

Pensieve Header: Solving the two F equations with perturbative hair, where F is written as an exponential. This time with a corrected F21 and with correct grading on the hair. Not working yet.

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SetDirectory ["C:\\drorbn\\AcademicPensieve\\Projects\\ScatterAndGlow"];
<< ScatterAndGlow.m

$CutoffDegree = 5;

flcoeffs = DeclareSeries[f1[x[1], x[2]], $CutoffDegree];
f2coeffs = DeclareSeries[f2[x[1], x[2]], $CutoffDegree];
PH[f1]
PH[f1[0, 0] + (f1[1, 0] x[1] + f1[0, 1] x[2]) z +
  (1/2 f1[2, 0] x[1]^2 + f1[1, 1] x[1] x[2] + 1/2 f1[0, 2] x[2]^2) z^2 +
  (1/6 f1[3, 0] x[1]^3 + 1/2 f1[2, 1] x[1]^2 x[2] + 1/2 f1[1, 2] x[1] x[2]^2 + 1/6 f1[0, 3] x[2]^3) z^3 +
  (1/24 f1[4, 0] x[1]^4 + 1/6 f1[3, 1] x[1]^3 x[2] + 1/4 f1[2, 2] x[1]^2 x[2]^2 +
  1/6 f1[1, 3] x[1] x[2]^3 + 1/24 f1[0, 4] x[2]^4) z^4 + O[z]^5]

F = S[Exp[a1 Ar[2, 1] + a2 Ar[1, 2] + Y[1, 2, 1, PH[f1]] + Y[1, 2, 2, PH[f2]]]];
F21 = S[Exp[
  a1 Ar[1, 2] + a2 Ar[2, 1] + Y[2, 1, 2, PH[f1]] + Y[2, 1, 1, PH[f2]]
] /. {x[1] -> x[2], x[2] -> x[1]}
];

lhs1 = Ar[3, 0] // S[Exp[Ar[1, 3] + Ar[2, 3]]] // F

```

A very large output was generated. Here is a sample of it:

$$\text{Ar}[3, 0] + Y[1, 2, 0, \text{PH}\left[-\frac{x[3]}{2} + a1 x[3] - a2 x[3]\right] z + \ll 5 \gg + O[z]^7] + \\
Y[1, 3, 0, \text{AH}\left[\frac{e^{x[2]} (-1 + e^{x[1]})}{x[1]}\right]] + Y[2, 3, 0, \text{AH}\left[\frac{-1 + e^{x[2]}}{x[2]}\right]]$$

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rhs1 = Ar[3, 0] // F // S[sigma[1, 3], sigma[2, 3]]

Ar[3, 0] + Y[1, 2, 0, AH[-(1 + e^{x[1]}) (-1 + e^{x[2]}) x[3] / (x[1] x[2])]] +
Y[1, 3, 0, AH[e^{x[2]} (-1 + e^{x[1]}) / x[1]]] + Y[2, 3, 0, AH[-1 + e^{x[2]} / x[2]]]

lhs2 = Ar[1, 0] // F21 // S[sigma[1, 2]] // S[Exp[1/2 Ar[1, 1]]] // S[Exp[1/2 Ar[2, 2]]]

