

Pensieve header: MS, manually compute matrix signature.

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In[ ]:= A = IdentityMatrix[2]
Out[ ]:=
{{1, 0}, {0, 1}}

In[ ]:= A1 = A[[2 ;;, 2 ;;]]
Out[ ]:=
{{1}}

In[ ]:= A1[[2 ;;, 2 ;;]]
Out[ ]:=
{}

In[ ]:= Outer[Times, {a, b, c}, {1, 2, 3}]
Out[ ]:=
{{a, 2 a, 3 a}, {b, 2 b, 3 b}, {c, 2 c, 3 c}}

In[ ]:= MS[{}] = 0;
MS[A_?MatrixQ] /; A == A^T := Module[{k, a, A1, j, l},
  {k} = FirstPosition[A[[1]], x_ /; x != 0, {None}];
  Switch[k,
    None, MS[A[[2 ;;, 2 ;;]]],
    1, (
      a = A[[1, 1]];
      Sign[a] + MS[
        A[[2 ;;, 2 ;;]] - Outer[Times, A[[2 ;;, 1]], A[[1, 2 ;;]] / a
      ]),
    _, (
      A1 = A; A1[[k]] /= A[[k, 1]]; A1[[All, k]] /= A[[1, k]];
      a = A1[[k, k]]; A1[[k]] -= a A1[[1]] / 2; A1[[All, k]] -= a A1[[All, 1]] / 2;
      For[j = 2, j <= Length@A, ++j, If[j != k,
        A1[[j]] -= A1[[j, 1]] A1[[k]] + A1[[j, k]] A1[[1]];
        A1[[ ;, j]] -= A1[[1, j]] A1[[ ;, k]] + A1[[k, j]] A1[[ ;, 1]]
      ]];
      l = Complement[Range@Length@A, {1, k}];
      MS[A1[[1, 1]]]
    )
  ]
]

In[ ]:= A = Table[RandomInteger[{-1, 1}], {7, 7}];
A = A + A^T;
{MatrixSignature[A], MS[A]}

Out[ ]:=
{1, 1}

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$$\text{In[]:= MS} \left[\begin{pmatrix} 0 & 4 & 0 & 6 & 5 \\ 4 & 4 & -1 & 6 & 3 \\ 0 & -1 & -2 & -3 & -2 \\ 6 & 6 & -3 & 2 & 1 \\ 5 & 3 & -2 & 1 & 0 \end{pmatrix} \right]$$

$$\gg \begin{pmatrix} 0 & 4 & 0 & 6 & 5 \\ 4 & 4 & -1 & 6 & 3 \\ 0 & -1 & -2 & -3 & -2 \\ 6 & 6 & -3 & 2 & 1 \\ 5 & 3 & -2 & 1 & 0 \end{pmatrix}$$

$$\gg \left\{ 2, \begin{pmatrix} 0 & 4 & 0 & 6 & 5 \\ 1 & 1 & -\frac{1}{4} & \frac{3}{2} & \frac{3}{4} \\ 0 & -1 & -2 & -3 & -2 \\ 6 & 6 & -3 & 2 & 1 \\ 5 & 3 & -2 & 1 & 0 \end{pmatrix} \right\}$$

$$\gg \left\{ 2, \begin{pmatrix} 0 & 1 & 0 & 6 & 5 \\ 1 & \frac{1}{4} & -\frac{1}{4} & \frac{3}{2} & \frac{3}{4} \\ 0 & -\frac{1}{4} & -2 & -3 & -2 \\ 6 & \frac{3}{2} & -3 & 2 & 1 \\ 5 & \frac{3}{4} & -2 & 1 & 0 \end{pmatrix} \right\}$$

$$\gg \begin{pmatrix} 0 & 1 & 0 & 6 & 5 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & \frac{1}{8} \\ 0 & -\frac{1}{4} & -2 & -3 & -2 \\ 6 & \frac{3}{4} & -3 & 2 & 1 \\ 5 & \frac{1}{8} & -2 & 1 & 0 \end{pmatrix}$$

$$\gg \left\{ 2, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -\frac{3}{2} & -\frac{3}{4} \\ 0 & 0 & -\frac{3}{2} & -7 & -\frac{7}{2} \\ 0 & 0 & -\frac{3}{4} & -\frac{7}{2} & -\frac{5}{4} \end{pmatrix} \right\}$$

$$\gg \begin{pmatrix} -2 & -\frac{3}{2} & -\frac{3}{4} \\ -\frac{3}{2} & -7 & -\frac{7}{2} \\ -\frac{3}{4} & -\frac{7}{2} & -\frac{5}{4} \end{pmatrix}$$

$$\gg \begin{pmatrix} -\frac{47}{8} & -\frac{47}{16} \\ -\frac{47}{16} & -\frac{31}{32} \end{pmatrix}$$

$$\gg \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Out[]:=

-1