{3, 10}]}]
Out[ ]=
249 True
```

Higher Comparisons

(Alt) In[]:=

```
Sum[ $\omega = e^{i \text{RandomReal}[\{0, 2\pi\}]}$ ; Bed[K,  $\omega$ ] == Kas[K,  $\omega$ ], {10}, {K, AllKnots[{11, 11}]}]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

(Alt) Out[]:=

5520 True

(Alt) In[]:=

```
Sum[ $\omega = e^{i \text{RandomReal}[\{0, 2\pi\}]}$ ; Bed[K,  $\omega$ ] == Kas[K,  $\omega$ ], {10}, {K, AllKnots[{12, 12}]}]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

(Alt) Out[]:=

21760 True

(Alt) In[]:=

```
Monitor[Sum[ $\omega = e^{i \text{RandomReal}[\{0, 2\pi\}]}$ ;
  Bed[K,  $\omega$ ] == Kas[K,  $\omega$ ], {K, AllKnots[{13, 13}]}, {i, 10}], {K, i}]
```

(Alt) Out[]:=

99880 True

(Alt) In[]:=

```
Monitor[Sum[ $\omega = e^{i \text{RandomReal}[\{0, 2\pi\}]}$ ;
  If[Bed[K,  $\omega$ ] != Kas[K,  $\omega$ ], Echo[{K,  $\omega$ }], True], {K, AllKnots[{14, 14}]}, {i, 10}], {K, i}]
```

KnotTheory: Loading precomputed data in KnotTheory/14A.dts.

KnotTheory: Loading precomputed data in KnotTheory/14N.dts.

(Alt) Out[]:=

469720 True

(Alt) In[]:=

```
Monitor[Sum[ $\omega = e^{i \text{RandomReal}[\{0, 2\pi\}]}$ ;
  If[Bed[K,  $\omega$ ] != Kas[K,  $\omega$ ], Echo[{K,  $\omega$ }], True], {K, AllKnots[{15, 15}]}, {i, 10}], {K, i}]
```

KnotTheory: Loading precomputed data in KnotTheory/15A.dts.

» {Knot[15, Alternating, 55264], 0.502387 - 0.864643 i}

KnotTheory: Loading precomputed data in KnotTheory/15N.dts.

» {Knot[15, NonAlternating, 90489], 0.497389 - 0.867528 i}

(Alt) Out[]:=

{2532928 True + Knot[15, Alternating, 55264] + Knot[15, NonAlternating, 90489],
(0.999776 - 1.73217 i) + 2532928 True}

In[]:= **NumberOfKnots[16, NonAlternating]**

Out[]:=

1008906

(Alt) In[]:=

```
Monitor[Sum[ $\omega = e^{i \text{RandomReal}[\{0, 2\pi\}]}$ ];
  If[Bed[K,  $\omega$ ]  $\neq$  Kas[K,  $\omega$ ], Echo[{K,  $\omega$ }], True], {K, AllKnots[{16, 16}]}, {i, 10}], {K, i}]
```

KnotTheory: Loading precomputed data in KnotTheory/16A.dts.

» {Knot[16, Alternating, 144399], $0.500001 + 0.866025 i$ }

KnotTheory: Loading precomputed data in KnotTheory/16N.dts.

» {Knot[16, NonAlternating, 225282], $0.50059 - 0.865685 i$ }

» {Knot[16, NonAlternating, 761158], $0.499916 + 0.866074 i$ }

(Alt) Out[]:=

```
{13887047 True + Knot[16, Alternating, 144399] + Knot[16, NonAlternating, 225282] +
  Knot[16, NonAlternating, 761158], (1.50051 + 0.866414 i) + 13887047 True}
```

From <https://oeis.org/A002863>:

```
{1  $\rightarrow$  0, 2  $\rightarrow$  0, 3  $\rightarrow$  1, 4  $\rightarrow$  1, 5  $\rightarrow$  2, 6  $\rightarrow$  3, 7  $\rightarrow$  7, 8  $\rightarrow$  21, 9  $\rightarrow$  49,
  10  $\rightarrow$  165, 11  $\rightarrow$  552, 12  $\rightarrow$  2176, 13  $\rightarrow$  9988, 14  $\rightarrow$  46972, 15  $\rightarrow$  253293,
  16  $\rightarrow$  1388705, 17  $\rightarrow$  8053393, 18  $\rightarrow$  48266466, 19  $\rightarrow$  294130458}
```