```

$$\begin{aligned}
& \text{Ar}[1, 0] + Y\left[1, 2, 0, \text{PH}\left[-a2 + \left(-a2 x[1] - \frac{1}{2} a1 a2 x[1] + f2[0, 0] x[1] - \frac{1}{2} a2^2 x[2]\right) z + \right. \right. \\
& \left. \left(-\frac{1}{2} a2 x[1]^2 - \frac{1}{2} a1 a2 x[1]^2 - \frac{1}{6} a1^2 a2 x[1]^2 + f2[0, 0] x[1]^2 + \frac{1}{2} a1 f2[0, 0] x[1]^2 + \right. \right. \\
& \left. \left. f2[0, 1] x[1]^2 - \frac{1}{2} a2^2 x[1] x[2] - \frac{1}{3} a1 a2^2 x[1] x[2] - \frac{1}{2} a2 f1[0, 0] x[1] x[2] + \right. \right. \\
& \left. \left. a2 f2[0, 0] x[1] x[2] + f2[1, 0] x[1] x[2] - \frac{1}{6} a2^3 x[2]^2\right) z^2 + \right. \\
& \left. \left(-\frac{1}{6} a2 x[1]^3 - \frac{1}{4} a1 a2 x[1]^3 - \frac{1}{6} a1^2 a2 x[1]^3 - \frac{1}{24} a1^3 a2 x[1]^3 + \frac{1}{2} f2[0, 0] x[1]^3 + \right. \right. \\
& \left. \left. \frac{1}{2} a1 f2[0, 0] x[1]^3 + \frac{1}{6} a1^2 f2[0, 0] x[1]^3 + f2[0, 1] x[1]^3 + \frac{1}{2} a1 f2[0, 1] x[1]^3 + \right. \right. \\
& \left. \left. \frac{1}{2} f2[0, 2] x[1]^3 - \frac{1}{4} a2^2 x[1]^2 x[2] - \frac{1}{3} a1 a2^2 x[1]^2 x[2] - \frac{1}{8} a1^2 a2^2 x[1]^2 x[2] - \right. \right. \\
& \left. \left. \frac{1}{2} a2 f1[0, 0] x[1]^2 x[2] - \frac{1}{3} a1 a2 f1[0, 0] x[1]^2 x[2] - \frac{1}{2} a2 f1[0, 1] x[1]^2 x[2] + \right. \right. \\
& \left. \left. a2 f2[0, 0] x[1]^2 x[2] + \frac{2}{3} a1 a2 f2[0, 0] x[1]^2 x[2] + \frac{1}{2} f1[0, 0] f2[0, 0] x[1]^2 x[2] - \right. \right. \\
& \left. \left. \frac{1}{2} f2[0, 0]^2 x[1]^2 x[2] + a2 f2[0, 1] x[1]^2 x[2] + f2[1, 0] x[1]^2 x[2] + \right. \right. \\
& \left. \left. \frac{1}{2} a1 f2[1, 0] x[1]^2 x[2] + f2[1, 1] x[1]^2 x[2] - \frac{1}{6} a2^3 x[1] x[2]^2 - \frac{1}{8} a1 a2^3 x[1] x[2]^2 - \right. \right. \\
& \left. \left. \frac{1}{3} a2^2 f1[0, 0] x[1] x[2]^2 - \frac{1}{2} a2 f1[1, 0] x[1] x[2]^2 + \frac{1}{2} a2^2 f2[0, 0] x[1] x[2]^2 + \right. \right. \\
& \left. \left. a2 f2[1, 0] x[1] x[2]^2 + \frac{1}{2} f2[2, 0] x[1] x[2]^2 - \frac{1}{24} a2^4 x[2]^3\right) z^3 + \right. \\
& \left. \left(-\frac{1}{24} a2 x[1]^4 - \frac{1}{12} a1 a2 x[1]^4 - \frac{1}{12} a1^2 a2 x[1]^4 - \frac{1}{24} a1^3 a2 x[1]^4 - \frac{1}{120} a1^4 a2 x[1]^4 + \right. \right. \\
& \left. \left. \frac{1}{6} f2[0, 0] x[1]^4 + \frac{1}{4} a1 f2[0, 0] x[1]^4 + \frac{1}{6} a1^2 f2[0, 0] x[1]^4 + \frac{1}{24} a1^3 f2[0, 0] x[1]^4 + \right. \right. \\
& \left. \left. \frac{1}{2} f2[0, 1] x[1]^4 + \frac{1}{2} a1 f2[0, 1] x[1]^4 + \frac{1}{6} a1^2 f2[0, 1] x[1]^4 + \frac{1}{2} f2[0, 2] x[1]^4 + \right. \right. \\
& \left. \left. \frac{1}{4} a1 f2[0, 2] x[1]^4 + \frac{1}{6} f2[0, 3] x[1]^4 - \frac{1}{12} a2^2 x[1]^3 x[2] - \frac{1}{6} a1 a2^2 x[1]^3 x[2] - \right. \right. \\
& \left. \left. \frac{1}{8} a1^2 a2^2 x[1]^3 x[2] - \frac{1}{30} a1^3 a2^2 x[1]^3 x[2] - \frac{1}{4} a2 f1[0, 0] x[1]^3 x[2] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} a_1 a_2 f_1[0, 0] x[1]^3 x[2] - \frac{1}{8} a_1^2 a_2 f_1[0, 0] x[1]^3 x[2] - \frac{1}{2} a_2 f_1[0, 1] x[1]^3 x[2] - \\
& \frac{1}{3} a_1 a_2 f_1[0, 1] x[1]^3 x[2] - \frac{1}{4} a_2 f_1[0, 2] x[1]^3 x[2] + \frac{1}{2} a_2 f_2[0, 0] x[1]^3 x[2] + \\
