

Pensieve header: Knot Signatures via the Goeritz Matrix.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Signatures"];
<< Common.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
 Read more at <http://katlas.org/wiki/KnotTheory>.

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Knot Signatures Using the Goeritz Matrix

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Formulas follow Gordon-Litherland. For checkerboard colouring, the region to the right of an odd-numbered arc is declared to be black.

```
In[ ]:=  $\mu[K_] := \text{Plus}@@(\text{PD}[K] /. \mathbf{x}_X \mapsto \{\text{PositiveQ}@\mathbf{x}, \text{OddQ}@\mathbf{x}[[1]]\} /. \{$ 
```

$$\{\text{True}, \text{True}\} \rightarrow 0, \{\text{True}, \text{False}\} \rightarrow 1, \{\text{False}, \text{True}\} \rightarrow -1, \{\text{False}, \text{False}\} \rightarrow 0$$

```
});
```

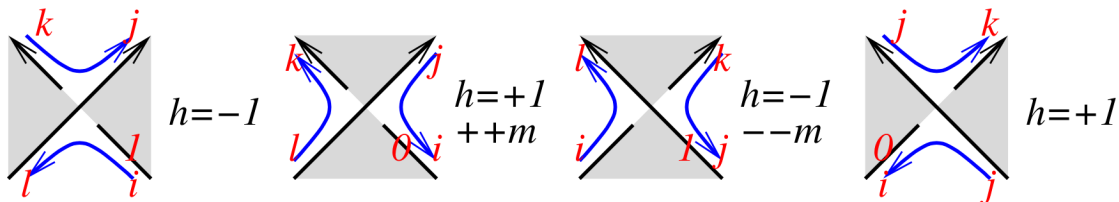
```
In[ ]:=  $\mu / @ \text{AllKnots}[\{3, 7\}]$ 
```

```
Out[ ]:=  $\{-3, 2, -5, -5, -4, -4, 3, -7, -7, 7, 7, -7, -5, -3\}$ 
```

Signature from Goeritz

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```
\includegraphics[width=\linewidth]{GoeritzConventions.pdf}
```



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```
In[ ]:= GoeritzSignature[K_] := Module[{m = 0, a, c = 0, ds, cs, is, A},
  ds = List@@PD[K] /.  $\mathbf{x} : \mathbf{X}[\mathbf{i}_-, \mathbf{j}_-, \mathbf{k}_-, \mathbf{l}_-] \mapsto \text{If}[\text{PositiveQ}@\mathbf{x},$ 
    If[OddQ@ $\mathbf{i}$ , { $\mathbf{a}_{-\mathbf{i}, \mathbf{l}}[++\mathbf{c}] \mathbf{a}_{\mathbf{k}, -\mathbf{j}}[++\mathbf{c}], -1$ },
      { $\mathbf{a}_{\mathbf{j}, \mathbf{i}}[++\mathbf{c}] \mathbf{a}_{-\mathbf{l}, -\mathbf{k}}[++\mathbf{c}], ++\mathbf{m}; 1$ }],
    If[OddQ@ $\mathbf{i}$ , { $\mathbf{a}_{-\mathbf{i}, -\mathbf{l}}[++\mathbf{c}] \mathbf{a}_{\mathbf{k}, \mathbf{j}}[++\mathbf{c}], --\mathbf{m}; -1$ },
      { $\mathbf{a}_{-\mathbf{j}, \mathbf{i}}[++\mathbf{c}] \mathbf{a}_{\mathbf{l}, -\mathbf{k}}[++\mathbf{c}], 1$ }]];
  cs = Times@@ds[All, 1] /.  $\mathbf{a}_{\mathbf{i}_-, \mathbf{j}_-}[\mathbf{x}_-] \mathbf{a}_{\mathbf{j}_-, \mathbf{k}_-}[\mathbf{y}_-] \mapsto \mathbf{a}_{\mathbf{i}, \mathbf{k}}[\mathbf{x}, \mathbf{y}] /. \mathbf{a}_-[\mathbf{x}_-] \mapsto \mathbf{a}[\mathbf{x}];$ 
  A = Table[0, Length@cs, Length@cs];
  Do[is = Position[cs, 2  $\mathbf{i} - \#$ ][[1, 1]] & /@ {1, 0};
    A[[is, is]] += ds[[ $\mathbf{i}$ , 2]] {{1, -1}, {-1, 1}},
    { $\mathbf{i}$ , Length[ds]}];
  MatrixSignature[A - m];
```

```
In[ ]:= GoeritzSignature@Knot[3, 1]
```

```
» { {a$7937-1,-5[1] a$79372,4[2], -1},
    {a$7937-3,-1[3] a$79374,6[4], -1}, {a$7937-5,-3[5] a$79376,2[6], -1} }
```

```
Out[ ]:= 2
```

```
In[ ]:= GoeritzSignature /@ AllKnots[{3, 8}]
```

```
Out[ ]:= {2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
          0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}
```

```
In[ ]:= -KnotSignature /@ AllKnots[{3, 8}]
```

```
Out[ ]:= {2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
          0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}
```

```
In[ ]:= Total[ (GoeritzSignature[#] == -KnotSignature[#]) & /@ AllKnots[{3, 10}]]
```

```
Out[ ]:= 249 True
```

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{\red\bf To do.} Tristram-Levine and Alexander in this language.