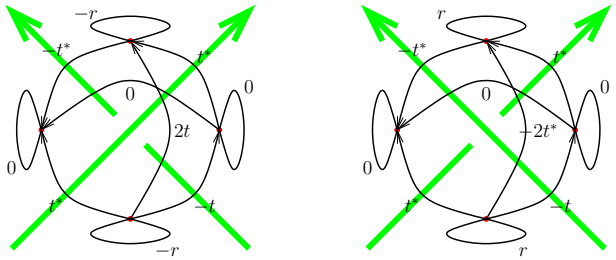
Commons

```
Once[<< KnotTheory`]
```

```
Loading KnotTheory` version
  of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

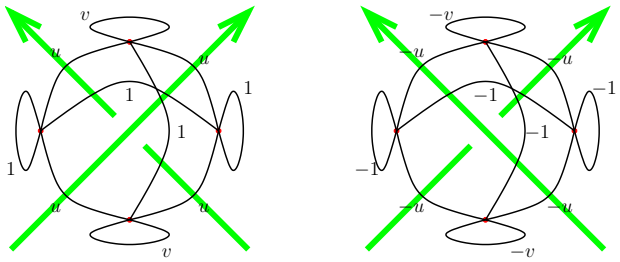
```
MatrixSignature[A_] :=
  Total[
   Sign[Select[Eigenvalues[A], Abs[#] > 10^-12 &]]];
Writhe[K_] := Sum[If[PositiveQ[x], 1, -1],
   {x, List @@ PD@K}];
```

The Bedlewo program




```
Bed[K_, ω_] :=
  Module[{t, r, XingsByArmpits, bends, faces, p, A, is},
   t = 1 - ω; r = t + t*;
   XingsByArmpits =
    List @@ PD[K] /. x : X[i_, j_, k_, l_] :>
      If[PositiveQ[x], X+[-i, j, k, -l], X-[-j, k, l, -i]];
   bends = Times @@ XingsByArmpits /.
     _[X][a_, b_, c_, d_] :> p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c};
   faces = bends //. p_{x__,y_} p_{y_,z__} :> p_{x,y,z};
   A = Table[0, Length@faces, Length@faces];
   Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
    A[[is, is]] += If[Head[x] === X+,
      {{-r, -t, 2t, t*}, {-t*, 0, t*, 0}, {2t*, t, -r, -t*}, {t, 0, -t, 0}},
      {{r, -t, -2t*, t*}, {-t*, 0, t*, 0}, {-2t, t, r, -t*}, {t, 0, -t, 0}}],
    {x, XingsByArmpits}];
   MatrixSignature[A]];
```

The Kashaev Program




```
Kas[K_, ω_] :=
  Module[{u, v, XingsByArmpits, bends, faces, p, A, is},
   u = Re[ω^1/2]; v = Re[ω]; (* so v=2u^2-1 *);
   XingsByArmpits =
    List @@ PD[K] /. x : X[i_, j_, k_, l_] :>
      If[PositiveQ[x], X+[-i, j, k, -l], X-[-j, k, l, -i]];
   bends = Times @@ XingsByArmpits /.
     _[X][a_, b_, c_, d_] :> p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c};
   faces = bends //. p_{x__,y_} p_{y_,z__} :> p_{x,y,z};
   A = Table[0, Length@faces, Length@faces];
   Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
    A[[is, is]] += If[Head[x] === X+,
      {{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}},
      -{{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}}],
    {x, XingsByArmpits}];
   (MatrixSignature[A] - Writhe[K]) / 2];
```

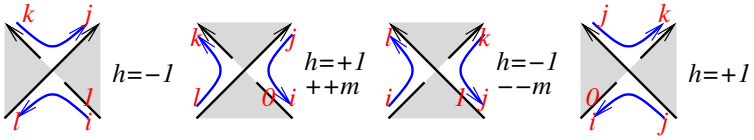
Comparisons

```
Sum[ω = e^(i RandomReal[{0,2π}]); Bed[K, ω] == Kas[K, ω],
 {10}, {K, AllKnots[{3, 10}]}]
```

```
2490 True
```

Knot Signatures Using the Goeritz Matrix

Formulas follow Gordon-Litherland. For checkerboard colouring, the region to the right of an odd-numbered arc is declared to be black.




```
GoeritzSignature[K_] :=
  Module[{m = 0, a, c = 0, ds, cs, is, A},
   ds = List @@ PD[K] /.
     x : X[i_, j_, k_, l_] :> If[PositiveQ@x,
       If[OddQ@i, {a_{-i,l}[++c] a_{k,-j}[++c], -1},
        {a_{j,i}[++c] a_{-l,-k}[++c], ++m; 1}],
       If[OddQ@i, {a_{-i,-l}[++c] a_{k,j}[++c], --m; -1},
        {a_{-j,i}[++c] a_{l,-k}[++c], 1}]];
   cs =
    Times @@ ds[[All, 1]] //.
      a_{i_,j_}[x__] a_{j_,k_}[y__] :> a_{i,k}[x, y] /.
     a__[x__] :> a[x];
   A = Table[0, Length@cs, Length@cs];
   Do[is = Position[cs, 2 i - #][[1, 1]] & /@ {1, 0};
    A[[is, is]] += ds[[i, 2]] {{1, -1}, {-1, 1}},
    {i, Length[ds]}];
```

To do. Tristram-Levine and Alexander in this language.