& \frac{2}{3} a_1 a_2 f_2[0, 0] x[1]^3 x[2] + \frac{1}{4} a_1^2 a_2 f_2[0, 0] x[1]^3 x[2] + \frac{1}{2} f_1[0, 0] f_2[0, 0] x[1]^3 x[2] + \\
& \frac{1}{3} a_1 f_1[0, 0] f_2[0, 0] x[1]^3 x[2] + \frac{1}{2} f_1[0, 1] f_2[0, 0] x[1]^3 x[2] - \\
& \frac{1}{2} f_2[0, 0]^2 x[1]^3 x[2] - \frac{1}{3} a_1 f_2[0, 0]^2 x[1]^3 x[2] + a_2 f_2[0, 1] x[1]^3 x[2] + \\
& \frac{2}{3} a_1 a_2 f_2[0, 1] x[1]^3 x[2] + \frac{1}{2} f_1[0, 0] f_2[0, 1] x[1]^3 x[2] - f_2[0, 0] f_2[0, 1] x[1]^3 x[2] + \\
& \frac{1}{2} a_2 f_2[0, 2] x[1]^3 x[2] + \frac{1}{2} f_2[1, 0] x[1]^3 x[2] + \frac{1}{2} a_1 f_2[1, 0] x[1]^3 x[2] + \\
& \frac{1}{6} a_1^2 f_2[1, 0] x[1]^3 x[2] + f_2[1, 1] x[1]^3 x[2] + \frac{1}{2} a_1 f_2[1, 1] x[1]^3 x[2] + \\
& \frac{1}{2} f_2[1, 2] x[1]^3 x[2] - \frac{1}{12} a_2^3 x[1]^2 x[2]^2 - \frac{1}{8} a_1 a_2^3 x[1]^2 x[2]^2 - \\
& \frac{1}{20} a_1^2 a_2^3 x[1]^2 x[2]^2 - \frac{1}{3} a_2^2 f_1[0, 0] x[1]^2 x[2]^2 - \frac{1}{4} a_1 a_2^2 f_1[0, 0] x[1]^2 x[2]^2 - \\
& \frac{1}{6} a_2 f_1[0, 0]^2 x[1]^2 x[2]^2 - \frac{1}{3} a_2^2 f_1[0, 1] x[1]^2 x[2]^2 - \frac{1}{2} a_2 f_1[1, 0] x[1]^2 x[2]^2 - \\
& \frac{1}{3} a_1 a_2 f_1[1, 0] x[1]^2 x[2]^2 - \frac{1}{2} a_2 f_1[1, 1] x[1]^2 x[2]^2 + \frac{1}{2} a_2^2 f_2[0, 0] x[1]^2 x[2]^2 + \\
& \frac{3}{8} a_1 a_2^2 f_2[0, 0] x[1]^2 x[2]^2 + \frac{2}{3} a_2 f_1[0, 0] f_2[0, 0] x[1]^2 x[2]^2 + \\
& \frac{1}{2} f_1[1, 0] f_2[0, 0] x[1]^2 x[2]^2 - \frac{1}{2} a_2 f_2[0, 0]^2 x[1]^2 x[2]^2 + \frac{1}{2} a_2^2 f_2[0, 1] x[1]^2 x[2]^2 + \\
& a_2 f_2[1, 0] x[1]^2 x[2]^2 + \frac{2}{3} a_1 a_2 f_2[1, 0] x[1]^2 x[2]^2 + \frac{1}{2} f_1[0, 0] f_2[1, 0] x[1]^2 x[2]^2 - \\
& f_2[0, 0] f_2[1, 0] x[1]^2 x[2]^2 + a_2 f_2[1, 1] x[1]^2 x[2]^2 + \frac{1}{2} f_2[2, 0] x[1]^2 x[2]^2 + \\
& \frac{1}{4} a_1 f_2[2, 0] x[1]^2 x[2]^2 + \frac{1}{2} f_2[2, 1] x[1]^2 x[2]^2 - \frac{1}{24} a_2^4 x[1] x[2]^3 - \\
& \frac{1}{30} a_1 a_2^4 x[1] x[2]^3 - \frac{1}{8} a_2^3 f_1[0, 0] x[1] x[2]^3 - \frac{1}{3} a_2^2 f_1[1, 0] x[1] x[2]^3 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} a_2 f_1[2, 0] x[1] x[2]^3 + \frac{1}{6} a_2^3 f_2[0, 0] x[1] x[2]^3 + \frac{1}{2} a_2^2 f_2[1, 0] x[1] x[2]^3 + \\
& \left. \frac{1}{2} a_2 f_2[2, 0] x[1] x[2]^3 + \frac{1}{6} f_2[3, 0] x[1] x[2]^3 - \frac{1}{120} a_2^5 x[2]^4 \right) z^4 + \\
& \left(-\frac{1}{120} a_2 x[1]^5 - \frac{1}{48} a_1 a_2 x[1]^5 - \frac{1}{36} a_1^2 a_2 x[1]^5 - \frac{1}{48} a_1^3 a_2 x[1]^5 - \frac{1}{120} a_1^4 a_2 x[1]^5 - \right. \\
& \frac{1}{720} a_1^5 a_2 x[1]^5 + \frac{1}{24} f_2[0, 0] x[1]^5 + \frac{1}{12} a_1 f_2[0, 0] x[1]^5 + \frac{1}{12} a_1^2 f_2[0, 0] x[1]^5 + \\
& \frac{1}{24} a_1^3 f_2[0, 0] x[1]^5 + \frac{1}{120} a_1^4 f_2[0, 0] x[1]^5 + \frac{1}{6} f_2[0, 1] x[1]^5 + \frac{1}{4} a_1 f_2[0, 1] x[1]^5 + \\
& \frac{1}{6} a_1^2 f_2[0, 1] x[1]^5 + \frac{1}{24} a_1^3 f_2[0, 1] x[1]^5 + \frac{1}{4} f_2[0, 2] x[1]^5 + \frac{1}{4} a_1 f_2[0, 2] x[1]^5 + \\
& \frac{1}{12} a_1^2 f_2[0, 2] x[1]^5 + \frac{1}{6} f_2[0, 3] x[1]^5 + \frac{1}{12} a_1 f_2[0, 3] x[1]^5 + \frac{1}{24} f_2[0, 4] x[1]^5 - \\
& \frac{1}{48} a_2^2 x[1]^4 x[2] - \frac{1}{18} a_1 a_2^2 x[1]^4 x[2] - \frac{1}{16} a_1^2 a_2^2 x[1]^4 x[2] - \frac{1}{30} a_1^3 a_2^2 x[1]^4 x[2] - \\
& \frac{1}{144} a_1^4 a_2^2 x[1]^4 x[2] - \frac{1}{12} a_2 f_1[0, 0] x[1]^4 x[2] - \frac{1}{6} a_1 a_2 f_1[0, 0] x[1]^4 x[2] - \\
& \frac{1}{8} a_1^2 a_2 f_1[0, 0] x[1]^4 x[2] - \frac{1}{30} a_1^3 a_2 f_1[0, 0] x[1]^4 x[2] - \frac{1}{4} a_2 f_1[0, 1] x[1]^4 x[2] - \\
& \frac{1}{3} a_1 a_2 f_1[0, 1] x[1]^4 x[2] - \frac{1}{8} a_1^2 a_2 f_1[0, 1] x[1]^4 x[2] - \frac{1}{4} a_2 f_1[0, 2] x[1]^4 x[2] - \\
& \frac{1}{6} a_1 a_2 f_1[0, 2] x[1]^4 x[2] - \frac{1}{12} a_2 f_1[0, 3] x[1]^4 x[2] + \frac{1}{6} a_2 f_2[0, 0] x[1]^4 x[2] + \\
& \frac{1}{3} a_1 a_2 f_2[0, 0] x[1]^4 x[2] + \frac{1}{4} a_1^2 a_2 f_2[0, 0] x[1]^4 x[2] + \frac{1}{15} a_1^3 a_2 f_2[0, 0] x[1]^4 x[2] + \\
& \frac{1}{4} f_1[0, 0] f_2[0, 0] x[1]^4 x[2] + \frac{1}{3} a_1 f_1[0, 0] f_2[0, 0] x[1]^4 x[2] + \\
& \frac{1}{8} a_1^2 f_1[0, 0] f_2[0, 0] x[1]^4 x[2] + \frac{1}{2} f_1[0, 1] f_2[0, 0] x[1]^4 x[2] + \\
& \frac{1}{3} a_1 f_1[0, 1] f_2[0, 0] x[1]^4 x[2] + \frac{1}{4} f_1[0, 2] f_2[0, 0] x[1]^4 x[2] - \frac{1}{4} f_2[0, 0]^2 x[1]^4 x[2] - \\
& \frac{1}{3} a_1 f_2[0, 0]^2 x[1]^4 x[2] - \frac{1}{8} a_1^2 f_2[0, 0]^2 x[1]^4 x[2] + \frac{1}{2} a_2 f_2[0, 1] x[1]^4 x[2] + \\
& \left. \frac{2}{3} a_1 a_2 f_2[0, 1] x[1]^4 x[2] + \frac{1}{4} a_1^2 a_2 f_2[0, 1] x[1]^4 x[2] + \frac{1}{2} f_1[0, 0] f_2[0, 1] x[1]^4 x[2] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} a_1 f_1[0, 0] f_2[0, 1] x[1]^4 x[2] + \frac{1}{2} f_1[0, 1] f_2[0, 1] x[1]^4 x[2] - \\
& f_2[0, 0] f_2[0, 1] x[1]^4 x[2] - \frac{2}{3} a_1 f_2[0, 0] f_2[0, 1] x[1]^4 x[2] - \\
& \frac{1}{2} f_2[0, 1]^2 x[1]^4 x[2] + \frac{1}{2} a_2 f_2[0, 2] x[1]^4 x[2] + \frac{1}{3} a_1 a_2 f_2[0, 2] x[1]^4 x[2] + \\
& \frac{1}{4} f_1[0, 0] f_2[0, 2] x[1]^4 x[2] - \frac{1}{2} f_2[0, 0] f_2[0, 2] x[1]^4 x[2] + \frac{1}{6} a_2 f_2[0, 3] x[1]^4 x[2] + \\
& \frac{1}{6} f_2[1, 0] x[1]^4 x[2] + \frac{1}{4} a_1 f_2[1, 0] x[1]^4 x[2] + \frac{1}{6} a_1^2 f_2[1, 0] x[1]^4 x[2] + \\
& \frac{1}{24} a_1^3 f_2[1, 0] x[1]^4 x[2] + \frac{1}{2} f_2[1, 1] x[1]^4 x[2] + \frac{1}{2} a_1 f_2[1, 1] x[1]^4 x[2] + \\
& \frac{1}{6} a_1^2 f_2[1, 1] x[1]^4 x[2] + \frac{1}{2} f_2[1, 2] x[1]^4 x[2] + \frac{1}{4} a_1 f_2[1, 2] x[1]^4 x[2] + \\
& \frac{1}{6} f_2[1, 3] x[1]^4 x[2] - \frac{1}{36} a_2^3 x[1]^3 x[2]^2 - \frac{1}{16} a_1 a_2^3 x[1]^3 x[2]^2 - \\
& \frac{1}{20} a_1^2 a_2^3 x[1]^3 x[2]^2 - \frac{1}{72} a_1^3 a_2^3 x[1]^3 x[2]^2 - \frac{1}{6} a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{4} a_1 a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{10} a_1^2 a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{6} a_2 f_1[0, 0]^2 x[1]^3 x[2]^2 - \\
& \frac{1}{8} a_1 a_2 f_1[0, 0]^2 x[1]^3 x[2]^2 - \frac{1}{3} a_2^2 f_1[0, 1] x[1]^3 x[2]^2 - \frac{1}{4} a_1 a_2^2 f_1[0, 1] x[1]^3 x[2]^2 - \\
& \frac{1}{3} a_2 f_1[0, 0] f_1[0, 1] x[1]^3 x[2]^2 - \frac{1}{6} a_2^2 f_1[0, 2] x[1]^3 x[2]^2 - \\
& \frac{1}{4} a_2 f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{3} a_1 a_2 f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{8} a_1^2 a_2 f_1[1, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{2} a_2 f_1[1, 1] x[1]^3 x[2]^2 - \frac{1}{3} a_1 a_2 f_1[1, 1] x[1]^3 x[2]^2 - \frac{1}{4} a_2 f_1[1, 2] x[1]^3 x[2]^2 + \\
& \frac{1}{4} a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{3}{8} a_1 a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{3}{20} a_1^2 a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{2}{3} a_2 f_1[0, 0] f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{2} a_1 a_2 f_1[0, 0] f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{6} f_1[0, 0]^2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{2}{3} a_2 f_1[0, 1] f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{2} f_1[1, 0] f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{3} a_1 f_1[1, 0] f_2[0, 0] x[1]^3 x[2]^2 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} f1[1, 1] f2[0, 0] x[1]^3 x[2]^2 - \frac{1}{2} a2 f2[0, 0]^2 x[1]^3 x[2]^2 - \frac{3}{8} a1 a2 f2[0, 0]^2 x[1]^3 x[2]^2 - \\
& \frac{1}{3} f1[0, 0] f2[0, 0]^2 x[1]^3 x[2]^2 + \frac{1}{6} f2[0, 0]^3 x[1]^3 x[2]^2 + \frac{1}{2} a2^2 f2[0, 1] x[1]^3 x[2]^2 + \\
& \frac{3}{8} a1 a2^2 f2[0, 1] x[1]^3 x[2]^2 + \frac{2}{3} a2 f1[0, 0] f2[0, 1] x[1]^3 x[2]^2 + \\
& \frac{1}{2} f1[1, 0] f2[0, 1] x[1]^3 x[2]^2 - a2 f2[0, 0] f2[0, 1] x[1]^3 x[2]^2 + \\
& \frac{1}{4} a2^2 f2[0, 2] x[1]^3 x[2]^2 + \frac{1}{2} a2 f2[1, 0] x[1]^3 x[2]^2 + \frac{2}{3} a1 a2 f2[1, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{4} a1^2 a2 f2[1, 0] x[1]^3 x[2]^2 + \frac{1}{2} f1[0, 0] f2[1, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{3} a1 f1[0, 0] f2[1, 0] x[1]^3 x[2]^2 + \frac{1}{2} f1[0, 1] f2[1, 0] x[1]^3 x[2]^2 - \\
& f2[0, 0] f2[1, 0] x[1]^3 x[2]^2 - \frac{2}{3} a1 f2[0, 0] f2[1, 0] x[1]^3 x[2]^2 - \\
& f2[0, 1] f2[1, 0] x[1]^3 x[2]^2 + a2 f2[1, 1] x[1]^3 x[2]^2 + \frac{2}{3} a1 a2 f2[1, 1] x[1]^3 x[2]^2 + \\
& \frac{1}{2} f1[0, 0] f2[1, 1] x[1]^3 x[2]^2 - f2[0, 0] f2[1, 1] x[1]^3 x[2]^2 + \\
& \frac{1}{2} a2 f2[1, 2] x[1]^3 x[2]^2 + \frac{1}{4} f2[2, 0] x[1]^3 x[2]^2 + \frac{1}{4} a1 f2[2, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{12} a1^2 f2[2, 0] x[1]^3 x[2]^2 + \frac{1}{2} f2[2, 1] x[1]^3 x[2]^2 + \frac{1}{4} a1 f2[2, 1] x[1]^3 x[2]^2 + \\
& \frac{1}{4} f2[2, 2] x[1]^3 x[2]^2 - \frac{1}{48} a2^4 x[1]^2 x[2]^3 - \frac{1}{30} a1 a2^4 x[1]^2 x[2]^3 - \\
& \frac{1}{72} a1^2 a2^4 x[1]^2 x[2]^3 - \frac{1}{8} a2^3 f1[0, 0] x[1]^2 x[2]^3 - \frac{1}{10} a1 a2^3 f1[0, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{8} a2^2 f1[0, 0]^2 x[1]^2 x[2]^3 - \frac{1}{8} a2^3 f1[0, 1] x[1]^2 x[2]^3 - \frac{1}{3} a2^2 f1[1, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{4} a1 a2^2 f1[1, 0] x[1]^2 x[2]^3 - \frac{1}{3} a2 f1[0, 0] f1[1, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{3} a2^2 f1[1, 1] x[1]^2 x[2]^3 - \frac{1}{4} a2 f1[2, 0] x[1]^2 x[2]^3 - \frac{1}{6} a1 a2 f1[2, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{4} a2 f1[2, 1] x[1]^2 x[2]^3 + \frac{1}{6} a2^3 f2[0, 0] x[1]^2 x[2]^3 + \frac{2}{15} a1 a2^3 f2[0, 0] x[1]^2 x[2]^3 +
\end{aligned}$$

$$\begin{aligned}
& \frac{3}{8} a^2 f1[0, 0] f2[0, 0] x[1]^2 x[2]^3 + \frac{2}{3} a^2 f1[1, 0] f2[0, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{4} f1[2, 0] f2[0, 0] x[1]^2 x[2]^3 - \frac{1}{4} a^2 f2[0, 0]^2 x[1]^2 x[2]^3 + \\
& \frac{1}{6} a^2 f2[0, 1] x[1]^2 x[2]^3 + \frac{1}{2} a^2 f2[1, 0] x[1]^2 x[2]^3 + \frac{3}{8} a^1 a^2 f2[1, 0] x[1]^2 x[2]^3 + \\
& \frac{2}{3} a^2 f1[0, 0] f2[1, 0] x[1]^2 x[2]^3 + \frac{1}{2} f1[1, 0] f2[1, 0] x[1]^2 x[2]^3 - \\
& a^2 f2[0, 0] f2[1, 0] x[1]^2 x[2]^3 - \frac{1}{2} f2[1, 0]^2 x[1]^2 x[2]^3 + \frac{1}{2} a^2 f2[1, 1] x[1]^2 x[2]^3 + \\
& \frac{1}{2} a^2 f2[2, 0] x[1]^2 x[2]^3 + \frac{1}{3} a^1 a^2 f2[2, 0] x[1]^2 x[2]^3 + \frac{1}{4} f1[0, 0] f2[2, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{2} f2[0, 0] f2[2, 0] x[1]^2 x[2]^3 + \frac{1}{2} a^2 f2[2, 1] x[1]^2 x[2]^3 + \\
& \frac{1}{6} f2[3, 0] x[1]^2 x[2]^3 + \frac{1}{12} a^1 f2[3, 0] x[1]^2 x[2]^3 + \frac{1}{6} f2[3, 1] x[1]^2 x[2]^3 - \\
& \frac{1}{120} a^2^5 x[1] x[2]^4 - \frac{1}{144} a^1 a^2^5 x[1] x[2]^4 - \frac{1}{30} a^2^4 f1[0, 0] x[1] x[2]^4 - \\
& \frac{1}{8} a^2^3 f1[1, 0] x[1] x[2]^4 - \frac{1}{6} a^2^2 f1[2, 0] x[1] x[2]^4 - \frac{1}{12} a^2 f1[3, 0] x[1] x[2]^4 + \\
& \frac{1}{24} a^2^4 f2[0, 0] x[1] x[2]^4 + \frac{1}{6} a^2^3 f2[1, 0] x[1] x[2]^4 + \frac{1}{4} a^2^2 f2[2, 0] x[1] x[2]^4 + \\
& \frac{1}{6} a^2 f2[3, 0] x[1] x[2]^4 + \frac{1}{24} f2[4, 0] x[1] x[2]^4 - \frac{1}{720} a^2^6 x[2]^5 \Big) z^5 + O[z]^6 \Big]
\end{aligned}$$

`rhs2 = Ar[1, 0] // S[Exp[Expand[1/2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] // F`

A very large output was generated. Here is a sample of it:

$$\begin{aligned}
& \text{Ar}[1, 0] + Y\left[1, 2, 0, \text{PH}\left[\left(-\frac{1}{2} - a^1\right) + \right. \right. \\
& \left. \left. \left(-\frac{1}{2} a^2 x[1] - \frac{1}{2} a^1 a^2 x[1] - f1[0, 0] x[1] + \frac{1}{8} (-x[1] - x[2]) - \frac{1}{2} a^1 x[2] - \frac{1}{2} a^1^2 x[2]\right) z + \right. \right. \\
& \left. \left. \left(\ll 39 \gg + \frac{f2[0, 0] x[1] x[2]^2}{2 (x[1] + x[2])} - \frac{a^1 x[2]^3}{8 (x[1] + x[2])} - \frac{a^1^2 x[2]^3}{4 (x[1] + x[2])} - \frac{1}{48} (x[1] + x[2])^2\right) z^2 + \right. \right. \\
& \left. \left. (\ll 1 \gg) z^3 + (\ll 1 \gg) z^4 + (\ll 1 \gg) z^5 + O[z]^6 \right] \right]
\end{aligned}$$

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```
eqs = {
  Coefficient[lhs1, Y[1, 2, 0]] == Coefficient[rhs1, Y[1, 2, 0]],
  Coefficient[lhs2, Y[1, 2, 0]] == Coefficient[rhs2, Y[1, 2, 0]]
}
```

A very large output was generated. Here is a sample of it:

$$\left\{ \text{PH} \left[\left(-\frac{x[3]}{2} + a1 x[3] - a2 x[3] \right) z + \left(\frac{1}{2} a1 x[1] x[3] + \frac{1}{2} a1 a2 x[1] x[3] - \frac{1}{2} a2^2 x[1] x[3] + f1[0, 0] x[1] x[3] + a1 x[2] x[3] + \frac{1}{2} a1^2 x[2] x[3] - \frac{1}{2} a2 x[2] x[3] - \frac{1}{2} a1 a2 x[2] x[3] + f2[0, 0] x[2] x[3] - \frac{a2 x[1]^2 x[3]}{2 (x[1]+x[2])} - \frac{a1 x[1] x[2] x[3]}{2 (x[1]+x[2])} - \frac{a2 x[1] x[2] x[3]}{2 (x[1]+x[2])} - \frac{a1 x[2]^2 x[3]}{2 (x[1]+x[2])} - \frac{1}{6} (x[1] + 2 x[2]) x[3] \right) z^2 + \dots \right] \right\}$$

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```
sol = HSolve[
  eqs,
  Join[{a1, a2}, f1coeffs, f2coeffs]
]
```

Solve::svars: Equations may not give solutions for all "solve" variables. >>

A very large output was generated. Here is a sample of it:

$$\left\{ \left\{ f1[4, 0] \rightarrow \frac{-2+7 a2^2+7 a2^4}{2520}, f1[3, 1] \rightarrow \frac{-83+164 a2-184 a2^2+176 a2^3+144 a2^4+11520 f1[0,2]-11520 a2 f1[0,2]+960 f2[1,0]+5760 a2 f2[1,0]-34560 f2[1,2]-23040 f2[1,3]}{23040}, \dots \right\} \right\}$$

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$$\left\{ \begin{aligned}
f1[4, 0] &\rightarrow \frac{-5 + 24 a^2 - 24 a^2 + 32 a^3 - 5760 f2[0, 4]}{5760}, \\
f1[3, 1] &\rightarrow \frac{1}{23040} (-83 + 164 a^2 - 184 a^2 + 176 a^3 + 144 a^4 + 11520 f1[0, 2] - \\
&\quad 11520 a^2 f1[0, 2] + 960 f2[1, 0] + 5760 a^2 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[1, 3]), \\
f1[2, 2] &\rightarrow \frac{1}{8640} (-21 + 62 a^2 - 106 a^2 + 84 a^3 + 36 a^4 + 3600 f1[0, 2] - \\
&\quad 5760 a^2 f1[0, 2] + 1440 a^2 f2[1, 0] - 8640 f2[2, 1] - 8640 f2[2, 2]), \\
f1[1, 3] &\rightarrow \frac{1}{23040} (-83 + 164 a^2 - 184 a^2 + 176 a^3 + 144 a^4 + 11520 f1[0, 2] - \\
&\quad 11520 a^2 f1[0, 2] + 960 f2[1, 0] + 5760 a^2 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[3, 1]), \\
f1[0, 4] &\rightarrow \frac{-5 + 24 a^2 - 24 a^2 + 32 a^3 - 5760 f2[4, 0]}{5760}, \\
f1[1, 2] &\rightarrow \frac{1}{34560} (-79 + 120 a^2 - 288 a^2 + 192 a^3 + 288 a^4 + \\
&\quad 17280 f1[0, 2] - 2880 f2[1, 0] + 11520 a^2 f2[1, 0] - 34560 f2[2, 1]), \\
f1[0, 3] &\rightarrow \frac{-35 + 28 a^2 + 12 a^2 - 24 a^3 + 96 a^4 + 4320 f1[0, 2] + 2880 a^2 f2[1, 0] - 17280 f2[1, 2]}{5760}, \\
f1[3, 0] &\rightarrow \frac{1}{240} a^2 (-1 + a^3), \\
f1[2, 1] &\rightarrow \frac{1}{34560} (-79 + 120 a^2 - 288 a^2 + 192 a^3 + 288 a^4 + \\
&\quad 17280 f1[0, 2] - 2880 f2[1, 0] + 11520 a^2 f2[1, 0] - 34560 f2[1, 2]), \\
f2[3, 0] &\rightarrow \frac{49 - 80 a^2 + 48 a^2 - 48 a^3 - 96 a^4 - 8640 f1[0, 2] - 5760 a^2 f2[1, 0] + 34560 f2[1, 2]}{11520}, \\
f2[0, 3] &\rightarrow \frac{-7 + 8 a^2 + 24 a^2 - 32 a^3 + 16 a^4}{3840}, \\
f1[1, 1] &\rightarrow \frac{3 + 10 a^2 - 25 a^2 + 720 f1[0, 2] - 360 f2[1, 0]}{1440}, \\
f2[1, 1] &\rightarrow \frac{9 - 20 a^2 - 10 a^2 - 1440 f1[0, 2] - 720 f2[1, 0]}{2880}, \\
f2[2, 0] &\rightarrow \frac{1}{288} (1 - 4 a^2 - 288 f1[0, 2]), \quad f1[2, 0] \rightarrow \frac{1}{360} (1 - 5 a^2), \\
f2[0, 2] &\rightarrow \frac{1 - 20 a^2 + 20 a^2}{1440}, \quad f1[0, 1] \rightarrow \frac{1}{96} (1 + 4 a^2 - 8 a^2 - 96 f2[1, 0]),
\end{aligned} \right.$$

$$\begin{aligned}
f1[1, 0] &\rightarrow \frac{1}{24} (a2 - a2^2), \quad f2[0, 1] \rightarrow \frac{1}{96} (1 - 4 a2^2), \\
f1[0, 0] &\rightarrow \frac{1}{12} (-1 + 3 a2), \quad f2[0, 0] \rightarrow \frac{1}{24} (-1 + 6 a2), \\
a1 &\rightarrow \frac{1}{2} (-1 + 2 a2) \}, \{ f1[4, 0] \rightarrow \frac{-1 - 1152 f2[0, 4]}{1152}, \\
f1[3, 1] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[1, 3]}{23040}, \\
f1[2, 2] &\rightarrow \frac{-7 + 1200 f1[0, 2] - 2880 f2[2, 1] - 2880 f2[2, 2]}{2880}, \\
f1[1, 3] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[3, 1]}{23040}, \\
f1[0, 4] &\rightarrow \frac{-1 - 1152 f2[4, 0]}{1152}, \quad f1[1, 2] \rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[2, 1]}{34560}, \\
f1[0, 3] &\rightarrow \frac{-7 + 864 f1[0, 2] - 3456 f2[1, 2]}{1152}, \quad f1[3, 0] \rightarrow 0, \\
f1[2, 1] &\rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[1, 2]}{34560}, \\
f2[3, 0] &\rightarrow \frac{49 - 8640 f1[0, 2] + 34560 f2[1, 2]}{11520}, \quad f2[0, 3] \rightarrow -\frac{7}{3840}, \\
f1[1, 1] &\rightarrow \frac{1}{480} (1 + 240 f1[0, 2] - 120 f2[1, 0]), \\
f2[1, 1] &\rightarrow \frac{1}{320} (1 - 160 f1[0, 2] - 80 f2[1, 0]), \quad f2[2, 0] \rightarrow \frac{1}{288} (1 - 288 f1[0, 2]), \\
f1[2, 0] &\rightarrow \frac{1}{360}, \quad f2[0, 2] \rightarrow \frac{1}{1440}, \quad f1[0, 1] \rightarrow \frac{1}{96} (1 - 96 f2[1, 0]), \quad f1[1, 0] \rightarrow 0, \\
f2[0, 1] &\rightarrow \frac{1}{96}, \quad f1[0, 0] \rightarrow -\frac{1}{12}, \quad f2[0, 0] \rightarrow -\frac{1}{24}, \quad a1 \rightarrow -\frac{1}{2}, \quad a2 \rightarrow 0 \} \}
\end{aligned}$$

Union[sol][[1]] /. a2 → 0

$$\left\{ \begin{aligned}
f1[4, 0] &\rightarrow \frac{-5 - 5760 f2[0, 4]}{5760}, \\
f1[3, 1] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[1, 3]}{23040}, \\
f1[2, 2] &\rightarrow \frac{-21 + 3600 f1[0, 2] - 8640 f2[2, 1] - 8640 f2[2, 2]}{8640}, \\
f1[1, 3] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[3, 1]}{23040}, \\
f1[0, 4] &\rightarrow \frac{-5 - 5760 f2[4, 0]}{5760}, f1[1, 2] \rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[2, 1]}{34560}, \\
f1[0, 3] &\rightarrow \frac{-35 + 4320 f1[0, 2] - 17280 f2[1, 2]}{5760}, f1[3, 0] \rightarrow 0, \\
f1[2, 1] &\rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[1, 2]}{34560}, \\
f2[3, 0] &\rightarrow \frac{49 - 8640 f1[0, 2] + 34560 f2[1, 2]}{11520}, \\
f2[0, 3] &\rightarrow -\frac{7}{3840}, f1[1, 1] \rightarrow \frac{3 + 720 f1[0, 2] - 360 f2[1, 0]}{1440}, \\
f2[1, 1] &\rightarrow \frac{9 - 1440 f1[0, 2] - 720 f2[1, 0]}{2880}, f2[2, 0] \rightarrow \frac{1}{288} (1 - 288 f1[0, 2]), \\
f1[2, 0] &\rightarrow \frac{1}{360}, f2[0, 2] \rightarrow \frac{1}{1440}, f1[0, 1] \rightarrow \frac{1}{96} (1 - 96 f2[1, 0]), \\
f1[1, 0] &\rightarrow 0, f2[0, 1] \rightarrow \frac{1}{96}, f1[0, 0] \rightarrow -\frac{1}{12}, f2[0, 0] \rightarrow -\frac{1}{24}, a1 \rightarrow -\frac{1}{2} \}
\end{aligned} \right.$$